Economics 246 — Fall 2006

International Macroeconomics

Problem Set 1

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Due:	**Wed, October 18**
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1 Productivity Shocks with Initially Unbalanced Current Accounts

There are two periods and two countries. Home produces with $Y = AF(K)$ and Foreign with $Y^* = A^*F(K^*)$, where A and A^* are productivity parameters. Both countries' representative agents have the same period utility so that $U_1 = u(C_1) + \beta u(C_2)$ and $U_1^* = u(C_1^*) + \beta u(C_2^*)$. Assume period utility $u(\cdot)$ to be *isoelastic*. International financial market clearing $S_1 + S_1^* = I_1 + I_1^*$ (or $CA_1 = -CA_1^*$) determines the world interest rate r.

1. State the intertemporal optimality conditions for production and show that an anticipated productivity shock dA_2/A_2 to production in Home changes equilibrium investment and the interest rate in the following way

$$r\frac{\mathrm{d}A_2}{A_2} + \left(\frac{\partial I_1}{\partial r}\right)^{-1} \mathrm{d}I_1 - \mathrm{d}r = 0.$$

   Derive the according equation for Foreign and dA_2^*/A_2^*.

2. State the intertemporal optimality conditions for consumption and totally differentiate them with respect to $\mathrm{d}C_1$, $\mathrm{d}C_1^*$, $\mathrm{d}r$, $\mathrm{d}A_2$ and $\mathrm{d}A_2^*$. Use $CA_1 = Y_1 - C_1 - I_1$ and $CA_1 = -CA_1^*$ to restate the total derivatives in terms of $\mathrm{d}CA_1$, $\mathrm{d}r$, $\mathrm{d}A_2$ and $\mathrm{d}A_2^*$.

3. Use the intertemporal optimality conditions for consumption, along with your results in 1 and 2, to show that an anticipated productivity shock dA_2/A_2 to production in Home changes the equilibrium current account level and the interest rate in the following way

$$D\,\frac{\mathrm{d}A_2}{A_2} - M\,\mathrm{d}r + V\,\mathrm{d}CA_1 = 0,$$

   for some functions V, D, and M. Derive the according equation for Foreign and dA_2^*/A_2^* (with V^*, D^*, M^*) but using $CA_1 = -CA_1^*$. Derive the signs of V, V^*, D and D^*. On what do the signs of M and M^* depend?

4. Show that anticipated productivity shocks to Home and Foreign have the following effect on the equilibrium interest rate and the Home current account:

$$\begin{aligned}
\mathrm{d}r &= \frac{1}{\frac{M}{V}+\frac{M^*}{V^*}}\left(\frac{D}{V}\frac{\mathrm{d}A_2}{A_2}+\frac{D^*}{V^*}\frac{\mathrm{d}A_2^*}{A_2^*}\right) \\
\mathrm{d}CA_1 &= -\frac{1}{\frac{V}{M}+\frac{V^*}{M^*}}\left(\frac{D}{M}\frac{\mathrm{d}A_2}{A_2}-\frac{D^*}{M^*}\frac{\mathrm{d}A_2^*}{A_2^*}\right).
\end{aligned}$$

   Prove that $\mathrm{d}r > 0$ for *isoelastic* utility.

5. Suppose Home runs a current account surplus $CA_1 > 0$ during period 1. How does an anticipated increase in Home productivity A_2 during period 2 affect the current accounts in period 1? Is the effect unambiguous? Why or why not? [*Hint*: You may want to use $\mathrm{d}r$ from 4 to derive $\mathrm{d}CA_1$.]

6. Suppose again Home runs a current account surplus $CA_1 > 0$ during period 1. How does an anticipated increase in Foreign productivity A_2^* during period 2 affect the current accounts in period 1? Is the effect unambiguous? Why or why not? [*Hint*: You may want to use $\mathrm{d}r$ from 4 to derive $\mathrm{d}CA_1$.]

7. Consider anticipated and equal proportional increases in Home and Foreign productivity $dA_2/A_2 = dA_2^*/A_2^*$. How does this change affect worldwide investment $I_1 + I_1^*$ during period 1? Does the answer depend on the elasticity of intertemporal substitution? Is the effect unambiguous? Why or why not?

2 Exponential Period Utility

There are two periods. A country's representative household has the exponential period utility function

$$u(C) = -\gamma \exp(-C/\gamma)$$

with $\gamma \in (0, \infty)$ and maximizes lifetime utility $U_1 = u(C_1) + \beta u(C_2)$ subject to

$$C_1 + RC_2 = Y_1 + RY_2 \equiv W,$$

where $R \equiv 1/(1+r)$ is the price of tomorrow's consumption in terms of today's consumption and W is initial wealth. The value of W depends on R.

1. Derive the Euler equation and solve it for C_2 as a function of C_1, R and β.

2. What is the optimal level of C_1 considering W, R and β as given?

3. Differentiate this consumption function of C_1 with respect to R (differentiate W with respect to R too) and show that

$$\frac{dC_1}{dR} = -\frac{C_1}{1+R} + \frac{Y_2}{1+R} + \frac{\gamma}{1+R}\big(1 - \ln(\beta/R)\big)$$

4. Derive the intertemporal elasticity of substitution of the exponential period utility $(-u'(C)/Cu''(C))$.

5. Use this result to show that the derivative dC_1/dR in part 3 can be expressed as
$$\frac{dC_1}{dR} = \frac{\sigma(C_2)\, C_2}{1+R} - \frac{C_2}{1+R} + \frac{Y_2}{1+R}.$$
Interpret the three additive terms in this derivative.

3 Stochastic Current Account Model

There are infinitely many periods. A country's representative household has the linear-quadratic period utility function

$$u(C) = C - \frac{a_0}{2}C^2$$

with $a_0 \in (0, \infty)$ and maximizes lifetime utility

$$U_t = \mathbb{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t} u\left(C_s\right)\right]$$

subject to

$$CA_s = B_{s+1} - B_s = rB_s + \tilde{Z}_s - C_s \quad \forall s \geq t$$

where $R \equiv 1/(1+r) = \beta$ and $\tilde{Z}_s(\equiv \tilde{Y}_s - \tilde{G}_s - \tilde{I}_s)$ is *random* net output.

1. Derive the stochastic Euler equations and show that C_t satisfies
$$C_t = rR\left((1+r)B_t + \sum_{s=t}^{\infty} R^{s-t}\mathbb{E}_t[\tilde{Z}_s]\right)$$

2. Show that $CA_t \equiv B_{t+1} - B_t = \tilde{Z}_t - \mathbb{E}_t[\hat{\tilde{Z}}_t]$, where the hat denotes the permanent level of the variable. The permanent level $\hat{X}$ of a random variable $\tilde{X}$ is defined as $\sum_{s=t}^{\infty} R^{s-t}\hat{\tilde{X}} \equiv \mathbb{E}_t\left[\sum_{s=t}^{\infty} R^{s-t}\tilde{X}_s\right]$.

3. Define $\Delta\tilde{Z}_s \equiv \tilde{Z}_{s+1} - \tilde{Z}_s$ and suppose that $\lim_{T\to\infty} R^T\mathbb{E}_t\left[\tilde{Z}_{t+T}\right] = 0$. Show that the current account follows a martingale, that is: show that current account innovations (unexpected changes to the current account) are unrelated to any past realizations of state variables.

   Hint: Show that the current account can be rewritten as
$$CA_t = -R\sum_{s=t}^{\infty} R^{s-t}\mathbb{E}_t\left[\Delta\tilde{Z}_s\right]$$
   for $\lim_{T\to\infty} R^T\mathbb{E}_t\left[\tilde{Z}_{t+T}\right] = 0$ and find $CA_t - \mathbb{E}_{t-1}\left[CA_t\right]$.

4. How is this finding related to Hall's (1978) result that consumption follows a martingale?

4 Current Account and Terms of Trade

In a small open economy, the representative individual maximizes the lifetime utility function

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(X_s^{\gamma} M_s^{1-\gamma}\right)^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where X is consumption of an exported good and M consumption of an imported good. The country completely specializes in production of the export good. The endowment of this good is constant at Y. The representative individual faces the fixed world interest rate $r = (1-\beta)/\beta$ in terms of the real consumption index $C = X^{\gamma} M^{1-\gamma}$ (so a loan of 1 real consumption unit today returns $1 + r$ real consumption units tomorrow). There is no investment or government spending.

1. Let p bet the price of the export goods in terms of the import good. So, a rise in p is an improvement in the terms of trade. Show that the consumption-based price index P in terms of imports is

$$P = p^{\gamma}/\gamma^{\gamma}(1-\gamma)^{1-\gamma}.$$

2. Show that the home country's current account identity is

$$B_{t+1} - B_t = rB_t + \frac{p_t(Y - X_t)}{P_t} - \frac{M_t}{P_t}.$$

What is the corresponding intertemporal budget constraint for the representative consumer?

3. Show that utility maximization (Marshallian demands for X_t and M_t) and expenditure minimization (Hicksian demands for X_t and M_t) both imply that $P_t C_t = p_t X_t + M_t$.

4. Derive the first-order conditions of the representative agents's intertemporal consumption problem. What are the optimal paths for C_t, X_t and M_t? For this purpose, express C_t in terms of the representative agent's present net wealth using the intertemporal budget constraint.

5. Suppose initial expectations are that p remains constant over time. There is an unexpected *temporary* fall in the terms of trade from p to $p' < p$. What is the effect on the current account $CA_t = B_{t+1} - B_t$ from part 2? What if p *permanently* drops to p'?

6. Now suppose foreign net wealth B is indexed to the import good M rather than to real consumption. Accordingly, let r denote the own-rate of interest on the import-denominated bond but assume again that $r = (1-\beta)/\beta$. How does a *temporary* drop in the terms of trade from p to $p' < p$ affect the current account now? How do you explain differences, if any, to part 5? [*Hint*: You might find it instructive to consider the effect of a one-percent change in p_t on p_t/P_t and the current account balance under either denomination.]