2030 (Lecture 2)

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1 Do people need deadlines?

Ariely and Wertenbroch (2002)

Proofreading tasks: "Sexual identity is intrinsically impossible," says Foucault; however, according to de Selby[1], it is not so much sexual identity that is intrinsically impossible, but rather the dialectic, and some would say the satsis, of sexual identity. Thus, D'Erlette[2] holds that we have to choose between premodern dialectic theory and subcultural feminism imputing the role of the observor as poet."

- Evenly spaced deadlines ($20 earnings)
- Self-imposed deadlines ($13 earnings)
  - subjects in this condition could self-impose costly deadlines ($1 penalty for each day of delay)
- End deadline ($5 earnings)

2 Self-control problems:

We say we do "too much" of the following activities:

- Watch TV
- Procrastinate
- Drink alcohol
- Fail to exercise
- Smoke cigarettes
- Eat unhealthfully
- Spend on credit cards

But are these self-reports to be trusted?

Immediacy seems to be implicated in many of our vices.

- Plans made at a distance appear to be less *impatient* than decisions made in the present.
- Our food experiment (pooled with undergraduate subjects):
  - Drink calories chosen for *today*: 419
  - Drink calories chosen for next *lecture*: 372
  - (Not significant: sample sizes 39 and 37)
- Why do I say that calories are a proxy for *impatience*?

Toy model:

Let c represent consumption (calories).

$$\begin{aligned} U(c_{t-1}, c_t, c_{t+1}) &= \left[\ln(c_t) - \frac{1}{2}c_{t-1}^2\right] \\ &\quad +\beta_1 \left[\ln(c_{t+1}) - \frac{1}{2}c_t^2\right] \\ &\quad +\beta_1\beta_2 \left[\ln(c_{t+2}) - \frac{1}{2}c_{t+1}^2\right] \end{aligned}$$

Optimal level of consumption:

$$\begin{aligned} \frac{1}{c_t} &= \beta_1 c_t \\ \frac{1}{c_{t+1}} &= \beta_2 c_{t+1} \end{aligned}$$

Rearranging yields,

$$\begin{aligned} \beta_1 &= \left(\frac{1}{c_t}\right)^2 \\ \beta_2 &= \left(\frac{1}{c_{t+1}}\right)^2 \end{aligned}$$

Consuming high quantities of calories, implies a low discount factor (or a high discount rate).

Here's another example:

Read, Loewenstein, and Kalyanaraman (1999)

Subjects given the opportunity to choose a movie video from a set of 24 titles: e.g., Four Weddings and a Funeral, Schindler's List.

- When choosing for today: 56% choose low-brow
- When choosing for next Monday: 37% choose low-brow
- When chooseing for the second Monday: 29% choose low-brow

Toy model revisited:

Let c represent consumption of culture.

$$
\begin{aligned}
U(c_{t-1}, c_t, c_{t+1}) \;=\; & \left[\ln(c_{t-1}) - \frac{1}{2}c_t^2\right] \\
& +\beta_1 \left[\ln(c_t) - \frac{1}{2}c_{t+1}^2\right] \\
& +\beta_1\beta_2 \left[\ln(c_{t+1}) - \frac{1}{2}c_{t+2}^2\right] + \quad \ldots
\end{aligned}
$$

Optimal level of consumption:

$$
\begin{aligned}
c_t &= \beta_1 \frac{1}{c_t} \\
c_{t+1} &= \beta_2 \frac{1}{c_{t+1}}
\end{aligned}
$$

Rearranging yields,

$$
\begin{aligned}
\beta_1 &= c_t^2 \\
\beta_2 &= c_{t+1}^2
\end{aligned}
$$

So consuming high quantities of culture, implies a high discount factor (or a low discount rate).

An example that cuts close to home:

Della Vigna and Malmendier (2004)

- Average cost of gym membership: $75 per month.
- Average number of visits per month: 4.
- Average cost per visit: $19.
- Cost of "pay-per-visit:" $10.

Other evidence:

- Oster and Scott-Morton (2004)
  - *People* sold on the newstand at a high price relative to subscription
  - *Foreign Affairs* sold on the newstand at a low price relative to subscription
  - But *People* is sold disproportionately on the newstand and *Foreign Affairs* is sold disproportionately by subscription.
- Wertenbroch (1998): people buy temptation goods in small packages, foregoing volume discounts
- Trope and Fischbach (2000): people are willing to set voluntary penalties on themselves for medical non-compliance

3 Intertemporal choice introduction

Check the item below that you would most prefer.

1 15 minute massage today.

2 20 minute massage tomorrow.

Check the item below that you would most prefer.

$1'$ 15 minute massage in 100 days.

$2'$ 20 minute massage in 101 days.

4 Discount functions and rates

- Discount function: $D(\tau)$.
- u utils in τ periods are worth $D(\tau)u$ utils today.
- Discount rate: rate of decline in discount function
$$-\frac{dD(\tau)/d\tau}{D(\tau)}$$
  - rate at which value declines with delay
- Exponential discounting: $D(\tau) = \delta^{\tau}$.
- For exponential case
$$-\frac{dD(\tau)/d\tau}{D(\tau)} = -\ln\delta \simeq 1 - \delta$$
- Exponential discount functions imply that discount rates do not change with horizon

Can discounting be exponential?

Not if you prefer 1 to 2 but also prefer $2'$ to $1'$.

Not if we discount utils tomorrow by 1%

(so the one-year discount factor is 0.99^{365})

- 100 utils in a year are worth 2.6 utils today.
- 100 utils in 10 years are worth 1×10^{-14} today.

5 Experimental evidence for quasi-hyperbolic discounting

Falling discount rates found with...

- money
- durable goods
- fruit juice
- sweets
- video rentals
- relief from noxious noise
- access to video games

5.1 A few (always imperfect) pieces of experimental evidence:

Revelation mechanism aligns incentives so subjects have an incentive to tell the truth (Becker, DeGroot, and Marschak)

For each of the choices below (rows $a - j$) circle the item that you most prefer. You should circle one item in EVERY row. You should assume that the item that you circle will be delivered to you on the associated date. For example, "$10 now," means I will give you $10 in class today.

After we collect these forms, we will randomly select one student. For that student, we will randomly select one row and pay that student what he/she chose in that randomly selected row. For example, imagine that we randomly select you and that we randomly select row j. Assume that you circled "$10 now" in row j. Then I will pay you $10 in class today.

(IMMEDIACY TREATMENT)

Please circle one item in EVERY row. (If you miss lecture on the day that you circle, I'll have the cash delivered to your House mailbox — same day.)

$a.$	$10 now	or	$15.00 "t+5"
$b.$	$10 now	or	$14.50 "t+5"
$c.$	$10 now	or	$14.00 "t+5"
$d.$	$10 now	or	$13.50 "t+5"
$e.$	$10 now	or	$13.00 "t+5"
$f.$	$10 now	or	$12.50 "t+5"
$g.$	$10 now	or	$12.00 "t+5"
$h.$	$10 now	or	$11.50 "t+5"
$i.$	$10 now	or	$11.00 "t+5"
$j.$	$10 now	or	$10.50 "t+5"

(DELAY TREATMENT)

Please circle one item in EVERY row. (If you miss lecture on the day that you circle, I'll have the cash delivered to your House mailbox — same day.)

$a.$	$10 on April 25	or	$15.00 on April 30
$b.$	$10 on April 25	or	$14.50 on April 30
$c.$	$10 on April 25	or	$14.00 on April 30
$d.$	$10 on April 25	or	$13.50 on April 30
$e.$	$10 on April 25	or	$13.00 on April 30
$f.$	$10 on April 25	or	$12.50 on April 30
$g.$	$10 on April 25	or	$12.00 on April 30
$h.$	$10 on April 25	or	$11.50 on April 30
$i.$	$10 on April 25	or	$11.00 on April 30
$j.$	$10 on April 25	or	$10.50 on April 30

- What amount makes you indifferent between \$10 "now" and \$$X$ at "t+5"? ($X = 11.206$)

$$\frac{10}{p_t} u'(c_t) = \frac{X}{p_{t+\tau}} \delta^\tau u'(c_{t+\tau})$$

  If $p_t \approx p_{t+\tau}$, $u'(c_t) \approx u'(c_{t+\tau})$, then

$$10 = \delta^\tau X$$

$$\begin{aligned} -\ln \delta &= \frac{1}{\tau} \ln X/10 \\ &= \frac{1}{5/365} \ln X/10 \\ &= 831\% \text{ per year} \end{aligned}$$

- What makes you indifferent between \$10 on April 25 and \$$X$ on April 30? ($X = 10.375$)

$$\begin{aligned} -\ln \delta &= \frac{1}{\tau} \ln X/10 \\ &= \frac{1}{5/365} \ln X/10 \\ &= 269\% \text{ per year} \end{aligned}$$

- This difference is significant at the 5% level.

- Of those in the "today" vs. "today+5 condition" 71% of respondents preferred immediate gratification if the tradeoff was \$10 vs. \$10.50.

- Of those in the "April 25" vs. "April 30" condition, 33% of respondents preferred immediate gratification if the tradeoff was \$10 vs. \$10.50.

- Respondents became less impatient when the horizon was moved further into the future.

- Confounds? Yes! You generate a list.

- Implied discount rates are “too high” for exponential discounters.

  - Suppose $-\ln\delta = 269\%$ or $-\ln\delta = 831\%$.

  - Then $\delta = 0.068$ or $\delta = 0.000246$.

  - The exponential discount model predicts indifference between \$10 today and $\delta \cdot \$X$ in a year.

  - $X \approx \$150$ or $\$41,000$.

- Intuitively, if you need a 3.75% return or a 12.06% return to wait five days.

- Then you would require a gross return of $(1.0375)^{73}$ or $(1.1206)^{73}$ to wait a year (5*73 days = 365 days).

- In experiments with current rewards, shifting out both rewards by the same amount of time lowers the implied discount rate (e.g., Kirby and Herrnstein, *Psychological Science*, 1996).

- For example, \$45 right now is preferred to \$52 in 27 days.

$$\begin{aligned} -\ln\delta &> \frac{1}{27/365}\ln 52/45 \\ &= 195\% \text{ per year} \end{aligned}$$

- But, \$45 in six days is inferior to \$52 in 33 days (now $-\ln\delta < 195\%$ per year).

Thaler (1981): hypothetical rewards.

- What amount makes you indifferent between \$15 today and $\$X$ in 1 month? ($X = 20$)

$$\begin{aligned} -\ln\delta &= \frac{1}{\tau}\ln X/15 \\ &= \frac{1}{1/12}\ln X/15 \\ &= 345\% \text{ per year} \end{aligned}$$

- What makes you indifferent between \$15 today and $\$X$ in ten years? ($X = 100$)

$$\begin{aligned} -\ln\delta &= \frac{1}{\tau}\ln X/15 \\ &= \frac{1}{10}\ln X/15 \\ &= 19\% \text{ per year} \end{aligned}$$

Benzion, Rapoport and Yagil (1989): hypothetical rewards.

- What amount makes you indifferent between \$40 today and $\$X$ in half a year? ($X = 50$)

$$40 = X\delta^{\tau}$$

so

$$\begin{aligned} -\ln\delta &= \frac{1}{\tau}\ln X/40 \\ &= \frac{1}{.5}\ln X/40 \\ &= 45\% \text{ per year} \end{aligned}$$

- What makes you indifferent between \$40 today and $\$X$ in four years? ($X = 90$)

$$\begin{aligned} -\ln\delta &= \frac{1}{\tau}\ln X/40 \\ &= \frac{1}{4}\ln X/40 \\ &= 20\% \text{ per year} \end{aligned}$$

6 Quasi-hyperbolic discounting

- Experimental evidence demonstrates that discount rates are higher in the short-run than in the long-run.

- More impatience trading off utils today vs. tomorrow than trading off utils on day 100 vs. day 101.

- In other words, subjects have a higher short-run discount rate (today vs. tomorrow) than their long-run discount rate (day 100 vs day 101).

- The *quasi-hyperbolic discount function* (Phelps and Pollak 1968, Laibson 1997):

$$D(\tau) = \begin{cases} 1 & \text{if} \quad \tau = 0 \\ \beta \cdot \delta^{\tau} & \text{if} \quad \tau \in \{1, 2, ...\} \end{cases} .$$

- We then can write the utility function as,

$$\begin{aligned} U_t &= u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + ... \\ &= u_t + \beta \left(\delta u_{t+1} + \delta^2 u_{t+2} + \delta^3 u_{t+3} + ... \right) \end{aligned}$$

- We tend to think that $\beta << 1$ and $\delta < 1$.

- E.g., $\beta = 2/3$ and $\delta = 0.95$ (with annual data).

Let's consider a special case that builds intuition.

- Assume that $\beta = \frac{1}{2}$ and $\delta \simeq 1$

$$D(\tau) = \{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...\}$$

- Intuition: relative to the current period, all future periods are worth less (weight $\frac{1}{2}$).

- All of the discounting takes place between the current period and the immediate future.

- There is no additional discounting between future periods.

- Reflects the property that most discounting occurs in the short-run.

- In the long-run we're relatively patient — utils tomorrow are just as valuable as utils the day after tomorrow.

7 Dynamic Inconsistency:

- Exercise has benefit today of -6.

- Exercise has delayed benefit of 8.

- Exercise today?

$$-6 + \frac{1}{2}(8) = -2.$$

- Exercise tomorrow!

$$0 + \frac{1}{2}(-6 + 8) = 1.$$

- But tomorrow you'll again want to postpone action (Akerlof 1992)

- Preferences are dynamically inconsistent iff optimal contingent plans change over time.

- *Dynamic consistency* means that early selves and later selves agree.

- In other words, the optimal contingent plans to do not change over time.

- So we can simply maximize at the beginning of time without worrying about later selves overturning the decisions of early selves.

But, in many domains, early selves and late selves don't agree:

- Next month, I'll quit smoking...

- Next week, I'll catch up on that required reading...

- Tomorrow morning, I'll wake up early and exercise...

- After Christmas, I'll go on a diet...

- In March, I'll pay my taxes...

- Next weekend, I'll send in those rebate forms...

- Next month, I'll join my 401(k) plan...

Early selves plan to "be good" (get up at 7AM to finish problem set)

Later self want "instant gratification" (keep hitting snooze button)

When discount functions are not exponential, the intertemporal choice model generates a conflict between early and late selves: dynamic inconsistency.

Dynamically inconsistent model predicts "self-control problems" like procrastination, laziness, addiction, etc...

8 Naifs and Sophisticates

- Naifs falsely believe that future selves will maximize today's preferences (Strotz 1957).
  - Solution concept: iterative maximization.
  - Prediction: never exercise (but join gym).
- Sophisticates have rational expectations (Strotz 1957)..
  - Solution concept: subgame perfect equilibrium.
  - Prediction: never exercise (and don't join gym).
- Partial naivite (O'Donoghue and Rabin, 2001)
  - Solution concept: subgame perfect equilibrium, using $\widehat{\beta}$ such that $\beta < \widehat{\beta} < 1$.
  - Note that naifs use $\widehat{\beta} = 1$ and sophisticates use $\widehat{\beta} = \beta$.

8.1 What's wrong with the naive and sophisticated models?

8.1.1 Naives

- Consider a naif with $\beta = \frac{1}{2}$ and $\delta = 1$.

- The naif has to finish a project by deadline T.

- In time period t, the (undiscounted) project costs $\left(\frac{3}{2}\right)^t$ utils to execute.

- When will the naif do the project?

From the current self's perspective, it's always better to postpone doing the project until next period:

$$\begin{aligned} \left(\frac{3}{2}\right)^t &> \beta\delta\left(\frac{3}{2}\right)^{t+1} \\ &= \frac{1}{2}\left(\frac{3}{2}\right)^{t+1} \\ &= \frac{3}{4}\left(\frac{3}{2}\right)^t \end{aligned}$$

When will the project be completed?

(Partial naives make the same kind of mistakes.)

8.1.2 Sophisticates:

Consider the same model as above.

When will a sophisticate do the project?

On a problem set you will prove the following two claims:

1. If T is even, then sophisticates will do the project in even periods (and not in odd periods).

2. If T is odd, then sophisticates will do the project in odd periods (and not in even periods).

9 Dynamic programming with dynamically inconsistent agents

- Consumption application (infinite horizon).

- Let c represent consumption

- Let x represent cash-on-hand

- Let $\tilde{y}$ represent iid stochastic income

- Let R represent gross interest rate

- So $x_{t+1} = R(x_t - c_t) + \tilde{y}_{t+1}$

- A (Markov) strategy is a map from state x to control c.

- Let V be the continuation-value function, W be the current-value function and C be the consumption function. Then:

$$\begin{aligned} V(x) &= U(C(x)) + \delta \, \mathsf{E}[V(R(x - C(x)) + y)] \\ W(x) &= U(C(x)) + \beta\delta \, \mathsf{E}[V(R(x - C(x)) + y)] \\ C(x) &= \underset{c}{\text{argmax}} \, U(c) + \beta\delta \, \mathsf{E}[V(R(x - c) + y)] \end{aligned}$$

- Envelope Theorem:

$$W'(x) = U'(C(x))$$

- First-order-condition:

$$U'(C(x)) = R\beta\delta \, \mathsf{E}\Big[V'(R(x - C(x)) + y)\Big]$$

- Identity linking V and W :

$$\beta V(x) = W(x) - (1 - \beta)U(C(x))$$

9.1 Problem is recursive

- Start with V.

- Find C:

$$C(x) = \underset{c}{\text{argmax}} \, U(c) + \beta\delta \, \mathsf{E}[V(R(x - c) + y)] \, .$$

- Find $\hat{V}$:

$$\hat{V}(x) = U(C(x)) + \delta \, \mathsf{E}[V(R(x - C(x)) + y)]$$

- In this way, generate an operator $T : V \mapsto \hat{V}$.

10 Generalized Euler Equation

We have

$$\begin{aligned} u'(c_t) &= R\beta\delta\,\mathsf{E}_t\Big[V'(x_{t+1})\Big] \\ &= R\delta\,\mathsf{E}_t\left[W'(x_{t+1}) - (1-\beta)u'(c_{t+1})\frac{dC_{t+1}}{dx_{t+1}}\right] \\ &= R\delta\,\mathsf{E}_t\left[u'(c_{t+1}) - (1-\beta)u'(c_{t+1})\frac{dC_{t+1}}{dx_{t+1}}\right]. \end{aligned}$$

Follows from FOC, differentiated identity, and envelope theorem. Simplifying,

$$u'(c_t) = R\,\mathsf{E}_t\left[\beta\delta\left(\frac{dC_{t+1}}{dX_{t+1}}\right) + \delta\left(1 - \frac{dC_{t+1}}{dX_{t+1}}\right)\right] u'(c_{t+1}).$$

See Harris and Laibson (2001).

Quasi-hyperbolics are highly patient when $\frac{dC_{t+1}}{dX_{t+1}} \simeq 0$, and highly impatient when $\frac{dC_{t+1}}{dX_{t+1}} \simeq 1$.

Calibration of steady state with no growth:

- $u(c) = \ln(c)$.
- In a standard exponential discounting model (i.e., $\beta = 1$), we have $\delta R = 1$, so the discount rate $(1-\delta)$ is approximately to the interest rate $(r = R - 1)$.
- What happens in the quasi-hyperbolic economy?
- Suppose $\beta = 2/3$ and $\delta = 0.975$, what is the steady state interest rate?

- If λ is the APC=MPC, then in steady state,

$$\begin{aligned} \frac{1}{\lambda} &= R\left[\beta\delta\lambda + \delta\left(1-\lambda\right)\right]\frac{1}{(1-\lambda)R\lambda} \\ 1 &= (1-\lambda)R \end{aligned}$$

- Calibrate $\beta = 2/3$, $\delta = 0.975$:

$$\begin{aligned} \lambda &= \frac{1-\delta}{1-\delta(1-\beta)} = 0.04. \\ r &\simeq \lambda = 0.04. \end{aligned}$$

- Why is the equilibrium interest rate so low?

11 Search and Procrastination

- See Akerlof (1992) and O'Donoghue and Rabin (1999) for early papers on Procrastination

- Today: Choi, Laibson, Madrian, and Metrick (2004)

- $0 < \beta < 1$

- $\delta = 1$

- Per period loss from delay L

- Stochastic action cost c_t drawn from a uniform distribution on the interval $[\underline{c}, \bar{c}]$

11.1 Sophisticates

Let W represent the current cost function (as above)

$$W(c) = \begin{cases} c & \text{if act} \\ \beta\left[L + EV(c')\right] & \text{if wait} \end{cases}. \qquad (1)$$

Let V represent the exponentially discounted continuation cost function (as above).

$$V(c) = \begin{cases} c & \text{if act tomorrow} \\ L + EV(c') & \text{if wait tomorrow} \end{cases}. \qquad (2)$$

Equilibrium is a "cutoff rule." Let this be c^*.

Agents must be indifferent in the current period between acting and waiting at the cutoff,

$$c^* = \beta\left[L + EV(c')\right]. \qquad (3)$$

Our problem can be reduced to two equations

$$\begin{aligned} c^* &= \beta\left[L + EV\right] \\ EV &= \int_{c \le c^*} c\,dF(c) + \int_{c > c^*} \left[L + EV\right] dF(c) \end{aligned}$$

and two unknowns: c^* and EV.

Proposition 11.1 *The equilibrium cutoff threshold is*

$$c^* = \frac{\underline{c} + \sqrt{\underline{c}^2\left[1 - (2-\beta)\beta\right] + 4\beta\left(1 - \frac{\beta}{2}\right)\left(\bar{c} - \underline{c}\right) L}}{2 - \beta}. \qquad (4)$$

11.2 Properties of c^*:

- How does c^* change with L, the flow cost?

- How does c^* change with β, the short-term discount factor?

- Is c^* above $\underline{c}$?

- Is c^* below $\bar{c}$?

11.3 Procrastination with sophisticates

- Let c^{**} be the *desired* future threshold. So,
$$c^{**} = c^*_{\beta=1} = \underline{c} + \sqrt{2\left(\bar{c} - \underline{c}\right)L}. \tag{5}$$

- How does c^{**} compare with $c^*_{\beta<1}$?

- What is the probability that an agent procrastinates in a given period?

- Calibration: $\underline{c} = 0$ and $\bar{c} = 1$.
$$c^{**} - c^* = \sqrt{2L}\left(1 - \sqrt{\frac{\beta/2}{1-\beta/2}}\right)$$

- If $\sqrt{2L} = 1$, exponentials always do it. If, $\beta = 2/3$ probability of procrastination is
$$c^{**} - c^* = 1 - \sqrt{\frac{\beta/2}{1-\beta/2}} = 1 - \sqrt{\frac{1}{2}} = 0.29.$$

11.4 Testing your intuition:

- Consider the boundary case in which $\bar{c} = \underline{c}$.
- What is the equilibrium action rule?

11.5 Procrastination with naives

- Let c_N^* be the equilibrium Naive threshold.
- You'll solve for this threshold on the problem set.
- Is c_N^* greater than or less than c^*?
- Is c_N^* greater than or less than $\underline{c}$?

12 Directions for Future Research:

- New neuro evidence linking β effects to the limbic system (McClure, Laibson, Loewenstein, and Cohen, 2004).

- What turns β on? Context dependent β effects.
  - Visceral (Loewenstein, 1996)
  - Cues (Laibson, 2001), Generalized temptation (Gul and Pesendorfer, 2002)

- See integrative two-brain models by Shefrin and Thaler (1981), Bernheim and Rangel (2004), Fudenberg and Levine (2004), Benhabib and Bisin (2004), and O'Donoghue and Loewenstein (2004).