The dynamic cost of ex post incentive compatibility in repeated games of private information

David A. Miller

Motivation

- Cooperation with private information requires communication
- But folk theorems (FLM, Miller) are not robust to:
  - Unstructured communication
  - No common prior
  - Spying/Higher order beliefs (Bergemann & Morris)
- Look for equilibria that are robust to these complications
  - Each stage must satisfy *ex post incentive compatibility* (EPIC)
  - Is efficiency attainable? —No.
  - Then what is optimal?

Contributions

- Formalize a *mechanism design approach* to repeated games
  - FLM, ABS
- Characterize *ex post perfect public equilibrium* (EPPPE)
  - Efficient average utilities not attainable
  - Computation of an optimum (linear programming)
- Characterize optimal EPPPEs in two-player allocation games:
  - Optimality $\Longrightarrow$ inefficient allocation
  - Private valuations: pooling, often stationary
  - Interdependent valuations $\Longrightarrow$ non-stationary
- Provide a new explanation for price wars in collusive equilibrium

1 Example: Multicolumns and equations

Game theory approach:

- Perfect public equilibrium (PPE)
- Strategies (best responses)
- Arbitrary messages
- Actions (observable)
- Voluntary transfers

Mechanism design approach:

- Recursive mechanism
- IC/IR constraints
- Direct revelation
- Mandatory actions
- Mandatory transfers

Out-of-context equation:

$$p_i^*(c) = \begin{cases} 1 \text{ if } c_i \leqslant c_{-i} \\ 2 \text{ if } c_i > c_{-i} \end{cases} \quad \text{and} \quad x_i(p^*(c)) = \begin{cases} 1 \text{ if } c_i < c_{-i} \\ 0 \text{ if } c_i > c_{-i} \\ \frac{1}{2} \text{ if } c_1 = c_2 \end{cases}$$

1.1 Example: Theorem

Theorem 1 ("Anti-folk" theorem). *For $\delta < 1$ sufficiently high, the surplus gap of an optimal EPPPE mechanism does not vary with δ.*

Proof outline. Given an outcome function x, optimal construction of continuation rewards, w, is fixed in present value terms; i.e., $\frac{\delta}{1-\delta}w$:

1. EPIC constrains $u_i(\theta, \theta; \langle x, t, w \rangle) = \pi_i(\theta, x(\theta)) + t_i(\theta) + \frac{\delta}{1-\delta} w_i(\theta)$;
2. The surplus gap is $\frac{\delta}{1-\delta}\big(\max_\theta [W(\theta)] - \mathbb{E}[W(\theta)]\big)$;
3. Individual rationality does not bind for sufficiently high δ. ∎