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Why is there money? Endogenous derivation of 'money' as the most liquid asset: a class of examples*

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Summary. The monetary character of trade, use of a common medium of exchange, is shown to be an outcome of an economic general equilibrium. Monetary structure can be derived from price theory in a modified Arrow-Debreu model. Two constructs are added: transaction costs and market segmentation in trading posts (with a separate budget constraint at each transaction). Commodity money arises endogenously as the most liquid (lowest transaction cost) asset. Government-issued fiat money has a positive equilibrium value from its acceptability for tax payments. Scale economies in transaction cost account for uniqueness of the (fiat or commodity) money in equilibrium.

Keywords and Phrases: Commodity money, Fiat money, Transaction cost, Scale economy, Double coincidence of wants.

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> "[An] important and difficult question...[is] not answered by the approach taken here: the integration of money in the theory of value..."
> Gerard Debreu, *Theory of Value* (1959)

1 Money in Walrasian general equilibrium

Consider four commonplace observations on the character of trade in virtually all economies:

(i) Trade is monetary. One side of almost all transactions is the economy's common medium of exchange.
(ii) Money is (virtually) unique. Though each economy has a 'money' and the 'money' differs among economies, almost all the transactions in most places most of the time use a single common medium of exchange.
(iii) 'Money' is government-issued fiat money, trading at a positive value though it conveys directly no utility or production.
(iv) Even transactions displaying a double coincidence of wants are transacted with money.[1]

Where economic behavior displays such uniformity, a general elementary economic theory should be able to account for the universal usages. But (i), (ii), and (iii) contradict the implications of a frictionless Walrasian general equilibrium model, and (iv) contradicts the conventional view of the role of money with regard to the double coincidence of wants. This essay presents a class of examples with a slight modification of the Arrow-Debreu general equilibrium model sufficient to derive points (i)–(iv) as outcomes. In doing so, this essay responds to a challenge expressed by Tobin (1980)

> Social institutions like money are public goods ... General equilibrium theory is not going to explain the institution of a monetary ... common means of payment.

Thus the examples below are intended to show that a general equilibrium model can explain endogenously from price theory the institution of a common monetary means of payment.[2] The price system itself designates 'money' and guides transactors to trade using 'money.' The model emphasizes complete markets and complete

[1] University of California faculty whose children are enrolled at the University pay fees in money, not in kind; Ford employees buying a Ford car pay in money, not in kind; Albertson's supermarket checkout clerks acquiring groceries pay in money, not in kind. This observation suggests that the focus on the absence of double coincidence of wants – as distinct from transaction costs – as an explanation for the monetization of trade may miss a significant part of the underlying causal mechanism.

[2] A bibliography of the issues involved in this inquiry appears in Ostroy and Starr (1990). In addition, note particularly Banerjee and Maskin (1996), Hellwig (2000), Howitt (2000), Howitt and Clower (2000), Iwai (1996), Kiyotaki and Wright (1989), Marimon et al. (1990), Rajeev (1999), Rey (2001), Trejos and Wright (1995), and Young (1998). The treatment of transaction costs in this essay (as opposed to the recent focus in the literature on search and random matching equilibria) resembles the general equilibrium models with transaction cost developed in Foley (1970); Hahn (1971); Starrett (1973), and Kurz (1974). The structure of bilateral trade here however is more detailed, with a budget constraint enforced on each transaction separately, so that the Foley, Hahn, and Starrett models do not immediately translate to the present setting.

information. Points (ii) and (iv) involve scale economies, nonconvex transaction costs; it will typically be difficult to develop general existence of equilibrium theorems – hence the use of examples.

It is well known that a frictionless Arrow-Debreu model cannot accomodate a role for money. The single budget constraint facing transactors in the model precludes a carrier of value between transactions. This essay is intended as a partial counterexample, demonstrating that minimal friction in trade is sufficient to induce the existence of money as a result, not an assumption. The monetary structure of the economy is derived from elementary price theory in a class of examples. Use of a common medium of exchange, a commodity money, is an outcome of the market equilibrium. Starting from a (non-monetary) Arrow-Debreu model, the monetary quality of the economic equilibrium is derived through the addition of market segmentation (with a separate budget constraint in each segment) and transaction costs. Multiplicity of budget constraints – requiring that goods acquired be paid for by delivery of equal value at each trade separately – creates a demand for media of exchange. Transaction costs imply differing bid and ask prices for each good. Liquidity is priced: its price is the bid/ask spread. The most liquid asset, the instrument that provides liquidity at lowest cost, will be chosen as the medium of exchange. Thus, the choice of a (possibly unique) ‘money’ is the outcome of optimizing behavior of economic agents in a market equilibrium. Fiat money – issued by government – derives its positive value from acceptability in payment of taxes; it becomes the medium of exchange from its low transaction cost. Uniqueness of (fiat or commodity) money follows from scale economy in transaction costs.

Section 3 of the paper presents the model of segmented markets with linear transaction costs without double coincidence of wants. Commodity money arises endogenously in market equilibrium. Section 4 demonstrates that the absence of double coincidence of wants is essential to monetization of trade in a linear model by considering the same problem with full double coincidence of wants. The result is a nonmonetary equilibrium. Section 6 considers a (nonconvex) transaction technology with scale economies. The examples there demonstrate that uniqueness of money (uniqueness of the endogenously chosen medium of exchange) results from scale economies in transaction costs. Further, Section 6 demonstrates that scale economies in transaction cost account for monetization of trade with a unique ‘money’ even when there is full double coincidence of wants. Section 7 considers government-issued fiat money whose value is supported by acceptability in payment of taxes. In a linear transaction cost model, fiat money’s (assumed) low transaction cost makes it the common medium of exchange. Alternatively, in a nonlinear model, scale economies in transaction cost and government’s large scale ensure that fiat money is the unique common medium of exchange.[3]

[3] Space does not permit a full discussion distinguishing random matching models of money, e.g. Kiyotaki and Wright (1989), Trejos and Wright (1995), from general equilibrium models with transaction cost, e.g. Foley (1970), Hahn (1971), Starrett (1973), Ostroy and Starr (1974), Iwai (1996), and this essay. The latter typically model more complete markets and greater reliance on the price system.

2 Formalizing menger's 'origin of money'

Why is there money? is one of the classic issues in the foundations of economic theory, with contributions extending from Smith's *Wealth of Nations*, to the present. *Money*, like *written language* and the *wheel*, is one of the fundamental discoveries of civilization. Nevertheless, despite the evident superiority of monetary trade over barter, there is a puzzling quality to monetary exchange. Monetary trade involves one party to a transaction giving up something desirable (labor, his production, a previous acquisition) for something useless (a fiduciary token or a commonly traded commodity for which he has no immediate use) in the hope of advantageously retrading this latest acquisition. An essential issue at the foundations of monetary theory is to articulate the elementary economic conditions that allow this paradox to be sustained as an individually rational market equilibrium.

Over a century ago, Carl Menger presented the paradox of monetary trade as a challenge to monetary theory and proposed an outline of its solution, a theory of liquidity as the basis of monetary theory, Menger (1892):

> It is obvious ... that a commodity should be given up by its owner ...for another more useful to him. But that every[one] ... should be ready to exchange his goods for little metal disks apparently useless as such...or for documents representing [them]...is...mysterious... why...is...economic man ...ready to accept a certain kind of commodity, *even if he does not need it,...* in exchange for all the goods he has brought to market[?] [Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution... Men ... exchange goods ... for other goods ... more saleable....[which] become *generally* acceptable media of exchange [emphasis in original].

Menger's proposed solution focused on the liquidity of commodities. A good is very *saleable* (liquid) in Menger's definition above, if the price at which a household can sell it (the market's prevailing bid price) is very near the price at which it can buy (the market's prevailing ask price). In this setting, price theory includes a theory of liquidity. The segmented market creates a demand for a carrier of value between transactions. Separate bid and ask prices represent transaction costs and put a price on liquidity: a good's bid/ask spread is the price of using it as a medium of exchange. Hence, a good with a uniformly narrow bid/ask spread is highly liquid – in Menger's word 'saleable' – and constitutes a natural 'money.' Price theory implies monetary theory. Liquidity creates monetization. This is the insight that will be formalized in the examples below.

Starting from the non-monetary Arrow-Debreu model, two additional structures are sufficient to give endogenous monetization in equilibrium: multiple budget constraints (one at each transaction, not just on net trade) and transaction costs. One way of formalizing multiple budget constraints is a trading post model. Thus, if there are N goods actively traded, there are $N(N-1)/2$ possible trading posts. That is the starting point of the examples below. The choice of which trading posts a typical household will trade at is part of the household optimization. The

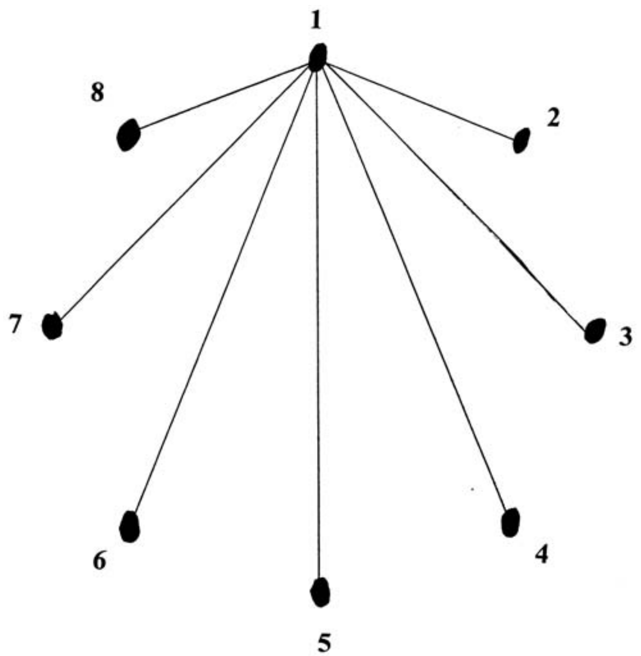

Figure 1. Monetary equilibrium with unique money

equilibrium structure of exchange is the array of trading posts that actually host active trade. The determination of which trading posts are active in equilibrium is endogenous to the model and characterizes the monetary character of trade. The equilibrium is monetary with a unique money if only $(N - 1)$ trading posts are active, those trading all goods against 'money.'

The examples below derive monetary equilibrium as a market equilibrium of optimizing agents based on elementary considerations of transaction cost. Household optimization includes deciding at which trading posts the household will trade. For a given mix of goods, trade is drawn to the lowest transaction cost trading posts. The question *Why is there money?* can then be answered by presenting sufficient conditions so that an equilibrium trading array has $N - 1$ active trading posts, those trading in a common medium of exchange versus the $N - 1$ other goods. This is illustrated in Figures 1 and 2. Each node in the figures represents a commodity. Active trade is represented by a chord between nodes. A barter economy will have chords among a wide variety of goods – one for each pair of goods where there is a household with a matching demand and supply (Fig. 2). A monetary economy with a unique money will be a sparser array. There will be one good so that the only chords are those linking that good to all others (Fig. 1). The question *why is there money?* is then reduced to asking for sufficient conditions so that the array of active trading posts in equilibrium looks like Figure 1 (spider-shaped) instead of Figure 2 (star-shaped).

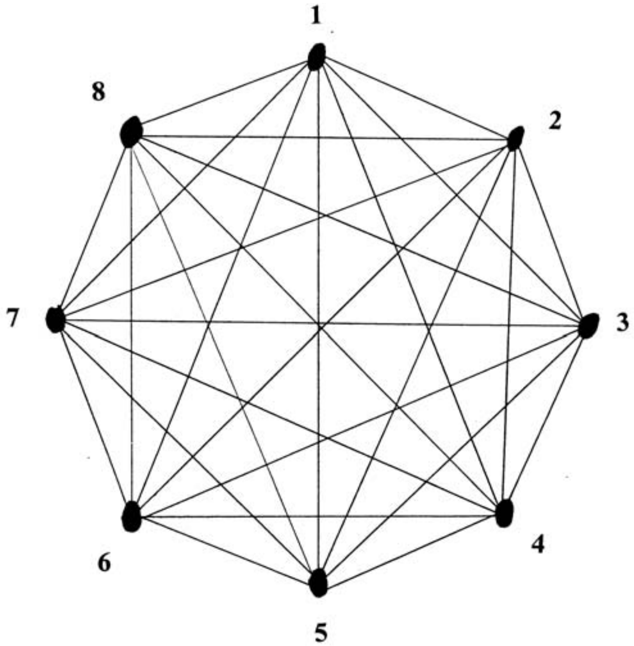

Figure 2. Barter equilibrium for H^D

3 Monetization comes from liquidity: monetary competitive equilibrium with linear transaction costs

The distinctive features of the model are (i) transactions exchange pairs of goods, (ii) budget constraints are enforced at each transaction separately, generating a role for a carrier of value between transactions (a medium of exchange), (iii) transaction costs are assumed to be linear in Sections 3 and 4 and nonconvex (displaying scale economies) in Sections 5, 6, and 7. In the linear transaction cost case without double coincidence of wants, the most liquid (lowest transaction cost) good becomes the common medium of exchange. There may be multiple media of exchange when there is a tie for lowest cost.

Let there be $N+1$ commodities, numbered 0,1,2,...N. They are traded in pairs – good i for good j – at specialized trading posts. The trading post for trade of good i versus good j (and vice versa) is designated $\{i, j\}$; trading post $\{i, j\}$ is identical to trading post $\{j, i\}$. Trading post $\{i, j\}$ is a business firm, the market maker in trade between goods i and j. $\{i, j\}$ actively buys and (re)sells both i and j. Trade as a resource using activity is modeled by describing the post's transaction costs. The notion of transaction technology summarizes costs that in an actual economy are incurred by retailers, wholesalers, individual firms and households. The bid/ask spread summarizes these costs to the model's transactors.

Specify a transaction cost function for these pairwise trading posts so that all transaction costs accrue in good 0. It is simplest to think of good 0 as the labor used in the transaction technology. This is obviously a restrictive convention, but it simplifies accounting for transaction costs. Trading post $\{i, j\}$ buys good 0 as an input to its transaction costs. The typical transactions of trading post $\{i, j\}$ will consist of purchases $y_i^{\{i,j\}B}$, $y_j^{\{i,j\}B}$, $y_0^{\{i,j\}B} \geq 0$, of i, j, and 0 respectively and sales $y_i^{\{i,j\}S}$, $y_j^{\{i,j\}S} \geq 0$ of i and j. In this section, we use the further simplifying assumption of linear transaction costs. The cost structure is generalized to non-convex costs in Sections 5, 6, and 7.

The transaction cost function for trading post $\{i, j\}$ is[4]

$$C^{\{i,j\}} = y_0^{\{i,j\}B} = \delta^i y_i^{\{i,j\}B} + \delta^j y_j^{\{i,j\}B} \qquad \text{(TCL)}$$

where δ^i, $\delta^j > 0$. In words, the transaction technology looks like this: Trading post $\{i, j\}$ makes a market in goods i and j, buying each good in order to resell it. It incurs transaction costs in good 0. These costs vary directly (in proportions δ^i, δ^j) with volume of trade. The transaction cost structure is separable in the two principal traded goods. The trading post $\{i, j\}$ buys good 0 to cover the transaction costs it incurs, paying for 0 in goods i and j. The transaction cost function $C^{\{i,j\}}$ is sufficiently flexible to distinguish transaction costs differing among commodities, including differences in durability, portability, recognizibility, divisibility.

The population of households is denoted H, consisting of a mix of subpopulations (with different tastes and endowments). A typical household $h\epsilon H$, has an endowment $r^h \epsilon R_+^N; r_n^h$ is h's endowment of good n. For simplicity in the examples below, each household is endowed with only one commodity. This is obviously inessential. h's utility function is $u^h(x) = u^h(x_0, x_1, ..., x_N)$.

It is convenient to arrange a subpopulation H^0 to provide good 0 (transaction labor). H^0's endowment of good 0 is characterized as

$\sum_{h\varepsilon H^0} r_0^h > \sum_{h\varepsilon H} \sum_{i=1}^{N} \delta^i r_i^h$. For typical $h \,\epsilon\, H^0, h'$s utility function is

$$u^h(x) = \sum_{i=0}^{N} x_i. \qquad \text{(U0)}$$

That is, a subpopulation H^0 owns all of the good 0; they have it in sufficient quantity to cover all the transaction costs in the economy that are likely to be incurred; their tastes treat all goods as perfect substitutes with MRS equal to unity. This unrealistic assumption is designed to make accounting for transaction costs particularly easy.

A typical household outside of H^0 may be denoted $h = [m, n]$ where m and n are integers between 1 and N (inclusive). m denotes the good with which h is endowed. n denotes the good h prefers. $[m, n]$'s utility function can then be taken to be

$$u^{[m,n]}(x) = \sum_{i=0, i\neq n}^{N} x_i + 3x_n. \qquad \text{(U1)}$$

[4] (TCL) is intended as a mnemonic for linear transaction cost.

$[m,n]$'s endowment, $r_m^{[m,n]}$, is specified as part of the description of the subpopulation.

Households formulate their trading plans deciding how much of each good to trade at each pairwise trading post. This leads to the rather messy notation:

$$b_\ell^{[m,n]\{i,j\}} = \text{planned purchase of good } \ell \text{ by household } [m,n] \text{ at trading post } \{i,j\}$$

$$s_\ell^{[m,n]\{i,j\}} = \text{planned sale of good } \ell \text{ by household } [m,n] \text{ at trading post} \{i,j\}$$

The bid prices (the prices at which the trading post will buy from households) at $\{i,j\}$ are $q_i^{\{i,j\}}, q_j^{\{i,j\}}$ for goods i and j respectively. The price of i is in units of j. The price of j is in units of i. The ask price (the price at which the trading post will sell to households) of j is the inverse of the bid price of i (and vice versa). That is, $\left(q_i^{\{i,j\}}\right)^{-1}$ and $\left(q_j^{\{i,j\}}\right)^{-1}$ are the ask prices of j and i at $\{i,j\}$. The trading post $\{i,j\}$ covers its costs by the difference between the bid and ask prices of i and j, that is, by the spread $\left(q_j^{\{i,j\}}\right)^{-1} - q_i^{\{i,j\}}$ and the spread $\left(q_i^{\{i,j\}}\right)^{-1} - q_j^{\{i,j\}}$. Transaction costs at the trading post are incurred in good 0. Post $\{i,j\}$ pays for 0 in i and j, acquired in trade through the difference in bid and ask prices. The bid price of 0 in terms of i is $q_{(i)0}^{\{i,j\}}$. The bid price of 0 in terms of j is $q_{(j)0}^{\{i,j\}}$.

Given $q_i^{\{i,j\}}, q_j^{\{i,j\}}$, for all $\{i,j\}$ household h then forms its buying and selling plans, in particular deciding which trading posts to use to execute his desired trades. Household $h \epsilon H$ faces the following constraints on its transaction plans:

$$\text{(T.i)} \quad b_n^{h\{i,j\}} > 0, \text{ only if } n = i,j; \; s_n^{h\{i,j\}} > 0, \text{ only if } n = i,j,0.$$

$$\text{(T.ii)} \quad b_i^{h\{i,j\}} \le q_j^{\{i,j\}} \cdot s_j^{h\{i,j\}}, \; b_j^{h\{i,j\}} \le q_i^{\{i,j\}} \cdot s_i^{h\{i,j\}} \text{ for each } \{i,j\}.$$

There is a slightly distinct version of (T.ii), (T.ii′), applying to households in H^0.

(T.ii′) For $h \epsilon H^0$, decompose $s_0^{h\{i,j\}}$ into nonnegative elements $s_{(i)0}^{h\{i,j\}}$ and $s_{(j)0}^{h\{i,j\}}$, so that $s_{(i)0}^{h\{i,j\}} + s_{(j)0}^{h\{i,j\}} = s_0^{h\{i,j\}}$, then we have $b_i^{h\{i,j\}} \le q_{(i)0}^{\{i,j\}} \cdot s_{(i)0}^{h\{i,j\}}$, and $b_j^{h\{i,j\}} \le q_{(j)0}^{\{i,j\}} \cdot s_{(j)0}^{h\{i,j\}}$ for each $\{i,j\}$.

$$(T.iii) \quad x_n^h = r_n^h + \Sigma_{\{i,j\}} b_n^{h\{i,j\}} - \Sigma_{\{i,j\}} s_n^{h\{i,j\}} \ge 0, 0 \le n \le N.$$

Note that condition (T.ii) [and (T.ii')] defines a budget balance requirement at the transaction level, implying the decentralized character of trade. Since the budget constraint applies to each pairwise transaction separately, there may be a demand for a carrier of value to move purchasing power between distinct transactions. h faces the array of bid prices $q_i^{\{i,j\}}, q_j^{\{i,j\}}$, and chooses $s_n^{h\{i,j\}}$ and $b_n^{h\{i,j\}}$, $n = i,j$ (and $n = 0$ for $h \epsilon H^0$), to maximize $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii). That is, h chooses which pairwise markets to transact in and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints.

The trading posts in Sections 3 and 4 have linear transaction technologies. A competitive equilibrium is an appropriate solution concept resulting in zero profits for the typical trading post (with the additional benefit that no account need be taken of distribution of profits). The threat of entry (by other similar trading post firms) rationalizes the competitive model, but for simplicity we take there to be a unique trading post firm making a market in goods i and j, denoted indiscriminately $\{i, j\} = \{j, i\}$.

A *competitive equilibrium* under (TCL) consists of $q_{(i)0}^{o\{i,j\}}$ $q_{(i)0}^{o\{i,j\}}$, $q_i^{o\{i,j\}}$, $q_j^{o\{i,j\}}$, $1 \le i, j \le N$, so that:

- For each household $h \in H$, there is a utility optimizing plan $b_n^{oh\{i,j\}}$, $s_n^{oh\{i,j\}}$, (subject to T.i, T.ii [or T.ii′ for $h \in H^0$], T.iii) so that $\Sigma_h b_n^{oh\{i,j\}} = y_n^{o\{i,j\}S}$, $\Sigma_h s_n^{oh\{i,j\}} = y_n^{o\{i,j\}B}$, for each $\{i, j\}$, each n, where
- $y_n^{o\{i,j\}S} \le y_n^{o\{i,j\}B}$, $n = i, j$.
- $y_0^{o\{i,j\}B}$ can be divided into two parts, $y_{(i)0}^{o\{i,j\}B} \ge 0, y_{(j)0}^{o\{i,j\}B} \ge 0$, so that $y_{(i)0}^{o\{i,j\}B} + y_{(j)0}^{o\{i,j\}B} = y_0^{o\{i,j\}B} = C^{\{i,j\}}$.
- $q_{(i)0}^{o\{i,j\}} y_{(i)0}^{o\{i,j\}B} \le y_i^{o\{i,j\}B} - q_j^{o\{i,j\}} y_j^{o\{i,j\}B}$.
  $q_{(j)0}^{o\{i,j\}} y_{(j)0}^{o\{i,j\}B} \le y_j^{o\{i,j\}B} - q_i^{o\{i,j\}} y_i^{o\{i,j\}B}$.
- $\delta^i + \delta^j q_i^{o\{i,j\}} = \left(q_{(i)0}^{o\{i,j\}}\right)^{-1} \left(1 - q_i^{o\{i,j\}} q_j^{o\{i,j\}}\right)$,
  $\delta^j + \delta^i\, q_j^{o\{i,j\}} = \left(q_{(j)0}^{o\{i,j\}}\right)^{-1} \left(1 - q_i^{o\{i,j\}} q_j^{o\{i,j\}}\right)$

The expression in the last bullet is a marginal cost pricing condition: the transaction cost (in good 0) of buying one unit of i and enough j to pay for it (pricing the 0 in good i) is equal to the amount of i left over after completing the trade in i and j. Similarly for trade in j.

An equilibrium is said to be *monetary* with a unique money, μ, if – for all households – good μ is the only good that a household will both buy and sell. An equilibrium will be monetary with multiple moneys, $\mu^1, \mu^2, ...$, if – for all households – $\mu^1, \mu^2, ...$ are the only goods that a household will both buy and sell.

Jevons (1875) reminds us that monetization of trade follows in part from the absence of a double coincidence of wants. In the present model, that logic is particularly powerful. Absence of coincidence of wants means that the typical traded good will be traded more than once in moving from endowment to consumption. Barter trade successfully rearranging the allocation to an equilibrium will transact an endowment first at the trading post where it is supplied and again at a distinct post where it is demanded. Hence monetary trade as an alternative (substituting retrade of money for the retrade of nonmonetary goods) can be undertaken without increasing total trading volume or transaction cost, even without scale economies. Conversely, when there is a full double coincidence of wants *and linear transaction cost*, equilibrium will be non-monetary even in the presence of a natural money (Section 4).

We now formalize the notion of the absence of double coincidence of wants. Let N be an integer, $N \ge 3$. For $m = 1, 2, ..., N$, and positive integers i, $1 \le i \le N-1$, let

$$m \oplus i = \begin{cases} m+i & \text{if } m+i \le N, \\ m+i-N & \text{if } m+i > N. \end{cases}$$

That is, $m \oplus i$ denotes $m+i$ mod N, skipping 0 (since good 0 is used primarily as an input to the transaction process). Recall that $[m, n]$ denotes a household endowed with good m, strongly preferring good n. Using the notation above, let $H^1 = \{[m, m \oplus 1] | m = 1, 2, ..., N; r_m^{[m,m\oplus 1]} = A > 0\}$. H^1 characterizes a population of N households with the same size of initial endowment, so that no pair of them have reciprocal matching endowments and preferences but so that their endowments in aggregate can be reallocated to make each one significantly better off (roughly by arranging the households clockwise in a circle ordered by endowment good and having each household $[m, m\oplus 1]$ send his endowment one place counterclockwise).

Example 3.1. Let the population of households be $H = H^0 \cup H^1$. Let $C^{\{i,j\}}$ be described by (TCL). Let $0 < \delta^i < 1/3$ and $0 < \delta^1 < \delta^i$, for $i = 2, 3, ...N$. Transaction costs are constant and non-trivial for all goods; they are significantly lower in good 1. Then there is a unique competitive equilibrium allocation (though a range of prices may support the unique real allocation of trades and consumptions). The equilibrium is a monetary equilibrium with good 1 as the unique 'money'.

Demonstration of Example 3.1. Using marginal cost pricing and market clearing, we have for each

$$\{i, j\}, i \ne j, 1 \le i, j \le N, q_{(i)0}^{\{i,j\}} = q_{(j)0}^{\{i,j\}} = 1, q_{i\oplus 1}^{\{i,i\oplus 1\}} = 1, q_i^{\{i,i\oplus 1\}} = \frac{1-\delta^i}{1+\delta^{i\oplus 1}},$$

and for $j \ne 1, i \oplus 1, q_i^{\{i,j\}} = 1 - \delta^i$, $q_j^{\{i,j\}} = 1 - \delta^j$; $q_i^{\{i,1\}} = \frac{1-\delta^i}{1+\delta^1}$, $q_1^{\{i,1\}} = 1$. $s_i^{[i,i\oplus 1]\{i,l\}} = A$, $b_1^{[i,i\oplus 1]\{i,1\}} = q_i^{\{i,1\}} A = s_1^{[i,i\oplus 1]\{i\oplus 1,1\}}$,
$b_{i\oplus 1}^{[i,i\oplus 1]\{i\oplus 1,1\}} = q_i^{\{i,1\}} q_1^{\{i\oplus 1,1\}} A$.

What's happening in Example 3.1? At first household $[i, i \oplus 1]$ goes to trading post $\{i, i \oplus 1\}$ offering i in exchange for $i\oplus 1$. But no one is coming to the trading post offering $i\oplus 1$. So good i is priced at a large discount at the post, reflecting the transaction costs of both i and $i\oplus 1$. On all other markets $\{i, j\}$ goods are priced to reflect their transaction costs, $q_i^{\{i,j\}} = 1 - \delta^i$. But at that pricing, since $\delta^1 < \delta^i$, it is advantageous for $[i, i\oplus 1]$ to trade through 1 as an intermediary. This follows since $(1-\delta^i)\cdot(1-\delta^1) > (1-\delta^i)\cdot(1-\delta^{i\oplus 1})$. This pricing creates a small shortage of 1 at each trading post (since small quantities of 1 are being retained at the post to cover 1's transaction costs) so prices are readjusted so that all of the discount in bid prices at $\{i, 1\}$ appears in the bid price of i. This results in $q_i^{\{i,1\}} = \frac{1-\delta^i}{1+\delta^1}$, $q_1^{\{i,1\}} = 1$. All trade of i for $i \oplus 1$ now goes through 1. Good 1 has become 'money,' the unique low transaction cost common medium of exchange.

In actual monetary economies we usually see a single 'money' as in Example 3.1. We'll argue in Section 5 that the reason for uniqueness of 'money' is scale economy. Does there have to be a reason for uniqueness? Yes. US dollars, pounds sterling, and euros, all have similar low transaction costs but in their separate markets they overwhelmingly dominate the mix of currencies used. Economic theory should have an explanation. Example 3.2 below emphasizes, by counterexample, that the nonconvexity in Section 5 is important. In Example 3.2, absent the non-convexity, when there's a tie for lowest transaction cost, there are many media of exchange in use. Is a tie realistic; isn't it a singularity? The example of dollars, sterling, and euros suggests that on the contrary, the notion of a tie for lowest transaction cost is a non-trivial event, so that uniqueness requires an explanation.

Example 3.2. Let the population of households be $H = H^0 \cup H^1$. Let $C^{\{i,j\}}$ be described by (TCL). Let $0 < \delta^1 = \delta^2 = \delta^3 < \delta^i < 1/3$, $i = 4, 5, ..., N$. Then there is a continuum of competitive equilibrium allocations with 1,2,3 acting as 'money' in proportions from 0% to 100%. Consumptions and utilities of all households are the same as in the equilibrium of Example 3.1.

Demonstration of Example 3.2. The marginal cost market-clearing pricing is identical to that in Example 3.1 with goods 2 and 3 priced similarly to good 1. The exception is trade between 'money''s where $q_1^{\{1,2\}} = 1 - \delta^1$, and similarly for 2,3, all of these bid prices being equal. The trading posts $\{i, 1\}$, $\{i, 2\}$, and $\{i, 3\}$, $i = 4, 5, ..., N$, (for trade in good i versus goods 1,2,3) are the trading posts with narrow bid/ask spreads since 1,2,3 have low transaction costs. Households can now divide their transactions among trading posts for goods 1, 2, and 3 versus all other goods in any proportion (though in equilibrium they will be the same proportions for all households). Markets clear.

The logic of Example 3.2 is merely the multi-money version of 3.1. Goods 1, 2, 3 are equally liquid and become media of exchange. They can be used however in any proportionate combination from 0% to 100% since absent economies of scale there is no reason further to specialize.

4 Absence of double coincidence of wants is essential to monetization in a linear model

Let $H^D = \{[m, n]|m, n = 1, 2, 3, ..., N, m \neq n\}$. H^D is distinctive in creating a population of households with fully complementary demands and supplies, full double coincidence of wants. We can use this population to illustrate the importance of the absence of double coincidence of wants to monetization in a linear model. Under the same conditions where monetary equilibria existed – and indeed were the only equilibria – in Examples 3.1 and 3.2 in the absence of double coincidence of wants, we can show that for H^D , with full double coincidence of wants, a barter equilibrium is the unique competitive equilibrium. Hence the classical focus on the absence of double coincidence of wants is confirmed; it is essential to monetization in a linear model. Note that this result depends on the linearity (or convexity) of transaction costs; if scale economies are present, then even with full double coincidence of wants, it may be resource saving to use a common medium of exchange with resulting high trading volumes.

Example 4.1. Let the population of households be $H = H^0 \cup H^D$. Let $C^{\{i,j\}}$ be described by (TCL). Let $0 < \delta^i < 1/3$ and $0 < \delta^1 < \delta^i$, for all $i, i = 0, 2, 3, ...N$. Transaction costs are constant and non-trivial for all goods but 1. Then there is a unique competitive equilibrium allocation. The equilibrium is non-monetary with active trade in all trading posts $\{i, j\}$, $1 \leq i, j \leq N$.

Demonstration of Example 4.1. For each $i, j, 1 \leq i, j \leq N$, $q_i^{\{i,j\}} = (1 - \delta^i)$, $q_j^{\{i,j\}} = (1 - \delta^j)$. $s_i^{[i,j]\{i,j\}} = A$, $b_j^{[i,j]\{i,j\}} = q_i^{\{i,j\}} A$, $s_j^{[j,i]\{i,j\}} = A$, $b_i^{[j,i]\{i,j\}} = q_j^{\{i,j\}} A$. Markets clear. The allocation is an equilibrium.

What's happening in Example 4.1? Direct barter trade works successfully in the presence of double coincidence of wants. For each household $[i, j]$ with a supply of one good and a demand for another, there is a precise mirror image $[j, i]$ in the population. They each go to the trading post $\{i, j\}$ where their common demands and supplies are traded. They trade, each incurring the cost of trading one good. Monetary trade is not advantageous since it requires twice the transactions volume – with corresponding cost – of direct barter trade (similar volumes for each non-monetary good and an equal volume of trade in the medium of exchange). Monetization of trade in equilibrium *in a linear model* depends on absence of double coincidence of wants.

5 Uniqueness of the medium of exchange: scale economies in transaction cost

Monetary trade is typically characterized by a unique medium of exchange or a small number of related media. How does this come about? Professor Tobin (1980) suggests that scale economies in transaction costs are essential:

> The use of a particular language or a particular money by one individual increases its value to other actual or potential users. Increasing returns to scale ... explains the tendency for one basic language or money to monopolize the field.

When monetization takes place, households supplying good i and demanding good j are induced to trade in a monetary fashion, first trading i for 'money' and then 'money' for j, by discovering that transaction costs are lower in this indirect trade than in direct trade of i for j. But as Example 3.2 points out, monetization of trade is no guarantee of uniqueness of the medium of exchange. Scale economies in transaction costs induce specialization in the medium of exchange function. High volume leads to low unit transaction costs (see also Rey, 2001; Starr and Stinchcombe, 1999). Scale economy is not a necessary condition for uniqueness of the medium of exchange in equilibrium (Example 3.1), but scale economy helps to ensure uniqueness (Example 6.1, below). If there are many equally low cost candidates for the medium of exchange, then scale economy in transaction costs will allow one to be endogenously chosen as the unique medium of exchange.

6 Monetization comes from liquidity again: monetary general equilibrium with unique money under average cost pricing of non-convex transaction costs

Scale economies in the transaction cost structure induce uniqueness of the equilibrium medium of exchange. As Professor Tobin (1959) tells us, "Why are some assets selected by a society as generally acceptable media of exchange while others are not? This is not an easy question, because the selection is self-justifying." Thus gold and dollar bills may have low transaction costs and be excellent candidates for medium of exchange, but if (despite high transaction cost) Yap Island stones are already the commonly chosen medium of exchange with high trading volume, then stones may have the lowest average transaction cost. The choice of Yap Island stones as the common medium of exchange is then self-justifying.

The nonconvex (scale economy) cost function for trading post $\{i, j\}$ is

$$C^{\{i,j\}} = y_0^{\{i,j\}B} = \min[\delta^i y_i^{\{i,j\}B}, \gamma^i] + \min[\delta^j y_j^{\{i,j\}B}, \gamma^j] \qquad \text{(TCNC)}^5$$

where $\delta^i, \delta^j, \gamma^i, \gamma^j > 0$. In words, the transaction technology looks like this: Trading post $\{i, j\}$ makes a market in goods i and j, buying each good in order to resell it. It incurs transaction costs in good 0. These costs vary directly (in proportions δ^i, δ^j) with volume of trade at low volume and then hit a ceiling after which they do not increase with trading volume. The specification in (TCNC) is an extreme case: zero marginal transaction cost beyond the ceiling. Adding additional linear terms would represent a more general case.

Since the trading posts in this economy have nonconvex transaction technologies, a competitive equilibrium is not an appropriate solution concept. The equilibrium notion used is an average cost pricing equilibrium resulting in zero profits for the typical trading post firm. The rationale for this choice of equilibrium concept is the threat of entry (by other similar firms) if any economic rent is actually earned. The presence of potential entrants and their actions is not explicitly modeled.

An *average cost pricing equilibrium* consists of $q_{(i)0}^{o\{i,j\}}, q_{(j)0}^{o\{i,j\}}, q_i^{o\{i,j\}}, q_j^{o\{i,j\}}$, $1 \le i, j \le N$, so that :

- For each household h, there is a utility optimizing plan $b_n^{oh\{i,j\}}, s_n^{oh\{i,j\}}$, (subject to T.i, T.ii [or T.ii′ for $h \epsilon H^0$], T.iii) so that $\Sigma_h b_n^{oh\{i,j\}} = y_n^{o\{i,j\}S}$, $\Sigma_h s_n^{oh\{i,j\}} = y_n^{o\{i,j\}B}$, for each $\{i, j\}$, each n, where
- $y_n^{o\{i,j\}S} \le y_n^{o\{i,j\}B}, n = i, j$.
- $y_0^{o\{i,j\}B}$ can be divided into two parts, $y_{(i)0}^{o\{i,j\}B} \ge 0, y_{(j)0}^{o\{i,j\}B} \ge 0$, so that $y_{(i)0}^{o\{i,j\}B} + y_{(j)0}^{o\{i,j\}B} = y_0^{o\{i,j\}B} = C^{\{i,j\}}$.
- $q_{(i)0}^{o\{i,j\}} \ y_{(i)0}^{o\{i,j\}B} = y_i^{o\{i,j\}B} - q_j^{o\{i,j\}} \ y_j^{o\{i,j\}B}$. $q_{(j)0}^{o\{i,j\}} \ y_{(j)0}^{o\{i,j\}B} = y_j^{o\{i,j\}B} - q_i^{o\{i,j\}} y_i^{o\{i,j\}B}$.

[5] (TCNC) is intended as a mnemonic for non-convex transaction cost.

Let κ be a positive integer, $2 < \kappa < (N/2)$. Let $H^{\kappa} = \{[m, m \oplus i] | m = 1, 2, ..., N; i = 1, 2, ..., \kappa; r_m^{[m,m\oplus i]} = A > 0\}$. H^{κ} is a set of κN households without double coincidence of wants. One way to visualize H^{κ}'s situation is to think of the households arrayed in a circle clockwise, each one's position designated by endowment. They can arrange a Pareto improving redistribution by each taking his endowment and sending it i places counterclockwise. However, reflecting the absence of double coincidence of wants, if each of the housheholds in H^{κ} goes to the trading post where his endowment is traded against his desired good, he finds himself alone. He's dealing on a thin market. The following Example 6.1 demonstrates that, with scale economies in transaction cost, virtually any good can become money; the designation is self-confirming.

Example 6.1. Let the population of households be $H = H^0 \cup H^{\kappa}$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $0 < \delta^i < +\infty$ all $i = 1, 2, ..., N$. Let $\frac{\gamma^i + \gamma^j}{\kappa A} < \frac{2}{3}$ and $\left(1 - \frac{\gamma^i + \gamma^j}{\kappa A}\right) > (1 - \delta^j)(1 - \delta^i)$ for all all $i \neq j$, $i, j = 1, 2...., N$. Then for each $i = 1, 2, ..., N$ there is a monetary average cost pricing equilibrium with good i as the unique 'money'.

Demonstration of Example 6.1. Choose an arbitrary $i = 1, 2, ..., N$ as 'money.' For all $j \neq i$, $j = 1, 2, ..., N$, let $q_i^{\{i,j\}} = 1$, $q_j^{\{i,j\}} = 1 - \frac{\gamma^i + \gamma^j}{\kappa A}$. For all j, and $k = 1, 2, ..., N, j \neq k \neq i, q_j^{\{j,k\}} = 1 - \delta^j, q_k^{\{j,k\}} = 1 - \delta^k$. For $1 \leq \ell \leq \kappa$, let $s_m^{[m,m\oplus\ell]\{i,m\}} = A$, $b_i^{[m,m\oplus\ell]\{i,m\}} = q_i^{\{i,m\}} A$, $s_i^{[m,m\oplus\ell]\{i,m\oplus\ell\}} = q_i^{\{i,m\}} A$, $b_{m\oplus\ell}^{[m,m\oplus\ell]\{i,m\oplus\ell\}} = q_i^{\{i,m\}} A$.

What's happening in Example 6.1? Virtually any good i can become money. Monetization comes from liquidity and – with scale economies – liquidity comes from trading volume. The economy is focusing on good i as its common medium of exchange. Since there are scale economies in transaction costs, high trading volume means low average cost with concommitant narrow bid/ask spread. The narrow bid/ask spread is the way the price system confirms and reinforces the choice of i as the medium of exchange. Trader $[m, m \oplus \ell]$ wants to trade good m for good $m \oplus \ell$. He could do so directly, but the transaction costs are heavy, reducing his return on the trade to $A(1 - \delta^m)$ $(1 - \delta^{m\oplus\ell})$ units of $m \oplus \ell$ after starting with A units of good m. The alternative is to trade good m for good i and then trade i for $m \oplus \ell$. This results in $A(1 - [(\gamma^i + \gamma^{m\oplus\ell})/\kappa A])$ units of $m \oplus \ell$. When κ is sufficiently large, that's a much greater return. Because of the narrow bid/ask spread on trade through i, every market with good i on one side attracts high trading volume, κ traders on each side of the market, the high trading volume needed to maintain good i's low bid/ask spreads. The scale economy means that the choice of good i as the common medium of exchange is self-confirming.

The difference between barter and monetary exchange is the contrast between a complex of many thin high transaction cost markets and an array of a smaller number of thick low transaction cost markets dealing in each good versus a unique common medium of exchange. The choice of medium of exchange is self-justifying. Any good i with sufficient scale economy in its transaction technology (with γ^i, the ceiling on its transaction costs, sufficiently low) can become the unique medium of exchange in equilibrium when trading volume κA is sufficiently high. Mint-standardized gold coins (with a low cost transaction technology) or Yap Island stones (high cost technology) may be 'money' depending on which is well established. Sufficient trading volume can confirm either choice.

Recall $H^D = \{[m,n] | m,n = 1,2,3,...,N, m \neq n, r_m^{[m,n]} = A > 0\}$. H^D is a set of $N(N-1)$ households with full double coincidence of wants. The following Example 6.2 demonstrates that even in the presence of double coincidence of wants, sufficient scale economies in transaction costs can lead to monetization of trade, the use of a common medium of exchange.

Example 6.2. Let the population of households be $H = H^0 \cup H^D$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $0 < \delta^i < +\infty$ all $i = 1,2,...,N$. For some i and all j, $1 \leq i,j \leq N$, $i \neq j$, let $\frac{\gamma^i+\gamma^j}{(N-1)A} < \frac{2}{3}$ and $\left(1 - \frac{\gamma^i+\gamma^j}{(N-1)A}\right) > (1 - \delta^j)$, $\left(1 - \frac{\gamma^i+\gamma^j}{(N-1)A}\right) > (1 - \delta^i)$. Then there is a monetary average cost pricing equilibrium with good i as the unique 'money.'

Demonstration of Example 6.2. For all $j \neq i, j = 1,2,...,N$, let $q_i^{\{i,j\}} = 1$, $q_j^{\{i,j\}} = 1 - \frac{\gamma^i+\gamma^j}{(N-1)A}$. For all j, and $k = 1,2,...,N$, $j \neq k \neq i$, $q_j^{\{j,k\}} = 1 - \delta^j$, $q_k^{\{j,k\}} = 1 - \delta^k$. Let $s_m^{[m,n]\{i,m\}} = A$, $b_i^{[m,n]\{i,m\}} = q_i^{\{i,m\}}A$, $s_i^{[m,n]\{i,n\}} = q_i^{\{i,m\}}A$, $b_n^{[m,n]\{i,n\}} = q_i^{\{i,m\}}A$.

What's happening in Example 6.2? Monetization comes from liquidity and – with scale economies – liquidity comes from trading volume. But how can monetization of trade occur where there is double coincidence of wants? The answer is scale economies. Trader $[m,n]$ wants to trade good m for good n. He could do so directly at post $\{m,n\}$, and he'd find a willing trading counterpart at the trading post, so he'd only have to pay for the transaction costs on one side of the trade. But the transaction costs are still substantial, reducing his return on the trade to $A(1-\delta^m)$ units of n after starting with A units of good m. The alternative is to trade good m for good i and then trade i for n. This results in $A(1 - [(\gamma^i + \gamma^n)/(N-1)A])$ units of n. When N is sufficiently large, that's a much greater return. Because of the narrow bid/ask spread on trade through i, every market with good i on one side attracts high trading volume, $N-1$ traders on each side of the market, the high trading volume needed to maintain good i's low bid/ask spreads. The scale economy means that the choice of good i as the common medium of exchange is self-confirming.[6]

7 Government-issued fiat money

In order to study fiat money we introduce a government with the unique power to issue fiat money. Fiat money is intrinsically worthless; it enters no one's utility

[6] For a network externality interpretation see Hahn (1997) which notes that in the presence of market set-up costs, each transactor in the market benefits from the participation of others. Young (1998) assumes the externality without additional explanation. Rey (2001) denotes this interaction the "thick markets externality.

function. But the government is uniquely capable of declaring it acceptable in payment of taxes. Adam Smith (1776) notes "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money…" (v. I, book 2, ch. 2). Abba Lerner (1947) comments, "The modern state can make anything it chooses generally acceptable as money...if the state is willing to accept the proposed money in payment of taxes."

Taxation – and fiat money's guaranteed value in payment of taxes – explains the positive equilibrium value of fiat money.[7] Scale economies explain its uniqueness as the medium of exchange.

As an economic agent, government is denoted G. Government sells tax receipts, the $N + 1^{\text{st}}$ good. It also sells good $N + 2$, an intrinsically worthless instrument, (latent) fiat money, that government undertakes to accept in payment of taxes, that is, in exchange for $N+1$. The typical household $[m, n]$ in H^1 or H^κ desires to purchase tax receipts to the extent it prefers not to have a quarrel with the government's tax authorities. Government sets a target tax receipt purchase by the taxayer of $\tau^{[m,n]}$. Then we rewrite $[m, n]$'s utility function as:

$$u^{[m,n]}(x) = \sum_{i=0, i\neq n}^{N} x_i + 3x_n - 10[\max[(\tau^{[m,n]} - x_{N+1}^{[m,n]}), 0]] \qquad (UT)$$

That is, household $[m, n]$ values paying his taxes with a positive marginal utility up to his tax bill $\tau^{[m,n]}$ and with zero marginal utility for tax payments thereafter. Government uses its revenue to purchase a variety of goods $n = 1, ..., N$, in the amount x_n^G.

Good $N+2$ good represents latent fiat money. Government, G, sells $N+1$ (tax receipts) for $N + 2$ at a fixed ratio of one-for-one. The trading post $\{N + 1, N + 2\}$ where tax receipts are traded for $N + 2$ operates with zero transaction cost. Acceptability in payment of taxes ensures $N + 2$'s positive value. If, in addition, $N + 2$ is assumed to have sufficiently low transaction cost, then it becomes the common medium of exchange. Thus the existence of a fiat money equilibrium with assumed low linear transaction cost is merely an application of Example 3.1 and need not be repeated here.

Government-issued fiat money is typically the unique common medium of exchange: in the US virtually all transactions are denominated in US dollars; in the UK virtually all (nonfinancial) transactions are denominated in pounds sterling. The virtual uniqueness of the monetary instrument is not merely a possibility; it seems to be a general fact. Dollars, euros, pounds sterling, and other government-issued fiat money's all seem to have similar low transaction costs. But in any single market economy precisely one of these instruments is likely to be the unique common medium of exchange. Example 7.1 harnesses scale economy to explain why fiat money is (almost universally) the unique common medium of exchange.

Particularly in the case of scale economies in the transactions technology, there is a strong tendency to multiple equilibria (recall Example 6.1). This creates an

[7] See also Li and Wright (1998) and Starr (1974).

interest in determining which of the several equilibria the economy will actually select. One solution to this problem is to posit an adjustment process to equilibrium that makes the choice. Hence we use the following *Tatonnement adjustment process for average cost pricing equilibrium:* Prices will be adjusted by an average cost pricing auctioneer. Specify the following adjustment process for prices.

Step 0. The starting point is somewhat arbitrary. In each pairwise market the bid/ask spread is set at average cost for low trading volume.

Cycle 1, step 1. Households compute their desired trades at the posted prices and report them for each pairwise market.

Step 2. Average costs (and average cost prices) are computed for each pairwise market based on the outcome of STEP 1. Prices are adjusted upward for goods in excess demand at a trading post, downward for goods in excess supply, with the bid/ask spread adjusted to average cost.

Cycle 2. Repeat Step 1 (at the new posted prices) and STEP 2.

Cycle 3, Cycle 4. .... repeat until the process converges.

This plausible adjustment process explains why government-issued fiat money becomes the unique common medium of exchange – and would do so even in the absence of legal tender rules. Government has two distinctive characteristics: it has the power to support the value of fiat money by making it acceptable in payment of taxes; it is a large economic presence undertaking a high volume of transactions in the economy. Hence, government can make its fiat money the common medium of exchange merely by using it as such. The scale economies implied will make fiat money the low transaction cost instrument and hence the most suitable medium of exchange, not just for government but for all transactors.

Example 7.1. Let the population of households be $H = H^0 \cup H^\kappa$. Let $u^{[m,n]}$ be described by (UT). Let $\tau^o > 0$ be a constant. Let $0 < \tau^{[m,n]} = \tau^o < A(1 - \delta^{N+2})(1 - \delta^m)$, all $[m, n] \epsilon H^\kappa$. Let $x_n^G = \kappa\tau^o q_{N+2}^{\{N+2,n\}}$ all $n = 1, 2, ..., N$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $(\gamma^{N+2}/\kappa\tau^o) < \delta^i < 1/3$ all $i = 1, 2, ..., N$. Then a monetary average cost pricing equilibrium with taxation with good $N + 2$ as ‘money’ is the unique limit point of the tatonnement adjustment.

Demonstration of Example 7.1.

Step 0. For $n \neq m$, set $q_n^{\{m,n\}} = (1 - \delta^n)$.

Cycle 1, Step 1.
For $i = 1, 2, ..., \kappa$, let $s_n^{[n,n\oplus i]\{n,n\oplus\}} = A - \left(\tau^o/q_n^{\{N+2,n\}}\right)$, $b_{n\oplus i}^{[n,n\oplus i]\{n,n\oplus i\}} = \left(A - \left(\tau^o/q_n^{\{N+2,n\}}\right)\right) q_n^{\{n,n\oplus i\}}$, $s_{N+2}^{[n,n\oplus i]\{N+2,N+1\}} = \tau^o = b_{N+1}^{[n,n\oplus i]\{N+2,N+1\}}$; $b_{N+2}^{[n,n\oplus i]\{N+2,n\}} = \tau^o$, $s_n^{[n,n\oplus i]\{N+2,n\}} = \tau^o/q_n^{\{N+2,n\}}$. For $n = 1, 2, ..., N$, let $s_{N+2}^{G\{N+2,n\}} = \kappa\tau^o$, $b_n^{G\{N+2,n\}} = \kappa\tau^o\ q_{N+2}^{\{N+2,n\}}$.

Cycle 1, Step 2.

For $n, m \neq N+2$, $n \neq m$, set $q_n^{\{m,n\}} = (1-\delta^n)$. $q_n^{\{N+2,n\}} = (1 - \min[\delta^n, \gamma^n/\kappa\tau^o])(1-\gamma^{N+2}/\kappa\tau^o)$, $q_{N+2}^{\{N+2,n\}} = 1$.

Cycle 2, Step 1.
For $n = 1, 2, ..., N$, let $s_{N+2}^{G\{N+2,n\}} = \kappa\tau^o$, $b_n^{G\{N+2,n\}} = \kappa\tau^o\ q_{N+2}^{\{N+2,n\}}$; $s_{N+1}^{G\{N+1,N+2\}} = N\kappa\tau^o$, $b_{N+2}^{G\{N+1,N+2\}} = N\kappa\tau^o$; $b_{N+1}^{[n,n\oplus i]\{N+2,N+1\}} = \tau^o$, $s_{N+2}^{[n,n\oplus i]\{N+2,N+1\}} = \tau^o$; $s_n^{[n,n\oplus i]\{N+2,n\}} = A$, $b_{N+2}^{[n,n\oplus i]\{n,N+2\}} = Aq_n^{\{N+2,n\}}$; $s_{N+2}^{[n,n\oplus i]\{n\oplus i,N+2\}} = Aq_n^{\{N+2,n\}} - \tau^o$, $b_{n\oplus i}^{[n,n\oplus i]\{n\oplus i,N+2\}} = \left(Aq_n^{\{N+2,n\}} - \tau^o\right) q_{N+2}^{\{n\oplus i,N+2\}}$.

Cycle 2, Step 2. For $n, m \neq N+2$, $n \neq m$, set $q_n^{\{m,n\}} = (1-\delta^n)$. $q_n^{\{N+2,n\}} = (1 - \min[\delta^n, \gamma^n/\kappa A])\ (1-\gamma^{\{N+2\}}/\kappa A)$, $q_{\{N+2\}}^{\{N+2,n\}} = 1$.

Cycle 3, Step 1. Repeat Cycle 2, Step 1.

Cycle 3, Step 2. Repeat Cycle 2, Step 2.
Convergence.

What's happening in Example 7.1? Scale economies are taking their course! Government expenditures in all goods markets in exchange for $N+2$ (and large household demand to acquire $N+2$ to finance tax payments) result in a large trading volume on the trading posts for good $N+2$ versus $n = 1, ..., N$. Volume is large enough that scale economies kick in. The average cost pricing auctioneer adjusts prices, the bid/ask spread, to reflect the scale economies. The bid/ask spreads incurred on trading m for $m \oplus i$ by way of good $N+2$ become considerably narrower than on trading m for $m \oplus i$ directly. The price system then directs each household to the market $\{m, N+2\}$ where its endowment is traded against good $N+2$. The household sells all its endowment there for $N+2$ and trades $N+2$ subsequently for tax payments and desired consumption. Scale economy has turned $N+2$ from a mere tax payment coupon into 'money,' the unique universally used common medium of exchange.

8 Conclusion

The monetary structure of trade in general equilibrium, the uniqueness of money, and the existence of a fiat money equilibrium can be derived from elementary price theory. The monetary character of trade, the existence of a common medium of exchange in economic equilibrium, can be logically derived from a (non-monetary) Arrow-Debreu Walrasian model through the addition of two constructs: segmented markets with multiple budget constraints (one at each transaction) and transaction costs. The multiplicity of budget constraints creates a demand for a carrier of value (medium of exchange) between transactions. Money (the common medium of exchange) arises endogenously as the most liquid (lowest transaction cost) asset. Government-issued fiat money derives its value from acceptability in payment of taxes. Uniqueness of the monetary instrument (fiat or commodity money) in equilibrium comes from scale economies in transaction cost.

Table 1. Equilibrium monetary structure

Demand structure	Returns to scale in transaction technology	
	Linear transaction technology	Increasing returns transaction technology
Absence of double Coincidence of wants	Monetary equilibrium where the low transaction cost instrument becomes 'money' (Example 3.1); Possibly multiple 'moneys' (Example 3.2)	Monetary equilibrium with unique 'money' (Example 6.1)
Absence of double Coincidence of wants with fiat money	Fiat money equilibrium if fiat money is the low transaction cost instrument (Apply Example 3.1)	Fiat money equilibrium ('money' is unique) when tax payments and government purchases are sufficiently large (Example 7.1)
Full double coincidence of wants	Nonmonetary equilibrium (Example 4.1)	Monetary equilibrium with unique 'money' (Example 6.2)

The taxonomy of cases developed is depicted in Table 1.

Absent double coincidence of wants, with linear transaction costs, a low transaction cost instrument is endogenously chosen as a medium of exchange. In the case of linear transaction costs, absence of double coincidence of wants is essential to monetary equilibrium. Alternatively scale economies in transaction cost (nonconvex transaction technology) lead to a corner solution, uniqueness of the common medium of exchange. Fiat money derives its positive value from acceptability in payment of taxes. Fiat money becomes the unique common medium of exchange when government taxation and purchases are sufficiently large that scale economies in transaction costs make it the low (average) transaction cost instrument.

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