

# The Extensive Margin of Exporting Products: A Firm-level Analysis\*

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August 3, 2011

## Abstract

We use a panel of Brazilian exporters, their products, and destination markets to document a set of regularities for multi-product exporters. Our data reveal that multi-product firms systematically export their top products across multiple destinations but their lowest-selling products ship in smaller amounts than the lowest-selling products of small exporters. To account for these regularities, we develop a model of firm-product heterogeneity with local entry costs that depend on exporter scope (the number of a firm's products in a market). Estimating this model for the within-firm sales distribution we find that firms face a strong decline in product sales with scope but also that market-specific entry costs drop fast. Counterfactual experiments with globally falling entry costs indicate that a large share of the simulated increase in trade is attributable to declines in the firm's entry cost for the first product.

**Keywords:** International trade; heterogeneous firms; multi-product firms; firm and product panel data; Brazil

**JEL Classification:** F12, L11, F14

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\*We thank David Atkin, Andrew Bernard, Thomas Chaney, Arnaud Costinot, Don Davis, Gilles Duranton, Jonathan Eaton, Elhanan Helpman, Kalina Manova, Gordon Hanson, Lorenzo Caliendo, Sam Kortum, Giovanni Maggi, Marc Melitz, Peter Neary, Jim Rauch, Steve Redding, Kim Ruhl, Peter Schott, Daniel Trefler and Jon Vogel as well as several seminar and conference participants for helpful comments and discussions. Roberto Álvarez kindly shared Chilean exporter and product data for the year 2000, for which we report comparable results to our Brazilian findings in an online Data Appendix at URL [econ.ucsd.edu/muendler/research](http://econ.ucsd.edu/muendler/research). We present an extension of our model to nested CES in an online Technical Appendix at the same URL. Oana Hirakawa and Olga Timoshenko provided excellent research assistance. Muendler and Arkolakis acknowledge NSF support (SES-0550699 and SES-0921673) with gratitude. A part of this paper was written while Arkolakis visited the University of Chicago, whose hospitality is gratefully acknowledged.

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# 1 Introduction

Multi-product firms dominate domestic markets and international trade (Bernard, Redding, and Schott (2011), Goldberg, Khandelwal, Pavcnik, and Topalova (2010)).<sup>1</sup> This preponderance has led to the conjecture that the addition of products within firms might be a significant margin of expansion for international trade.

Previous analysis has been largely descriptive and qualitative. In this paper we quantify the contribution of multi-product exporters to a trade expansion. To do so, we develop a new structural approach within a general equilibrium framework. We document empirically that multi-product exporters dominate markets with their top products but most of their products contribute little to overall trade. In line with this regularity, our simulations suggest that new products of incumbent exporters typically contribute much less to bilateral trade than new exporters. These results are corroborated in a number of empirical exercises that assess the predictions of our setup.

A rigorous quantification of the relevance of multi-product exporters requires both the establishment of salient empirical facts on the product sales distribution within firms, to discern alternative explanations, and the formulation of a consistent theoretical framework. Our paper takes on both tasks. First, on the empirical side, we use novel Brazilian exporter data to infer every exporter's individual product sales by destination and the *exporter scope* (the exporter's number of products at every destination). We establish a robust set of stylized observations within and across markets.<sup>2</sup> In every market, a small number of products accounts for much of a firm's exports and these top products are systematically more likely than the firm's other products to also ship to additional markets. This regularity is at variance with theories of international trade that assume that a firm's product sales are random in every market (Bernard, Redding, and Schott (2011) and Armenter and Koren (2010)).<sup>3</sup> Our data also show that, within each destination, wide-scope exporters sell larger amounts of their top selling products compared to narrow-scope exporters. However, the lowest-selling products of wide-scope exporters sell in small amounts, some for merely a few hundred dollars. Moreover,

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<sup>1</sup>Bernard, Jensen, and Schott (2009) show for U.S. trade data in 2000, for instance, that firms that export more than five products at the HS 10-digit level make up 30 percent of exporting firms but account for 97 percent of all exports. In our Brazilian exporter data for 2000, 25 percent of all manufacturing exporters ship more than ten products at the internationally comparable HS 6-digit level and account for 75 percent of total exports. Similar findings are shared by Iacovone and Javorcik (2008) for Mexico and Álvarez, Faruq, and López (2007) for Chile.

<sup>2</sup>To assess robustness for another country, we use panel data of Chilean exporters in 2000 (see Álvarez, Faruq, and López 2007). We find the regularities confirmed and estimates to be similar, and report them in our online Data Appendix.

<sup>3</sup>Bernard, Redding, and Schott (2011) develop a multi-product firm model with products that have idiosyncratic country-specific demand shocks.

the lowest-selling products of wide-scope exporters sell in systematically smaller amounts than the lowest-selling products of narrow-scope exporters. This fact motivates the existence of local entry costs that depend on firm scope, largely ignored by previous theories.

Second, we develop a parsimonious model consistent with these observations by embedding an Eckel and Neary (2010) production setup into a conventional demand system with constant elasticity of substitution (CES). In particular, the model rests on a single source of firm heterogeneity (productivity) and firms face declining efficiency in supplying their less successful products away from their core competency (see Eckel and Neary (2010) and Mayer, Melitz, and Ottaviano (2011)). The setup implies that a firm's product sales are correlated across its destinations in accordance with the data but also in contrast to other modelling frameworks of multi-product exporters.<sup>4</sup> Our model accommodates the possible cases of both economies and diseconomies of scope in local entry costs.<sup>5</sup> The model offers a tractable generalization of the Melitz (2003) framework, as augmented by Chaney (2008), to multiple products where the firm decides along three export margins: its presence at export destinations, its exporter scope at a destination, and its individual product sales at the destination.

We fit our model's structural equation for within-firm product sales to the data with a flexible maximum-likelihood estimator and obtain novel estimates of entry cost parameters. We compare the predictions of the estimated model to our empirical facts in the cross-firm dimension. The comparison shows that the model approximates well the scope and sales distributions, generating the observed positive association between exporter scope and average product sales across firms. Estimates point to a strong decline in product efficiency with scope. Thus, our model implies that only highly productive firms choose wide scope. Estimated local entry costs exhibit economies of scope for the introduction of additional products within a market. Therefore, our model also implies that small-scale products are only shipped by wide-scope firms, consistent with the fact that wide-scope exporters sell their lowest-selling products in minor amounts.

Having parameterized the model for Brazil, a country close to the world median in exports per capita, we simulate a 25-percent reduction in entry costs and their effect on global trade.<sup>6</sup> We dis-

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<sup>4</sup>Incomplete correlation can readily be built into our model, using random sales shocks per product as developed by Eaton, Kortum, and Kramarz (2010). The benchmark specification in Bernard, Redding, and Schott (2011) makes product heterogeneity market specific so that product sales are uncorrelated across markets. For product sales covariation to be built into that framework, a correlated distribution of product efficiencies would need to be specified.

<sup>5</sup>Seminal references on economies of scope are Panzar and Willig (1977) and (1981). Formally, there are economies of scope if the cost function satisfies  $C(x + y) < C(x) + C(y)$  (if the cost function is subadditive).

<sup>6</sup>After rounds of tariff reductions and drops in transport costs, local entry costs from technical barriers to trade and regulatory protection are thought to be major remaining impediments to trade (Baldwin 2000, Maskus and Wilson 2001). The World Bank estimates in its *Global Economic Prospects 2004* report that trade facilitation would increase world trade

tinguish between a decline in firm entry costs for the first product and a decline in entry costs for an exporter's additional products. Most of the simulated trade increase is due to falling entry costs for the first product—such as one-shot beachhead costs for information acquisition, the setup of certified and accredited testing facilities, investments in technology acquisition for export development, and perhaps brand marketing costs. In contrast, trade is less sensitive to falling entry costs for subsequent products—such as compliance with an individual product's technical requirements, mandatory or voluntary product safety standards, and packaging and labelling procedures, or expenditures for extending marketing and the distribution network to additional products.

These findings suggest that opportunities for reducing entry costs with respect to product standards and technical regulations are perhaps more limited than earlier opportunities from reducing variable trade costs. Overall, a simulated 25-percent reduction in entry costs only results in a less than 1-percent welfare increase. We confirm the small response at the extensive margin of exporting products also for a simulated 25-percent drop in variable trade costs. The within-firm extensive margin of adding products is relatively important for small and medium size exporters but not at the wide-scope exporters that dominate trade.

While intentionally parsimonious, our model is qualitatively consistent with the observed regularities under a set of mild, and empirically confirmed, parameter restrictions. Under the common assumption of Pareto distributed firm productivity, our model preserves desirable predictions of previous trade theory in the aggregate: the model generates a total sales distribution across firms that is Pareto-shaped in the upper tail as in Chaney (2008), and at the country level the model results in a general equilibrium gravity relationship resembling the one in Anderson and van Wincoop (2003) and Eaton and Kortum (2002).

At the micro level, product sales distributions exhibit richer features than we aim to capture. Mayer, Melitz, and Ottaviano (2011) analyze properties of the within-firm sales distribution through simultaneous country-pair and product-pair comparisons and explain them with an Eckel and Neary (2010) production setup under a varying demand elasticity.<sup>7</sup> Eckel, Iacovone, Javorcik, and Neary (2010) study a firm's endogenous investment in product appeal.<sup>8</sup> Amador and Oromolla (2008) and

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by approximately \$377 billion overall, to which an improvement in the regulatory environment would contribute \$83 billion and services sector infrastructure and e-business usage another \$154 billion (World Bank 2003). The World Trade Organization's *World Trade Report 2008* asserts that "with respect to product standards, technical regulations and sanitary and phytosanitary (SPS) measures, considerable opportunities exist for reducing trade costs" (WTO 2008, p. 149).

<sup>7</sup>Feenstra and Ma (2008), Nocke and Yeaple (2006) and Dhingra (2010) study multi-product exporters but do not generate a within-firm sales distribution, which lies at the heart of our analysis.

<sup>8</sup>Exporter scope is socially optimal in these models, as in ours. Thomas (2011), in contrast, documents inefficient

Morales, Sheu, and Zahler (2011) study the sequential entry of multi-product firms into additional export markets, which exhibits path dependence. Our framework emphasizes long-term relationships, similar to earlier trade models, and consequently employs a cross section of exporter data.

Arkolakis, Costinot, and Rodríguez-Clare (2010) show for a wide family of models, which includes ours, that conditional on identical observed trade flows these models predict identical ex-post welfare gains irrespective of firm turnover and product-market reallocation. Their findings also imply, however, that models in that family differ in their predictions for trade flows and welfare with respect to ex-ante changes in fixed entry costs. Our model provides market-specific micro-foundations for these entry costs. The model's tractable setup can be used to compute the impact of rich policy experiments on trade flows and welfare.

The organization of this paper is in six more sections. In Section 2 we describe the data and present key regularities. We introduce the general model in Section 3 and show how it generates the regularities. In Section 4 we derive equilibrium and bilateral trade under a Pareto distribution. We obtain structural estimates of entry cost parameters in Section 5, and simulate their cross sectional predictions. Section 6 applies these estimates to simulate a drop in entry costs. Section 7 concludes.

## 2 Data

Our Brazilian exporter data derive from the universe of customs declarations for merchandise exports during the year 2000 by any firm. From these customs records, we construct a three-dimensional panel of exporters, their respective destination countries, and their export products at the Harmonized System (HS) 6-digit level. We briefly discuss the data sources and characteristics, and then present three main stylized facts that emerge from the data.

### 2.1 Data sources and sample characteristics

In our original exports data from SECEX (*Secretaria de Comércio Exterior*), product codes are 8-digit numbers (under the common Mercosur nomenclature), of which the first six digits coincide with the first six HS digits. We aggregate the data to the HS 6-digit product and firm level so that the

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variation in firm scope for detergent manufacturers across local markets in Western Europe and proposes an agency approach to product adoption.

resulting dataset is widely comparable to data for other countries.<sup>9</sup> To relate our data to product-market information for destination countries and their sectors, we map the HS 6-digit codes to ISIC revision 2 at the two-digit level and link our data to World Trade Flow (WTF) data for the year 2000 (Feenstra, Lipsey, Deng, Ma, and Mo 2005). In 2000, our SECEX data for manufactured merchandise sold by Brazilian firms from any sector (including commercial intermediaries) reaches a coverage of 95.9 percent of Brazilian exports in WTF.

We restrict our sample to manufacturing firms and their exports of manufacturing products, removing intermediaries and their commercial resales of manufactures. The restriction to manufacturing firms and their manufactured products makes our findings closely comparable to Eaton, Kortum, and Kramarz (2004) and Bernard, Redding, and Schott (2011), for example. The group of manufacturing firms covers a substantial fraction of exports (81.7 percent of the WTF manufactures exports).<sup>10</sup> The resulting manufacturing firm sample has 10,215 exporters shipping 3,717 manufacturing products at the 6-digit HS level to 170 foreign destinations, and a total of 162,570 exporter-destination-product observations. Multi-product exporters sell more than 90 percent of all exports from Brazil.

## 2.2 Three regularities

To describe the extensive margin of exporting products, we look at the number of products that a firm sells at each destination as well as a firm’s individual product sales. We decompose a firm  $\omega$ ’s total exports to destination  $d$ ,  $t_d(\omega)$ , into the number of products  $G_d(\omega)$  sold at  $d$ —the *exporter scope* in  $d$ —and the average sales per export product  $a_d(\omega) \equiv t_d(\omega)/G_d(\omega)$  in  $d$ —the *exporter scale* in  $d$ . We elicit three major stylized facts from the data at three levels of aggregation, moving from less to more aggregation. The first fact characterizes the distribution of sales of products within the firm, depicted in Figure 1.<sup>11</sup>

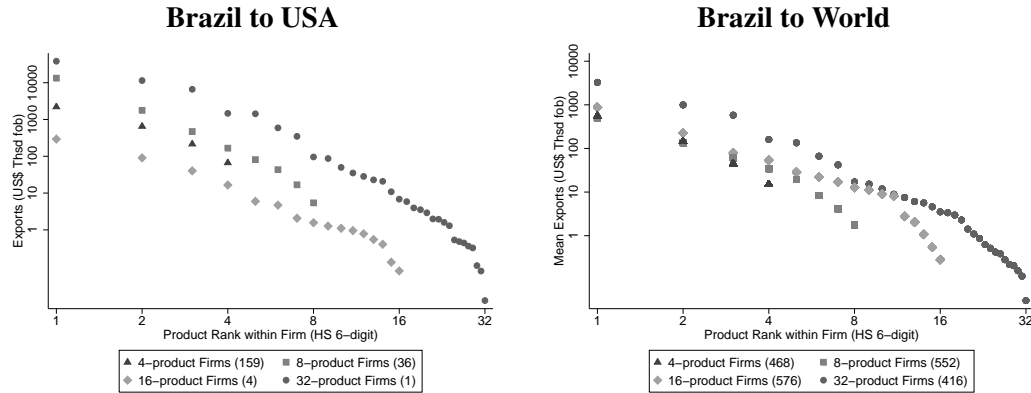
### **Fact 1** *Within firms and destinations,*

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<sup>9</sup>Our online Data Appendix documents that our findings are similar at the common Mercosur nomenclature 8-digit level, which is comparable to the HS 8-digit level.

<sup>10</sup>Exporter behavior in Brazil is strikingly similar to that in leading export countries such as France and the United States (see our online Data Appendix). Appendix D.1 reports summary statistics and documents the dominance of multi-product exporters in total exports. In our online Data Appendix we also report findings from the complementary group of commercial intermediary firms and their exports of manufactures.

<sup>11</sup>Bernard, Redding, and Schott (2011) present evidence for U.S. firms’ sales worldwide comparable to the right-hand side graph in Figure 1. Our analysis shifts attention to regularities by destination.



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. World average in right-hand graph from pooling destinations where firms in a given exporter scope group ship.

Figure 1: Within-firm Sales Distribution

(a) *wide-scope exporters sell large amounts of their top-selling products, with exports concentrated in few products, and*

(b) *wide-scope exporters sell small amounts of their lowest-selling products.*

To plot the Figure we consider firms with the same number of products and rank the products of each firm from top-selling (rank 1) to lowest-selling at a given destination. We then take the average across these firms of each product at a given product rank and plot the logarithm of this value against the logarithm of the rank of the product. The Figure depicts the results for manufacturers that sell exactly 4, 8, 16 or 32 products to Brazil's top export destination in 2000, the United States, or worldwide over all destinations. The worldwide Figures here treat the rest of the world as if it were a single destination (individual plots are similar destination by destination).

The elasticity of individual product sales with respect to the rank of the product is about -2.8 in the United States and -2.6 worldwide, implying that sales fall sharply with rank. Fact 1a describes the concentration of sales. For shipments to the United States, the top-selling product (rank 1) sells on average US\$ 38 million at 32-product firms but only US\$ 2.2 million at 4-product firms.<sup>12</sup> Notice that the contribution of the top-selling products in the total sales of firms is large: for firms with 32

<sup>12</sup>There is considerable small-sample variability within single destinations so that top-product sales may not always increase between firms with increasing scope. In Figure 1 for the United States, for instance, the (four) 16-product firms exhibit untypically low top-product sales compared to 8-product firms, whereas the (nine) 17-product firms do exhibit higher top-product sales compared to the (twenty-two) 9-product firms as expected. Destination aggregates do not exhibit such small-sample variability.

products in the USA, the top three products account, on average, for more than 85 percent of their total sales. This number is 76 percent for the world as one destination.

Fact 1b states that wide-scope exporters tolerate lower sales for their lowest-selling products than narrow-scope exporters. For instance, as illustrated in Figure 1, the lowest-selling product of 32-products exporters to the United States sells on average for merely US\$ 12 in 2000 (rank 32) and 16-products exporters ship just US\$ 77 of their lowest-selling product (rank 16). In contrast, the lowest-selling product of 8 and 4-products exporters (ranks 8 and 4) sells for US\$ 5,400 and US\$ 67,000 respectively.

To document the systematic nature of this feature in Figure 1, we regress the lowest-ranked product's log sales  $p_{dG}(\phi)x_{dG}(\phi)$  on a firm's log exporter scope  $G_{\phi d}$  in a market, conditioning on fixed effects for firm  $\phi$  and destination  $d$ :

$$\ln p_{dG}(\phi)x_{dG}(\phi) = -2.065 \ln G_{\phi d}.$$

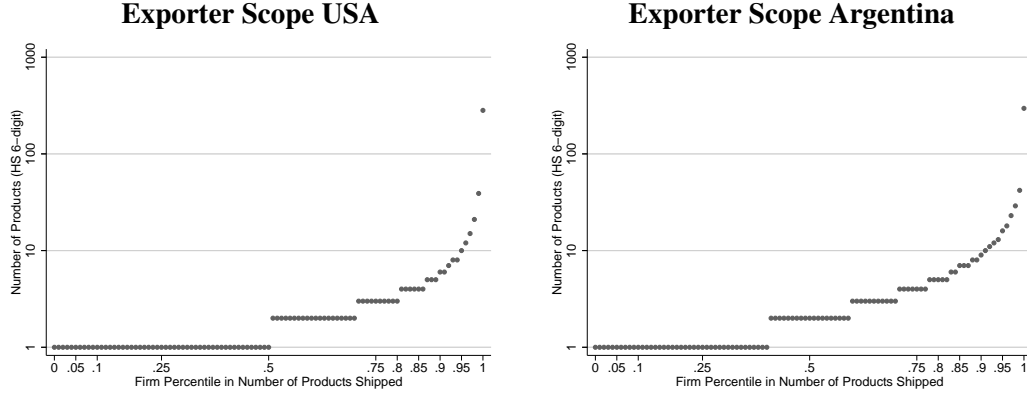
(.015)

The  $R^2$  is .391 (standard error in parentheses clustered at firm level) for 170 destinations and 10,215 firms (46,208 observations). The estimate shows that sales of the lowest-selling product fall with an elasticity of 2.1 as exporter scope in a market widens. The finding is inconsistent with models of multi-product firms where product entry costs are fixed or constant, such as Bernard, Redding, and Schott (2011) or Mayer, Melitz, and Ottaviano (2011), and motivates our choice of product-specific local entry costs. Fact 1b is also closely related to our later simulation findings that falling entry costs induce more trade mostly through entry of new exporters with their first product and that falling barriers to product entry raise trade less than similar relative declines in variable trade costs.

**Fact 2** *Within destinations, there are few wide-scope and many narrow-scope firms.*

To graph the exporter scope distribution, we rank firms according to their exporter scopes in a destination market. The upper panel of Figure 2 plots exporter scope against the scope percentiles for Brazil's top two exporting markets, the United States and Argentina. These plots too are similar for most Brazilian destinations. For instance, the median Brazilian exporter sells one or two products per destination and the mean number of products is around three to four products in individual destinations (see also Table D.1 in the Appendix). Exporter scope is a discrete variable but the overall shape of the distributions approximately resembles that of a power-law distributed variable.

This fact is reminiscent of the findings of Eaton, Kortum, and Kramarz (2010) that in each destination there are few large-sales firms but many small-sales firms. In fact, in our Brazilian data plots



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at HS 6-digit level. In the lower panel, the left-most observations are all exporters; at the next percentile are exporter observations with sales in the top 99 percentiles; up to the right-most observations with exporters whose sales are in the top percentile.

Figure 2: **Exporter Scope and Export Sales Distributions**

of total exports against the total exports percentiles look very similar to the one generated by Eaton, Kortum, and Kramarz (2010).<sup>13</sup> Models that strive to explain the presence of multi-product exporters should also be able to explain the dominant presence of exporters that sell a few products.

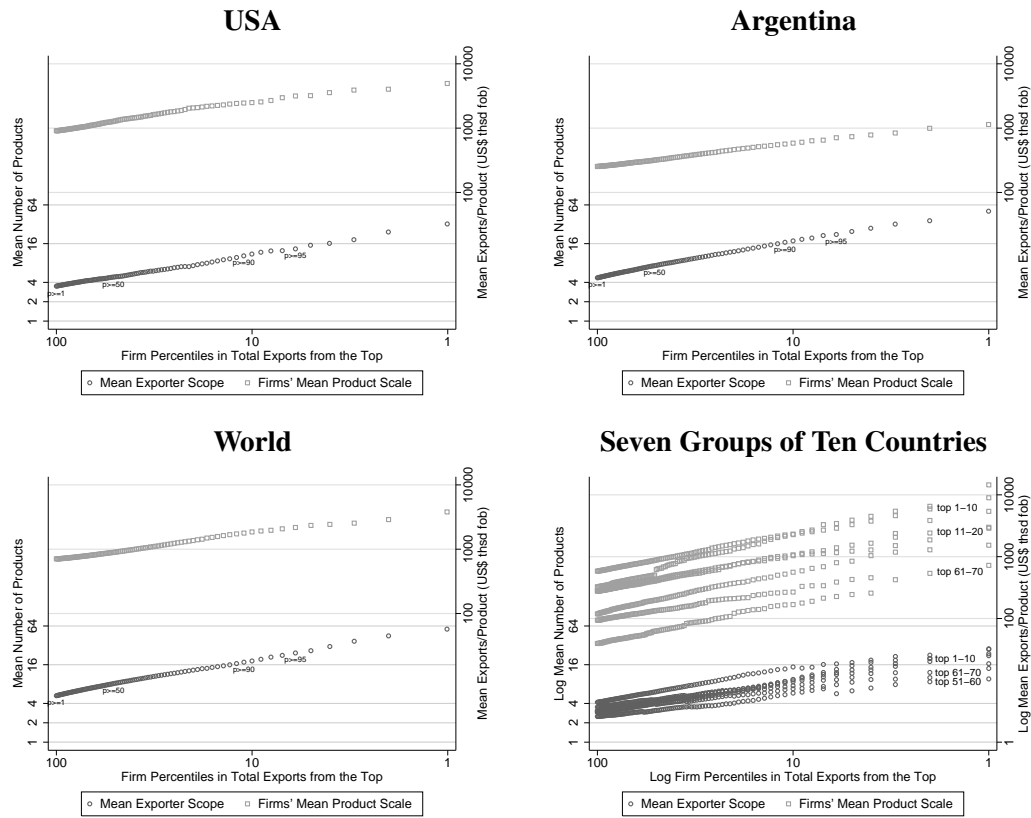
**Fact 3** *Within destinations, mean exporter scope and mean exporter scale are positively associated.*

One might expect from Fact 1b that wide-scope firms would have low average sales per product because they adopt more products with minor sales. The opposite is the case. In Figure 3 we plot firms' mean scope and scope-weighted mean exporter scale at a destination against these firms' rank in total exports at that destination.<sup>14</sup>

On the horizontal axis, we group firms at or above a given total exports percentile. At the origin, we cumulate all firms and plot their mean scope  $\bar{G}_{sd}$  and the scope-weighted mean exporter scale  $\bar{a}_{sd}$  so that the product of the two means yields mean total sales  $\bar{t}_{sd}$ . Then we move upwards in the total-exports ranking of firms, percentile by percentile, dropping from the sample all those firms that are below the next higher total-exports percentile and depict mean exporter scope and mean

<sup>13</sup>Introducing the marketing cost mechanism of Arkolakis (2010) is a straightforward extension of our model and would allow us to match the size distribution of smaller firms as well. For our focus is on the multi-product firm, we abstain from an exploration of small-firm deviations.

<sup>14</sup>Scope-weighted mean exporter scale is  $[\sum_{\omega} G_d(\omega)a_d(\omega)]/\sum_{\omega} G_d(\omega) = \sum_{\omega} t_d(\omega)/\sum_g G_d(\omega)$ . For unweighted mean exporter scale, a similar positive association as depicted in Figure 3 arises. We present those Figures and numerous additional results in our online Data Appendix.



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: World graph is based on pooling all markets. The groups-of-ten graph shows 70 markets (with 100 or more Brazilian exporters), where markets are first ranked by total sales and then lumped to seven groups of ten countries by total-sales rank. Products at the Harmonized-System 6-digit level. Left-most observations are all exporters; at the next percentile are exporter observations with sales in the top 99 percentiles; up to the right-most observations with exporters whose sales are in the top percentile.

Figure 3: Mean Exporter Scale and Mean Exporter Scope

average scale for the higher-ranked group of firms. By construction, the means in Figure 3 are a linear decomposition of the mean exports of firms at a certain percentile.

The log mean scope and log mean scale both increase in the firms' log percentile. The increases are close to linear in the two export markets United States and Argentina and, on average, in the world (treating all destinations as a single market). This log-linear pattern is also visible in groups of ten similarly ranked destinations (we rank the 70 destinations with at least 100 Brazilian exporters by total sales and then lump them to seven groups of ten countries).

Overall, Figure 3 strongly suggests that there is a systematically positive relationship between *average* exporter scale and exporter scope. This quantitative finding is useful to distinguish across theories or across different parameterizations of the same theory. For example, it is easy to show that the Bernard, Redding, and Schott (2011) model, specified with Pareto distributed product sales within the firm, implies that average firm scale is independent of firm scope. Our model will generate a strictly positive relationship between average scale and scope under the empirically relevant parametrization.

### **2.3 Product shipments across destinations**

Before we turn to a firm-level model, we present evidence that a firm's successful products in one market are also its leading products in other markets and that those successful products reach a larger number of markets. These systematic patterns are inconsistent with a simple balls-and-bins model of trade where products are randomly assigned to different destinations, which addresses the constructive critique of Armenter and Koren (2010) in our context. These patterns are also inconsistent with the benchmark version of the stochastic firm-product model of Bernard, Redding, and Schott (2011) where the sales of products are uncorrelated across markets. Instead, these patterns justify the use of a model where firms have some systematic sales advantage for some products, and together with the previous facts, motivate our modelling choice of "core competency" and product-specific entry costs.

For this empirical exercise we use the United States and Argentina as our respective reference countries and look across a firm's further destinations. The United States is Brazil's top export destination in 2000, Argentina is the second most important Brazilian export destination. We present three different types of evidence. First, within a firm, the leading products in the reference market have systematically higher sales than its other products market by market. For each given HS 6-digit product that a firm sells in the United States we correlate the firm-product's U.S. rank with the firm-

Table 1: Overlaps between Reference Countries and Rest of World by Product Rank

Product rank in Ref. country	Reference country: USA				Reference country: Argentina			
	Overlap (1)	Overlap top prd. (2)	#Dest./ firm (3)	#Firms (4)	Overlap (5)	Overlap top prd. (6)	#Dest./ firm (7)	#Firms (8)
1	.83	.83	8.9	2,280	.77	.77	7.8	3,071
2	.54	.77	13.0	1,033	.54	.76	10.7	1,672
4	.36	.73	18.9	368	.38	.67	14.2	797
8	.34	.69	24.1	137	.30	.63	18.5	307
16	.26	.59	24.3	63	.24	.54	22.6	136
32	.24	.53	30.2	22	.22	.50	29.7	48
64	.15	.49	38.9	10	.29	.40	35.9	19
128	.13	.69	42.4	5	.11	.33	43.8	12

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Destination counts in columns 3 and 7 are mean numbers of destinations to which firms with at least as many products as reported for a rank ship. Overlap in columns 1 and 5 is the proportion of destinations that a product of reported rank reaches relative to the overall destination counts (in columns 3 and 7). Overlap in columns 2 and 6 is the proportion of destinations that the top-selling product of firms with at least as many products as reported for a rank reaches relative to the overall destination counts (in columns 3 and 7). Products at the HS 6-digit level, ranked by decreasing export value within firm in reference country. Sample restricted to firm-products that ship to reference country and at least one other destination.

product's rank elsewhere. We find a correlation coefficient of .747 and a Spearman's rank correlation coefficient of .837. For Argentina as reference country, we find even higher coefficients of .785 and .860, respectively.

Second, lower ranked products reach systematically fewer destinations. Table 1 documents that the number of destinations where a firm ships a product drops with the product's rank in the reference country. Consider the top-ranked product in the United States for the 2,280 firms that ship at least one product to the United States (including single-product exporters to the United States). These firms reach 8.9 destinations on average and their top-selling product ships to 83 percent of the destinations that the firms reach with any product. Firms that sell at least two products in the United States reach on average 13.0 destinations but their second-ranked product only ships to a fraction of 54 percent of the destinations reached with any product. This fraction drops to 36 percent for the fourth-ranked product for firms with at least 4 products in the United States and to 13 percent for the 128th ranked product. Similar results are true for Argentina as reference country.

Finally, there is evidence that export scale per product is positively associated with exporter scope at the individual firm-product level, within industries and destinations (and not just across groups of firms as Fact 3 showed). For our sample of manufacturing exporters and their individual manufactured

products, a regression of the log sales per product in a market on the seller’s log exporter scope in the same market, controlling for industry and destination fixed effects, returns a coefficient estimate that is positive (0.072) and statistically significantly different from zero at the one-percent level (clustering at industry level). Wide-scope exporters therefore also receive systematically higher revenues for each individual product at a destination. This finding refutes the hypothesis that a firm is a random collection of products. For a random collection of products, the exporter scale would be independent of the exporter scope in a market.<sup>15</sup>

We now turn to a model of exporting that generates the three stylized facts, and then revisit the data to empirically evaluate the derived relationships.

### 3 A Model of Exporter Scope and Exporter Scale

Our model rests on a single source of firm heterogeneity. Firms sell one or multiple products in the markets where they enter. There are three key ingredients: a firm’s overall productivity that affects all products of the firm worldwide; firm-product specific efficiency that determines individual product sales worldwide; and local fixed entry costs that depend on the number of products that a firm sells in each destination market.

#### 3.1 Consumers

There are  $N$  countries. We label the source country of an export shipment with  $s$  and the export destination with  $d$ . There is a measure of  $L_d$  consumers at destination  $d$ . Consumers have symmetric preferences with a constant elasticity of substitution  $\sigma$  over a continuum of varieties. In our multi-product setting, a conventional “variety” offered by a firm  $\omega$  from source country  $s$  to destination  $d$  is the product composite

$$X_{sd}(\omega) \equiv \left( \sum_{g=1}^{G_{sd}(\omega)} x_{sdg}(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where  $G_{sd}(\omega)$  is the number of products that firm  $\omega$  sells in country  $d$  and  $x_{sdg}(\omega)$  is the quantity of product  $g$  that consumers consume. In marketing terminology, the product composite is often called a

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<sup>15</sup>If firms drew their product sizes from the same distribution (even if this distribution were truncated so that only a fraction of the firm’s products made it to a given market), then the scale of each firm-product would not be related to the firm’s scope.

firm's product line or product mix. We assume that every product line is uniquely offered by a single firm, but a firm may ship different product lines to different destinations.

The consumer's utility at destination  $d$  is

$$\left( \sum_{k=1}^N \int_{\omega \in \Omega_{kd}} X_{kd}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{for } \sigma > 1, \quad (1)$$

where  $\Omega_{kd}$  is the set of firms that ship from source country  $k$  to destination  $d$ . For simplicity we assume that the elasticity of substitution across a firm's products is the same as the elasticity of substitution between varieties of different firms.<sup>16</sup> It is straightforward to generalize the model to consumer preferences with two nests (see Appendix C for a brief outline and our companion paper Arkolakis and Muendler (2010)). If the products in the inner nest, which contains a firm's products, are closer substitutes than the product lines across firms are, then a firm's additional products cannibalize the sales of its infra-marginal products. The cannibalization effect would not substantively alter our main theoretical results and estimation equations.

The representative consumer earns a wage  $w_d$  from inelastically supplying her unit of labor endowment to producers in country  $d$  and receives a per-capita dividend distribution  $\pi_d$  equal to her share  $1/L_d$  in total profits at national firms. We denote total income with  $Y_d = (w_d + \pi_d)L_d$ . The consumer's first-order conditions of utility maximization imply a product demand

$$x_{sdg}(\omega) = \left( \frac{p_{sdg}}{P_d} \right)^{-\sigma} \frac{T_d}{P_d}, \quad (2)$$

where  $p_{sdg}$  is the price of product  $g$  in market  $d$  and we denote by  $T_d$  the total spending of consumer in country  $d$ . In the calibration, we will allow for the possibility that total spending  $T_d$  is different from country output  $Y_d$  so that we use different notation for the two terms. We define the corresponding ideal price index  $P_d$  as

$$P_d \equiv \left[ \sum_{k=1}^N \int_{\omega \in \Omega_{kd}} \sum_{g=1}^{G_{kd}(\omega)} p_{kdg}(\omega)^{-(\sigma-1)} d\omega \right]^{-\frac{1}{\sigma-1}}. \quad (3)$$

## 3.2 Firms

Following Chaney (2008), we assume that there is a continuum of potential producers of measure  $J_s$  in each source country  $s$ . Productivity is the only source of firm heterogeneity so that, under the model

<sup>16</sup>Allanson and Montagna (2005) adopt a similar nested CES form to study the product life-cycle and market structure, and Atkeson and Burstein (2008) use a similar nested CES form in a heterogeneous-firms model of trade but do not consider multi-product firms.

assumptions below, firms of the same type  $\phi$  from country  $s$  face an identical optimization problem in every destination  $d$ . Since all firms with productivity  $\phi$  will make identical decisions in equilibrium, it is convenient to name them by their common characteristic  $\phi$  from now on.

A firm of type  $\phi$  chooses the number of products  $G_{sd}(\phi)$  to sell to a given market  $d$ . The firm makes each product  $g \in \{1, 2, \dots, G_{sd}(\phi)\}$  with a linear production technology, employing local labor with efficiency  $\phi_g$ . When exported, a product incurs a standard iceberg trade cost so that  $\tau_{sd} > 1$  units must be shipped from  $s$  for one unit to arrive at destination  $d$ . We normalize  $\tau_{ss} = 1$  for domestic sales. Note that this iceberg trade cost is common to all firms and to all firm-products shipping from  $s$  to  $d$ .

Without loss of generality we order each firm's products in terms of their efficiency so that  $\phi_1 \geq \phi_2 \geq \dots \geq \phi_{G_{sd}}$ . Ranking products by consumer appeal would generate isomorphic results for within-firm product sales heterogeneity. Under this convention we write the efficiency of the  $g$ -th product of a firm  $\phi$  as

$$\phi_g \equiv \frac{\phi}{h(g)} \quad \text{with } h'(g) > 0. \quad (4)$$

We normalize  $h(1) = 1$  so that  $\phi_1 = \phi$ . We think of the function  $h(g) : [0, +\infty) \rightarrow [1, +\infty)$  as continuous and differentiable but we will consider its values at discrete points  $g = 1, 2, \dots, G_{sd}$ .<sup>17</sup>

The assumption of declining firm-product efficiencies implies that a firm will enter export market  $d$  with the most efficient product first and then expand its scope moving up the marginal-cost ladder product by product. Therefore some firm's products will sell systematically more across markets (as empirically documented in Section 2.3 above). The assumption of declining firm-product efficiency is a common assumption in multi-product models of exporters. Similar models are Eckel and Neary (2010), who define the product with the highest efficiency as the "core competency" of the firm, and Mayer, Melitz, and Ottaviano (2011). Nocke and Yeaple (2006), in contrast, assume that wider scope reduces efficiency for all infra-marginal products.

Related to the marginal-cost schedule  $h(g)$  we define firm  $\phi$ 's product efficiency index in market  $d$  as

$$H(G_{sd}) \equiv \left( \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}. \quad (5)$$

This efficiency index decreases with exporter scope and will play an important role in the firm's optimality conditions for scope choice.

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<sup>17</sup>Considering the function in its whole domain allows us to express various conditions in a general form. The function  $h(g)$  could be considered destination specific but such generality would introduce degrees of freedom that are not required for our analysis.

The firm faces a product-destination specific *incremental local entry cost*  $f_{sd}(g)$  in terms of labor at destination  $d$ . The firm faces this cost as it widens its exporter scope in a destination  $d$ . The cost is zero at zero scope and strictly positive otherwise:<sup>18</sup>

$$f_{sd}(0) = 0 \quad \text{and} \quad f_{sd}(g) > 0 \quad \text{for all } g = 1, 2, \dots, G_{sd}, \quad (6)$$

where  $f_{sd}(g)$  is a continuous function in  $[1, +\infty)$ .

The incremental local entry cost  $f_{sd}(g)$  accommodates fixed costs of production (e.g. with  $0 < f_{ss}(g) < f_{sd}(g)$ ). In a market, the incremental local entry costs  $f_{sd}(g)$  may increase or decrease with exporter scope. But a firm's local entry costs

$$F_{sd}(G_{sd}) = \sum_{g=1}^{G_{sd}} f_{sd}(g)$$

necessarily increase with exporter scope  $G_{sd}$  in country  $d$  because  $f_{sd}(g) > 0$ .<sup>19</sup> We assume that the incremental local entry costs  $f_{sd}(g)$  require labor from the destination country  $d$  so that  $F_{sd}(G_{sd})$  is homogeneous of degree one in  $w_d$ . Combined with the preceding varying firm-product efficiencies, this local entry cost structure allows us to endogenize the exporter scope choice at each destination.

In summary, there are two scope-dependent cost components in our model, the marginal cost schedule  $h(g)$  and the incremental local entry cost  $f_{sd}(g)$ . Suppose for a moment that the incremental local entry cost is constant in market  $d$  and independent of  $g$  with  $f_{sd}(g) = f_{sd}$ . Then a firm in our model faces diseconomies of scope in market  $d$  because the marginal-cost schedule  $h(g)$  strictly increases with the product index  $g$ . But, if incremental local entry costs decrease sufficiently strongly with  $g$ , there could be overall economies of scope in market  $d$ .

Before we proceed to firm optimization, we introduce a parameterized example for these functions that will allow us to quantitatively match the patterns that we observe in the Brazilian data. We set

$$\begin{aligned} f_{sd}(g) &= f_{sd} \cdot g^\delta \quad \text{for } \delta \in (-\infty, +\infty) \quad \text{and} \\ h(g) &= g^\alpha \quad \text{for } \alpha \in [0, +\infty). \end{aligned} \quad (7)$$

The choice of these two functions is guided by the log-linear relationships observed in Section 2.2. Introducing the example at this stage will allow us to develop intuition for the role of the parameters  $\delta$

<sup>18</sup>Brambilla (2009) adopts a related specification but its implications are not explored in an equilibrium firm-product model.

<sup>19</sup>This specification accommodates a potentially separate firm-level entry cost (sometimes referred to as a one-time beachhead cost), which can be subsumed in the first product's fixed local entry cost. The only requirement is that our later assumptions on the shape of the local entry cost schedule are satisfied. In continuous product space with nested CES utility, in contrast, local entry costs must be non-zero at zero scope because a firm would otherwise export to all destinations worldwide (see Arkolakis and Muendler 2010, Bernard, Redding, and Schott 2011).

and  $\alpha$ . The parameter  $\delta$  is scope elasticity of local fixed entry cost. The product  $\alpha(\sigma-1)$  is the scope elasticity of product efficiency. Depending on the estimated sign of the sum  $\delta + \alpha(\sigma-1)$ , incremental entry costs in our model will increase or decrease in scope. The estimated value of  $\alpha(\sigma-1)$  will determine how fast the sales drop for additional firm products.

### 3.2.1 Firms

A firm with a productivity  $\phi$  from country  $s$  faces the following optimization problem for selling to destination market  $d$

$$\pi_{sd}(\phi) = \max_{G_{sd}, p_{sdg}} \sum_{g=1}^{G_{sd}} \left( p_{sdg} - \tau_{sd} \frac{w_s}{\phi/h(g)} \right) \left( \frac{p_{sdg}}{P_d} \right)^{-\sigma} \frac{T_d}{P_d} - F_{sd}(G_{sd}).$$

The firm's first-order conditions with respect to individual prices  $p_{sdg}$  imply product prices

$$p_{sdg}(\phi) = \tilde{\sigma} \tau_{sd} w_s h(g)/\phi \quad (8)$$

with an identical markup over marginal cost  $\tilde{\sigma} \equiv \sigma/(\sigma-1) > 1$  for  $\sigma > 1$ .<sup>20</sup>

For profit maximization with respect to exporter scope to be well defined, we assume the following condition to be satisfied.

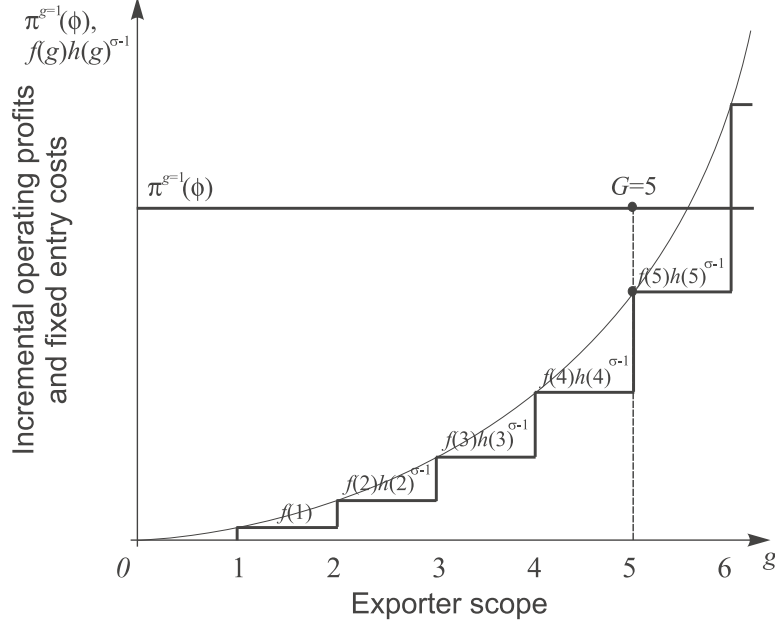
**Assumption 1** (Strictly increasing combined incremental scope costs). *Combined incremental scope costs  $z_{sd}(G) \equiv f_{sd}(G) h(G)^{\sigma-1}$  strictly increase in exporter scope  $G$ .*

Under this assumption, and given the pricing decision from equation (8), the optimal choice for  $G_{sd}(\phi)$  is the largest  $G \in \{0, 1, \dots\}$  such that operating profits from that product equal (or still exceed) the incremental local entry costs:

$$\begin{aligned} \left( \frac{P_d \phi}{\tilde{\sigma} \tau_{sd} w_s} \right)^{\sigma-1} \frac{T_d}{\sigma} \frac{1}{h(G)^{\sigma-1}} &\geq f_{sd}(G) \iff \\ \pi_{sd}^{g=1}(\phi) \equiv \left( \frac{P_d \phi}{\tilde{\sigma} \tau_{sd} w_s} \right)^{\sigma-1} \frac{T_d}{\sigma} &\geq f_{sd}(G) h(G)^{\sigma-1} \equiv z_{sd}(G), \end{aligned} \quad (9)$$

where  $\pi_{sd}^{g=1}(\phi)$  are the operating profits from the core product. In our parameterized example the assumption simply requires that the sum  $\delta + \alpha(\sigma-1)$  is larger than zero since  $z_{sd}(G) = G^{\delta+\alpha(\sigma-1)}$ .

<sup>20</sup>Similarly, in continuous product space (Arkolakis and Muendler 2010) the optimal markup does not vary with exporter scope for constant elasticities of substitution under monopolistic competition (even in the general case of different elasticities of substitution  $\varepsilon \neq \sigma$ ).



Note: Operating profits for the core product are  $\pi^{g=1}(\phi) = [P\phi/(\tilde{\sigma}\tau w)]^{\sigma-1}T/\sigma$ . Combined incremental scope costs  $z(g) \equiv f(g)h(g)^{\sigma-1}$  strictly increase in  $g$  by Assumption 1, with  $f(0) = 0$  and  $h(1) = 1$ .

Figure 4: **Optimal Exporter Scope**

Figure 4 depicts the choice of optimal exporter scope. A firm keeps widening its exporter scope as long as adding products does not reduce total profits. Equivalently, a firm keeps widening scope as long as incremental scope costs  $z_{sd}(g) = f_{sd}(G)h(G)^{\sigma-1}$  are weakly less than the firm's core operating profits  $\pi_{sd}^{g=1}(\phi)$ . In this optimality condition, incremental local entry cost and costs from declining product efficiency enter multiplicatively and their product must increase in scope for a well defined optimum to exist. Assumption 1 is therefore comparable to a second-order condition (for perfectly divisible scope in the continuum version of the model, Assumption 1 is equivalent to the second order condition). When Assumption 1 holds we will say that a firm faces *overall diseconomies of scope*.

We can express the condition for optimal scope more intuitively and evaluate the optimal scope of different firms. Firm  $\phi$  exports from  $s$  to  $d$  iff  $\pi_{sd}(\phi) \geq 0$ . At the break-even point  $\pi_{sd}(\phi) = 0$ , the firm is indifferent between selling its first product to market  $d$  or not selling at all. Equivalently, reformulating the break-even condition and using the above expression for minimum profitable scope, the productivity threshold  $\phi_{sd}^*$  for exporting at all from  $s$  to  $d$  is given by

$$(\phi_{sd}^*)^{\sigma-1} \equiv \frac{\sigma f_{sd}(1)}{T_d} \left( \frac{\tilde{\sigma} \tau_{sd} w_s}{P_d} \right)^{\sigma-1}. \quad (10)$$

In general, using the above definition, we can define the productivity threshold  $\phi_{sd}^{*,G}$  such that firms with  $\phi \geq \phi_{sd}^{*,G}$  sell at least  $G_{sd}$  products as

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} = \frac{z_{sd}(G)}{f_{sd}(1)} \left(\phi_{sd}^*\right)^{\sigma-1}, \quad (11)$$

where  $z_{sd}(G) \equiv f_{sd}(G) h(G)^{\sigma-1}$  and under the convention that  $\phi_{sd}^* \equiv \phi_{sd}^{*,1}$ . Note that if Assumption 1 holds then  $\phi_{sd}^* < \phi_{sd}^{*,2} < \phi_{sd}^{*,3} < \dots$  so that more productive firms introduce more products in a given market. As a result,  $G_{sd}(\phi)$  is a step-function that weakly increases in  $\phi$ .

Using consumer demand (2) and the above definitions, we can write individual product sales as

$$\begin{aligned} p_{sdg}(\phi) x_{sdg}(\phi) &= \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*}\right)^{\sigma-1} h(g)^{-(\sigma-1)} \\ &= \sigma z_{sd}(G_{sd}(\phi)) \left(\frac{\phi}{\phi_{sd}^{*,G}}\right)^{\sigma-1} h(g)^{-(\sigma-1)} \end{aligned} \quad (12)$$

and, summing over  $g$ , total sales as

$$t_{sd}(\phi) = \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*}\right)^{\sigma-1} H(G_{sd}(\phi))^{-(\sigma-1)}. \quad (13)$$

The following proposition summarizes the findings.

**Proposition 1** *If Assumption 1 holds, then for all  $s, d \in \{1, \dots, N\}$*

- *exporter scope  $G_{sd}(\phi)$  is positive and weakly increases in  $\phi$  for  $\phi \geq \phi_{sd}^*$ ;*
- *total firm exports  $t_{sd}(\phi)$  are positive and strictly increase in  $\phi$  for  $\phi \geq \phi_{sd}^*$ .*

**Proof.** The first statement follows directly from the discussion above. The second statement follows because  $H(G_{sd}(\phi))^{-(\sigma-1)}$  strictly increases in  $G_{sd}(\phi)$  and  $G_{sd}(\phi)$  weakly increases in  $\phi$  so that  $t_{sd}(\phi)$  strictly increases in  $\phi$  by (13). ■

The firm's equilibrium choices for total sales  $t_{sd}(\phi)$  and the number of sold products  $G_{sd}(\phi)$  determine its *exporter scale* in market  $d$ ,

$$a_{sd}(\phi) \equiv \frac{t_{sd}(\phi)}{G_{sd}(\phi)} = \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*}\right)^{\sigma-1} \frac{H(G_{sd}(\phi))^{-(\sigma-1)}}{G_{sd}(\phi)}, \quad (14)$$

conditional on exporting from  $s$  to  $d$ . In Section 2, we analyzed the data for the scope-weighted mean exporter scale. An individual firm's exporter scale  $a_{sd}(\phi)$  is tightly related to mean exporter scale: if

$a_{sd}(\phi)$  is a monotonic function of productivity then scope-weighted mean exporter scale is a monotonic function of productivity.<sup>21</sup> In our model, it is easy to work with  $a_{sd}(\phi)$  so we will characterize its analytical properties to describe scope-weighted mean exporter scale. Under an additional restriction,  $a_{sd}(\phi)$  increases in firm productivity  $\phi$  and therefore also in firm total sales:

**Case C1** (Strong overall diseconomies of scope). *Combined incremental scope costs  $z_{sd}(G) \equiv f_{sd}(G) h(G)^{\sigma-1}$  strictly increase in  $G$  with an elasticity*

$$\frac{\partial \ln z_{sd}(G)}{\partial \ln G} > 1.$$

Case C1 is more stringent than Assumption 1 in that the restriction not only requires  $z_{sd}$  to increase with  $G$  but that the increase be more than proportional. In terms of our parameterized example it requires that  $\delta + \alpha(\sigma - 1) > 1$ . We can now state the following result.<sup>22</sup>

**Proposition 2** *If  $z_{sd}(G)$  satisfies Case C1, then exporter scale  $a_{sd}(\phi)$  strictly increases in  $\phi$  at the thresholds  $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}, \dots, \phi_{sd}^{*,G}$ .*

**Proof.** See Appendix A.1. ■

This proposition is particularly informative in situations where  $f_{sd}(g)$  is a strictly decreasing function. In such situations a highly productive firm adds many low-selling products because the firm can generate additional profits from these products as  $f_{sd}(g)$  declines. It is therefore possible in such situations that wide-scope firms would have low exporter scale. Case C1, however, suffices to guarantee that scale increases even if  $f_{sd}(g)$  is a strictly decreasing function: it implies that the efficiencies of marginal products decline so fast that only highly productive firms introduce them. These productive firms have high sales for their top selling products, which means that their overall scale is larger.<sup>23</sup>

The model can also parsimoniously generate the concentration of a firm's sales in its core products. To do that we need to introduce an additional sufficient restriction on  $h(g)$ .

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<sup>21</sup>To see this note that  $(t(\phi) + x) / (G(\phi) + y) \leq x/y \iff t(\phi)/G(\phi) \leq x/y$ . Hence, if  $t/G$  declines, then excluding lower percentiles in scope-weighted mean exporter scale leads to increases in its value.

<sup>22</sup>The proposition demonstrates that the function  $a_{sd}(\phi)$  generically increases in  $\phi$ . This statement is not true for all  $\phi$ . The reason is that the choice of products (the denominator of the function  $a_{sd}(\phi)$ ) is a step function that depends on combined incremental scope costs  $z_{sd}(G)$ . The summation in the numerator of  $a_{sd}(\phi)$  is also a step function but one that only depends on  $h(G)$ , thus rendering stronger statements about  $a_{sd}(\phi)$  elusive.

<sup>23</sup>Case C1 is a sufficient condition for the proposition. Examples can be found where Case C1 fails but  $a_{sd}(\phi)$  generically still increases in  $\phi$ . The result that scale increases with scope is not trivial and does not necessarily generalize to other setups. In the Bernard, Redding, and Schott (2011) multi-product model, for instance, it can be shown that  $a_{sd}(\phi)$  is constant under a Pareto distribution of product-specific demand shocks.

**Case C2** (Bounded firm-product efficiency). *The marginal-cost schedule  $h(\cdot)$  results in bounded firm-product efficiency*

$$\lim_{G \rightarrow \infty} H(G)^{-(\sigma-1)} = \sum_{g=1}^{\infty} h(g)^{-(\sigma-1)} \in (0, +\infty).$$

A number of conventional real analysis tests (e.g. the root test or the ratio test, see Rudin 1976, ch. 3) can be used to determine whether the sum converges by looking at the limiting terms  $h(g)^{-(\sigma-1)}$  as  $g \rightarrow \infty$ . Formally, when this sum converges, the minimum share of a product  $g'$  is bounded from below by the finite number  $h(g')^{-(\sigma-1)} / \sum_{g=1}^{+\infty} h(g)^{-(\sigma-1)}$ . Intuitively, Case C2 implies that the “core” products account for a significant share of total sales, which remain bounded even if many additional products are added.

## 4 Model Equilibrium and Model Predictions

To derive clear predictions for the model equilibrium we specify a Pareto distribution of firm productivity following Helpman, Melitz, and Yeaple (2004) and Chaney (2008). A firm’s productivity  $\phi$  is drawn from a Pareto distribution with a source-country dependent location parameter  $b_s$  and a shape parameter  $\theta$  over the support  $[b_s, +\infty)$  for  $s = 1, \dots, N$ . The cumulative distribution function of  $\phi$  is  $\Pr = 1 - (b_s)^\theta / \phi^\theta$  and the probability density function is  $\theta(b_s)^\theta / \phi^{\theta+1}$ , where more advanced countries are thought to have a higher location parameter  $b_s$ . Therefore the measure of firms selling to country  $d$ , that is the measure of firms with productivity above the threshold  $\phi_{sd}^*$ , is

$$M_{sd} = J_s(b_s)^\theta / (\phi_{sd}^*)^\theta. \quad (15)$$

The probability density function of the conditional distribution of entrants is given by

$$\mu_{sd}(\phi) = \begin{cases} \theta(\phi_{sd}^*)^\theta / \phi^{\theta+1} & \text{if } \phi \geq \phi_{sd}^*, \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

### 4.1 Equilibrium and the gravity equation of trade

Under the Pareto assumption we can compute several aggregate statistics for the model. We denote aggregate bilateral sales of firms from  $s$  to country  $d$  with  $T_{sd}$ . The corresponding average sales per firm are defined as  $\bar{T}_{sd}$ , so that  $T_{sd} = M_{sd}\bar{T}_{sd}$  and

$$\bar{T}_{sd} = \int_{\phi_{sd}^*}^{\infty} t_{sd}(\phi) \mu_{sd}(\phi) d\phi. \quad (17)$$

Similarly, we define average local entry costs as

$$\bar{F}_{sd} = \int_{\phi_{sd}^*} F_{sd}(G_{sd}(\phi)) \mu_{sd}(\phi) d\phi.$$

To compute  $\bar{T}_{sd}$  in terms of fundamentals we need two further necessary assumptions.

**Assumption 2** (Pareto probability mass in low tail). *The Pareto shape parameter satisfies  $\theta > \sigma - 1$ .*

**Assumption 3** (Bounded local entry costs and product efficiency). *Incremental local entry costs and product efficiency satisfy  $\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} \in (0, +\infty)$ , where  $\tilde{\theta} \equiv \theta/(\sigma - 1)$ .*

Assumptions 2 and 3 guarantee that average sales per firm are positive and finite.

**Proposition 3** *Suppose Assumptions 1, 2 and 3 hold. Then for all  $s, d \in \{1, \dots, N\}$ , average sales per firm are a constant multiple of average local entry costs:*

$$\bar{T}_{sd} = \frac{\tilde{\theta} \sigma}{\tilde{\theta} - 1} f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} = \frac{\tilde{\theta} \sigma}{\tilde{\theta} - 1} \bar{F}_{sd}, \quad (18)$$

where  $\tilde{\theta} \equiv \theta/(\sigma - 1)$ .

**Proof.** See Appendix A.2. ■

The share of total local entry costs in total exports  $\bar{F}_{sd}/\bar{T}_{sd}$  only depends on the model's parameters  $\theta$  and  $\sigma$ , even though local entry costs vary by source and destination country. Despite firm-product heterogeneity, bilateral average sales can therefore be summarized with a function only of the parameters  $\theta$  and  $\sigma$  and the properties of average local entry costs  $\bar{F}_{sd}$ .

Finally, we can use definition (15) of  $M_{sd}$  together with definition (10) of  $\phi_{sd}^*$  and expression (18) for average sales to derive bilateral expenditure shares of country  $d$  on products from country  $s$

$$\lambda_{sd} = \frac{M_{sd} \bar{T}_{sd}}{\sum_k M_{kd} \bar{T}_{kd}} = \frac{J_s(b_s)^\theta (w_s \tau_{sd})^{-\theta} f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd}}{\sum_k J_k(b_k)^\theta (w_k \tau_{kd})^{-\theta} f_{kd}(1)^{-\tilde{\theta}} \bar{F}_{kd}}, \quad (19)$$

where  $\tilde{\theta} \equiv \theta/(\sigma - 1)$ , and  $f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd} = \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$  by equation (18).

Remarkably, the elasticity of trade with respect to variable trade costs is  $-\theta$ , as in Eaton and Kortum (2002) and Chaney (2008).<sup>24</sup> Thus, our framework is consistent with bilateral gravity. The

<sup>24</sup>In our model, the elasticity of trade with respect to trade costs is the negative Pareto shape parameter, whereas it is the negative Fréchet shape parameter in Eaton and Kortum (2002).

difference between our model, in terms of aggregate bilateral trade flows, and the framework of Eaton and Kortum (2002) is that fixed costs affect bilateral trade similar to Chaney (2008). Beyond previous work, we provide a micro-foundation as to how entry cost components affect aggregate bilateral trade through the weighted sum  $\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$ . Our model therefore offers a tool to evaluate the responsiveness of overall trade to changes in individual entry cost components.

The partial elasticity  $\eta_{\lambda, f(g)}$  of trade with respect to a product  $g$ 's entry cost component is  $-(\tilde{\theta}-1)$  times the product's share in the weighted sum. To assess the relative importance of the extensive margin of exporting products, relative to firm entry with the core product, we can compare elasticities using the ratio

$$\frac{\eta_{\lambda, f(g)}}{\eta_{\lambda, f(1)}} = \frac{f_{sd}(g)^{-(\tilde{\theta}-1)} h(g)^{-\theta}}{f_{sd}(1)^{-(\tilde{\theta}-1)}}, \quad (20)$$

which becomes  $g^{-\delta(\tilde{\theta}-1)-\alpha\theta}$  in our parametrization. The power is strictly negative if and only if  $\delta + \alpha(\sigma-1) > \delta/\tilde{\theta}$ . It therefore depends on the sign and quantitative magnitude of  $\delta$  whether the elasticity of trade with respect to an additional product's fixed cost is higher or lower than the elasticity of firm entry.

We can also compute mean exporter scope in a destination. For the average number of products to be finite we will need the necessary assumption that

**Assumption 4** (Strongly increasing combined incremental scope costs). *Combined incremental scope costs satisfy  $\sum_{G=1}^{\infty} z_{sd}(G)^{-\tilde{\theta}} \in (0, +\infty)$ .*

This assumption is in general more restrictive than Assumption 1. It requires that combined incremental scope costs  $Z(G)$  increase in  $G$  at a rate asymptotically faster than  $1/\tilde{\theta}$  (a result obtained using the ratio rule, see Rudin 1976, ch. 3).

Mean exporter scope in a destination is<sup>25</sup>

$$\bar{G}_{sd} = \int_{\phi_{sd}^*} G_{sd}(\phi) \theta \frac{(\phi_{sd}^*)^{\theta}}{(\phi)^{\theta+1}} d\phi = f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} z_{sd}(G)^{-\tilde{\theta}}. \quad (21)$$

For our parameterized example, the expression implies that mean exporter scope is invariant to destination market size. This implication resonates with the evidence of highly robust scope distributions

<sup>25</sup>The expression is derived using that

$$\bar{G}_{sd} = \int_{\phi_{sd}^*} G_{sd}(\phi) \theta \frac{(\phi_{sd}^*)^{\theta}}{(\phi)^{\theta+1}} d\phi = (\phi_{sd}^*)^{\theta} \theta \left[ \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \phi^{-(\theta+1)} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} 2\phi^{-(\theta+1)} d\phi + \dots \right].$$

Completing the integration, rearranging terms and using equation (11), we obtain (21).

across destinations of different sizes as presented in Section 2.2.<sup>26</sup>

We turn to the model's equilibrium. Notice that total manufacturing sales of a country  $s$  equal its total sales across all destinations:

$$Y_s = \sum_{k=1}^N \lambda_{sk} T_k. \quad (22)$$

Additionally, Proposition 3 implies that a country's total spending on fixed local entry costs is a constant (source country invariant) share of bilateral exports. This result implies that the share of wages and profits in total income is constant (source country invariant) and given by

$$w_s L_s = \frac{\tilde{\theta}\sigma - 1}{\tilde{\theta}\sigma} Y_s \quad \text{and} \quad \pi_s L_s = \frac{1}{\tilde{\theta}\sigma} Y_s. \quad (23)$$

See Appendix A.4 for a derivation.

This concludes the presentation of equilibrium conditions when trade is balanced ( $Y_d = T_d$ ). We will relax the assumption of balanced trade in our calibration and defer the discussion of the full solution.

## 4.2 Structural equations and relation to empirical regularities

We summarize the assumptions necessary for equilibrium existence under our parameterized example in Table 2.<sup>27</sup> We will now derive a series of structural equations that will be used in the estimation and to illustrate the quantitative predictions of the model. The use of Assumptions 1 through 4 will guarantee that these relationships are well defined and Cases C1 and C2 will help us connect these relationships to the empirical regularities that we uncover.

The optimal number of products for firms with  $\phi \geq \phi_{sd}^*$  is given by (9) and can be written as

$$G_{sd}(\phi) = \text{integer} \left\{ (\phi / \phi_{sd}^*)^{\frac{\sigma-1}{\delta + \alpha(\sigma-1)}} \right\}. \quad (24)$$

<sup>26</sup>To directly test that mean exporter scope is largely unresponsive to market size we present this relationship in Figure D.1 (Appendix D.1). This implication as well as the robust scope and sales distributions is related to the Pareto distribution assumption. Another implication of the Pareto assumption is that the relationship in the model between the number of exporters to a destination and the destination market size becomes linear in logs—a salient feature of both the Brazilian and the French data (see Eaton, Kortum, and Kramarz 2004).

<sup>27</sup>Assumption 4 implies Assumption 1 but it depends on the sign of  $\delta$  whether Assumption 3 implies Assumption 1 (or Assumption 4). The necessary conditions for equilibrium existence can be summarized compactly with

$$\min \left\{ \delta(\tilde{\theta} - 1), \delta\tilde{\theta} \right\} + \alpha\theta > 1 \quad \text{and} \quad \tilde{\theta} > 1.$$

By parametrization (7), the combined fixed cost function  $f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd}(\nu) \equiv f_{sd}(1)^{\tilde{\theta}-1} \sum_{G=1}^{\infty} (G)^{-\nu}$  contains a Riemann zeta function  $\zeta(\nu) \equiv \sum_{G=1}^{\infty} G^{-\nu}$  for a real parameter  $\nu \equiv \tilde{\theta}[\delta + \alpha(\sigma-1)] + \delta$ .

Table 2: Parametric Functional Forms

Condition		Parameter values
Ass. 1	Strictly increasing combined incremental scope costs	$\delta + \alpha(\sigma - 1) > 0$
Ass. 2	Pareto probability mass in low tail	$\theta > \sigma - 1$
Ass. 3	Bounded local entry costs	$\delta + \alpha(\sigma - 1) > (\delta + 1)/\tilde{\theta}$
Ass. 4	Strongly increasing combined incremental scope costs	$\delta + \alpha(\sigma - 1) > 1/\tilde{\theta}$
C1	Strong overall diseconomies of scope	$\delta + \alpha(\sigma - 1) > 1$
C2	Bounded firm-product efficiency	$\alpha(\sigma - 1) > 1$

Note: Functional forms  $f_{sd}(g) = f_{sd} \cdot g^\delta$  and  $h(g) = g^\alpha$  by (7).

Using this relationship and equation (12) we can express optimal sales of the  $g$ -th product in market  $d$  for a firm  $\phi$  as a function of the total number of products that the firm sells in  $d$ :

$$p_{sdg}(\phi)x_{sdg}(\phi) = \sigma f_{sd}(1) G_{sd}(\phi)^{\delta + \alpha(\sigma - 1)} \left( \frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma - 1} g^{-\alpha(\sigma - 1)}. \quad (25)$$

Total sales of a firm with  $G_{sd}$  products are given by

$$t_{sd}(\phi) = \sigma f_{sd}(1) G_{sd}(\phi)^{\delta + \alpha(\sigma - 1)} \left( \frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma - 1} H(G_{sd}(\phi))^{-(\sigma - 1)} \quad (26)$$

where  $H(G_{sd}(\phi))^{-(\sigma - 1)} = \sum_{g=1}^{G_{sd}(\phi)} g^{-\alpha(\sigma - 1)}$ , while exporter scale becomes

$$a_{sd}(\phi) = \sigma f_{sd}(1) G_{sd}(\phi)^{\delta + \alpha(\sigma - 1) - 1} \left( \frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma - 1} H(G_{sd}(\phi))^{-(\sigma - 1)}. \quad (27)$$

Note that the restriction  $\delta + \alpha(\sigma - 1) > 1$  (Case C1) is a sufficient condition for  $a_{sd}(\phi)$  to increase with  $G_{sd}$  but not a necessary one since  $H(G_{sd})^{-(\sigma - 1)}$  also increases in scope.

Using these structural equations we relate the model's predictions to Facts 1 through 3 as presented in Section 2.2. By Fact 1a (Figure 1), a firm's sales within a destination are concentrated in few core products. In the model, the degree of concentration is regulated by how fast  $h(g)$  increases with  $g$  (the elasticity  $\alpha(\sigma - 1)$ , which we will estimate in the next section). Thus, this fact is intimately related to Case C2. Note that Figure 1 also implies that wide-scope exporters sell more of their top-selling products than firms with few products. The model's equation (25) matches this fact under Assumption 1. Finally, Fact 1b that wide-scope exporters sell their lowest-ranked products for small amounts also follows from the model's equation (25). The equation implies that the sales of the

lowest-ranked product  $g = G_{sd}$  fall with scope if and only if local entry costs decline with additional products.

Fact 2 (Figure 2) documents a high frequency of narrow-scope exporters. The model's predictions for the scope distribution of exporters depend critically on the functional forms that we specify for  $f_{sd}(g)$  and  $h(g)$ . Our parametrization (7) implies that  $G_{sd}(\phi)$  is Pareto distributed in the upper tail with shape parameter  $\tilde{\theta}[\delta + \alpha(\sigma - 1)]$ .

As regards conditions under which the model generates a positive relationship between exporter scope and scope-weighted mean exporter scale (Fact 3, Figure 3), the restriction of Case C1 on the combined incremental scope costs  $z_{sd}(G)$  suffices for this to happen (Proposition 2). Case C1 implies that exporter scale  $a_{sd}(\phi)$  and exporter scope  $G_{sd}(\phi)$  are generically positively associated, so scope-weighted mean exporter scale  $\bar{a}_{sd}(\phi)$  and scope are also positively related.

Predictions regarding the overall sales distribution are less dependent on the functional form choices for  $f_{sd}(g)$  or  $h(g)$ . Case C2 is a sufficient condition for equation (26) to exhibit a Pareto distribution in the upper tail. In that case total firm exports are Pareto distributed with shape parameter  $-\tilde{\theta}$  in the upper tail, which is reminiscent of results in the Chaney (2008) and Eaton, Kortum, and Kramarz (2010) models.<sup>28</sup>

We now apply the model to the Brazilian data and estimate parameters of the functional forms in (7). Our estimation target is the within-firm product sales distribution, using the same data that we used to generate Figure 1. To then evaluate the model's performance, we use the within-firm estimates to simulate cross-firm relationships regarding exporter scope and exporter scale as in Figures 2 and 3.

## 5 Estimation

Individual product sales are directly related to exporter scope and the product's rank within the firm under equation (25). This is the basis of our estimation. We augment the equation by a multiplicative sales disturbance  $\epsilon_{sdg}$ , which may be due to unanticipated demand shocks after pre-determined scope

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<sup>28</sup> To see that sales are Pareto distributed notice that the sales percentile  $\Pr$  of a firm with productivity  $\phi$  is given by  $1 - \Pr = (\phi_{sd}^*/\phi)^\theta$ . We can therefore express equation (13) as

$$1 - \Pr = t_{sd}(\phi)^{-\tilde{\theta}} \left( \sigma f_{sd}(1) H(G_{sd}(\phi))^{-(\sigma-1)} \right)^{\tilde{\theta}}.$$

Since a firm's product efficiency index  $H(\cdot)$  converges to a constant as  $t_{sd}(\phi) \rightarrow \infty$  and  $\Pr \rightarrow 1$ , this expression implies that sales are Pareto distributed in the upper tail with a shape parameter  $\tilde{\theta}$ .

choice, or optimization and measurement error. We estimate the equation in its log form:

$$\begin{aligned} \ln p_{sdg}(\phi)x_{sdg}(\phi) &= [\delta + \alpha(\sigma - 1)] \ln G_{sd}(\phi) - \alpha(\sigma - 1) \ln g \\ &+ (\sigma - 1) \ln \left( \phi / \phi_{sd}^{*,G} \right) + \ln \sigma f_{sd}(1) + \ln \epsilon_{sdg}(\phi). \end{aligned} \quad (28)$$

This equation is the product-level counterpart to Figure 1 (Fact 1). Figure 1 presents the cross-firm average of product sales for products with the same rank.

For a given destination, the identification of parameter  $\alpha(\sigma - 1)$  relies on the variation of product sales across products with different ranks. The parameter  $\delta$  is identified from sales of products with a given rank across firms with different exporter scope. Pooling destinations allows us to identify  $\delta$  even as we control for a full set of industry-destination fixed effects. We keep single-product firms in our estimation sample but their presence only contributes to the identification of fixed effects.

We estimate the terms  $\ln \sigma f_{sd}(1)$  with industry-destination fixed effects. Beyond the disturbance  $\ln \epsilon_{sdg}$ , there are two more unobserved components in the estimation equation. The first unobserved component,  $\ln \sigma$ , is common to all firms and is captured by the constant term. The second unobserved component,  $(\sigma - 1) \ln(\phi / \phi_{sd}^{*,G})$ , varies by firm and destination. Our theory implies that  $(\sigma - 1) \ln(\phi / \phi_d^{*,G}) = -(1/\tilde{\theta}) \ln(1 - \text{Pr}_{d\phi}^G)$ . To measure  $1 - \text{Pr}_{d\phi}^G$ , we compute a Brazilian firm's local total-exports percentile among the Brazilian exporters with minimum exporter scope  $G$  and include the log percentile as a regressor.<sup>29</sup>

We first estimate equation (28) with ordinary least squares under industry-destination fixed effects and apply two-way clustering to the standard errors (Cameron, Gelbach, and Miller 2011) at the industry-destination and product-rank levels. Clustering standard errors at the product-rank level accommodates potential cross-destination correlations of product sales, given our earlier observation (Section 2.3) that firms sell the same product with a similar rank across markets. We then allow the coefficients  $\delta_i$ ,  $\alpha_i(\sigma - 1)$  and  $f_i$  to be random coefficients for each industry  $i$ , maintaining all other coefficients constant, and estimate the accordingly generalized equation with maximum restricted likelihood (REML). We report the best linear unbiased predictors across 259 *CNAE* manufacturing industries (standard errors clustered at the industry-destination level).

Table 3: Individual Product Sales

	OLS-FE		REML
	(1)	(2)	(3)
Log # Products	1.332 (.232)	1.217 (.243)	1.279 (.001)
Log Product Rank	-2.634 (.129)	-2.646 (.152)	-2.690 (.001)
Log local Exports Percentile <sup>a</sup>		-1.823 (.180)	-1.745 (.006)
<b>Scope elast. of local entry cost (<math>\delta</math>)</b>	-1.302 (.106)	-1.429 (.093)	-1.411 (.001)
<b>Scope elast. of prod. efficiency (<math>\alpha(\sigma-1)</math>)</b>	2.634 (.129)	2.646 (.152)	2.690 (.001)
Observations	162,570	147,891	147,891
Industry-destination panels	10,129	9,906	
$R^2$ (within)	.562	.692	

<sup>a</sup>Log of firm's local total-exports percentile among exporters with minimum exporter scope.

Source: SECEX 2000, manufacturing firms and their manufactured products.

Notes: Products at the Harmonized-System 6-digit level. Standard errors in parentheses, two-way clustered at the industry-destination and product-rank levels in OLS-FE (Cameron, Gelbach, and Miller 2011).

OLS-FE fixed effects estimation based on regression equation

$$\ln p_{d\phi g} x_{d\phi g} = [\delta + \alpha(\sigma-1)] \ln G_{d\phi} - [\alpha(\sigma-1)] \ln g_{d\phi} + \ln \sigma f_{id}(1) - (1/\tilde{\theta}) \ln(1 - \Pr_{d\phi}^G) + \ln \epsilon_{d\phi g}$$

under industry-destination fixed effects for firm  $\phi$  in industry  $i$  exporting  $G_{d\phi}$  products, each with rank  $g$ , to destination  $d$ . REML (maximum restricted likelihood) estimation of fixed and random coefficients based on

$$\ln p_{d\phi g} x_{d\phi g} = [\delta_i + \alpha_i(\sigma-1)] \ln G_{d\phi} - [\alpha_i(\sigma-1)] \ln g_{d\phi} + \ln \sigma f_i f_d(1) - (1/\tilde{\theta}) \ln(1 - \Pr_{d\phi}^G) + \ln \epsilon_{d\phi g}$$

with random coefficients  $\delta_i$ ,  $\alpha_i(\sigma-1)$  and  $f_i$  (reported BLUP estimates across 259 CNAE manufacturing industries).

## 5.1 Estimates

Table 3 documents that different specifications result in robust estimates of  $\delta + \alpha(\sigma-1)$  and  $\alpha(\sigma-1)$  for Brazilian manufacturers and their manufacturing products. Across specifications in Table 3,  $\delta$  falls in the range from  $-1.30$  to  $-1.43$  and  $\alpha(\sigma-1)$  in the range from  $2.63$  to  $2.69$ . The magnitude of the  $\delta$  estimate implies that incremental local entry costs drop at an elasticity of around  $-1.4$  when manufacturers introduce additional products in a market with a presence (column 3). But firm-product efficiency drops off even faster with an elasticity of around  $2.7$ . Adding the two fixed scope cost coefficients suggests that there are net overall diseconomies of scope with a scope elasticity of  $1.3$ .

<sup>29</sup>Note that the inclusion of a firm's local sales percentiles relative to its Brazilian peers absorbs idiosyncratic firm-destination demand shocks (for an alternative treatment see Crozet, Head, and Mayer 2009).

The coefficient estimates also suggest that both Case C1 and C2 are satisfied in our data.<sup>30</sup>

To assess the robustness of our estimates we perform two more estimation exercises. First, we aggregate the data as in Figure 1 (Fact 1) and fit the according regression equation with non-linear least squares under literature-guided calibrations of the free parameter  $\tilde{\theta}$  (see Appendix 5.2). Regardless of free parameter choice, we find for  $\delta + \alpha(\sigma - 1)$  estimates of 1.60 to 1.61 and for  $\alpha(\sigma - 1)$  estimates of 2.55, close to the estimates in Table 3. Second, we move on to estimate parameters also from the cross section of firms. Note that the slopes of the graphs in Figure 3 (Fact 3) are equal to the respective Pareto shape parameters and observe that our parametrization implies that  $G_{sd}(\phi)$  is Pareto distributed in the upper tail with shape parameter  $\tilde{\theta}[\delta + \alpha(\sigma - 1)]$  by equation (24) and  $a_{sd}(\phi)$  is Pareto distributed in the upper tail with shape parameter  $\tilde{\theta}[\delta + \alpha(\sigma - 1)]/[\delta + \alpha(\sigma - 1) - 1]$  by equation (27). So the ratio of the two Pareto shape parameters also yields an estimate of  $\delta + \alpha(\sigma - 1)$ . For the United States and Argentina in Figure 3, for instance, we fit the graphs to linear relationships as implied by the Pareto distribution and find estimates for  $\delta + \alpha(\sigma - 1)$  of 1.88 and 1.66, respectively, with  $R^2$  above 97 percent in the individual regression. These are reasonably close to the implied estimates of  $\delta + \alpha(\sigma - 1)$  between 1.22 and 1.33 in Table 3.

Supportive evidence on our estimated relationships comes from empirical studies in industrial organization. As regards declining product sales with wider scope, there is evidence that production costs increase for firms that introduce more products (e.g. Bayus and Putsis 1999 for PCs). Our finding of economies of scope in market-specific entry costs echoes related evidence of falling marketing costs with scope in the consumer goods industry (e.g. Morgan and Rego 2009, who define a market segment by NAICS operating code similar to our definition of an HS-6 digit product) and falling distribution costs in the finance industry (e.g. Cummins, Weiss, Xie, and Zi 2010). Beyond a quantification of diseconomies of scope, we are interested in their relationship to exporter entry and implications for the firm size distribution and global trade.

Using our estimates, the power in the partial elasticity ratio (20) is strictly negative because we find  $\delta + \alpha(\sigma - 1) > 0 > \delta/\tilde{\theta}$ . The partial elasticity of trade with respect to an additional product's fixed cost is therefore lower than the elasticity with respect to the core product. In other words, our estimates imply that product entry at multi-product exporters should matter less than firm entry with the core product. We will return to the magnitudes in our simulation and consider the importance of each margin for overall trade.

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<sup>30</sup>We do not use the coefficient estimate on the log local exports percentile to calibrate  $1/\tilde{\theta}$  because the estimate might be affected by unobserved dimensions of firm heterogeneity outside our model.

## 5.2 Quantitative predictions

To illustrate the model’s quantitative predictions, we need to specify two more parameters:  $\tilde{\theta}$  and, given that parameter, either  $\theta$  or  $\sigma$ . Specifying  $\tilde{\theta}$  suffices to determine the firm-level predictions (Facts 1-3) because  $\tilde{\theta}$ ,  $\delta$  and  $\alpha(\sigma - 1)$  regulate sales per product and the optimal scope of firms (see equations (24) and (25)). Specifying the parameter  $\theta$  (or  $\sigma$  given  $\tilde{\theta}$ ) is important for the aggregate predictions of the model (trade and welfare gains) and thus for our counterfactual experiments.

We first obtain a value for the Pareto shape parameter  $\tilde{\theta}$ , which regulates the increase in exporter size. To estimate this coefficient we regress log mean export sales on the log exporter percentile and a constant for the 70 destinations to which at least 100 Brazilian firms ship. The model implies that the coefficient on the log exporter percentile is  $-1/\tilde{\theta}$  (equation (13)).<sup>31</sup> We find  $\tilde{\theta} = 1.21$  as the average estimate over all 70 destinations. In prior research, Luttmer (2007) and di Giovanni and Levchenko (2010) use  $\tilde{\theta} = 1.06$  as reported by Axtell (2001) for the sales of U.S. firms, which is close to Zipf’s Law. Arkolakis (2010) estimates  $\tilde{\theta} = 1.49$  using the the Melitz (2003) model with Pareto distributed productivity (Chaney 2008).<sup>32</sup>

For the elasticity parameter  $\theta$  we experiment with two different values given the existence of different estimates in the literature. Anderson and van Wincoop (2004) report a range of implied  $\theta$  estimates between four and nine. As our high value, we use  $\theta = 8$ , which is close to the preferred estimate in Eaton and Kortum (2002). As our low value, we use  $\theta = 5$ , which is closer to the estimates of Eaton, Kortum, and Kramarz (2010) and Simonovska and Waugh (2010). For the scope elasticities of local entry costs and product-introduction costs, we use the estimates from Table 3 (column 3):  $\delta = -1.41$  and  $\alpha(\sigma - 1) = 2.69$ . Table 4 collects our benchmark parameter values.

Based on our coefficient estimates on the within-firm sales distribution (Fact 1), we evaluate the calibrated model for Facts 2 and 3. We view these predictions as over-identifying checks on the

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<sup>31</sup>To be precise, equation (13) implies a relationship that also depends on  $H(G_{sd})$  (see footnote 28). We show in Appendix A.5 that an exact regression, reflecting this correction term, is one of mean log export sales on the log exporter percentile, on a constant, and on mean log  $H(G_{sd})^{-(\sigma-1)}$  by destination. We find  $\tilde{\theta} = 1.18$  on average across countries but use  $\tilde{\theta} = 1.21$  from the simple regression subsequently because it resembles the conventional approach that does not control for scope. (The simple regression also results in less dispersion of coefficient estimates across countries.) The reason for the minor difference between the exact and the simple regression is that, under Case C2, the term  $H(G_{sd})^{-(\sigma-1)}$  quickly converges to a constant and has a minor effect. This is also reflected in the good fit of the average sales distribution under the estimate  $\tilde{\theta} = 1.21$  (Figure 5).

<sup>32</sup>The observation that the model exhibits a good fit with a lower  $\tilde{\theta}$  than the estimate from more aggregate trade data (as in Arkolakis 2010) is indicative of unmodelled within-firm heterogeneity (in addition to unmodelled firm heterogeneity as in Arkolakis 2010). For we strive to explain multi-product firms’ optimal choices, we abstract from idiosyncratic sales shocks per product and choose a value of  $\theta$  at the midpoint of the range of existing estimates.

Table 4: Benchmark Parameter Values

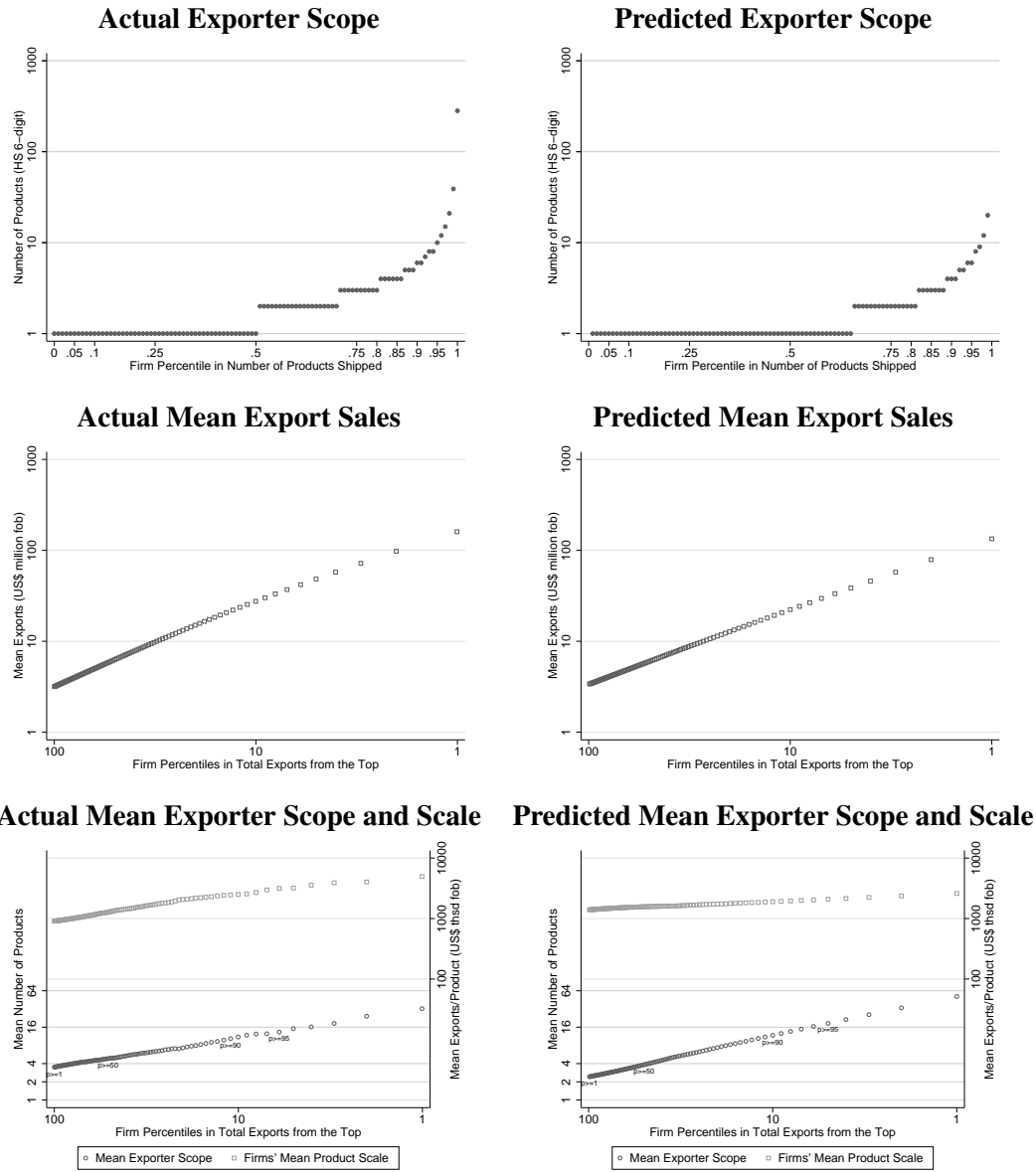
Parameter		Value	Source
Scope elasticity of local entry costs	$\delta$	-1.41	Estimate (Table 3, col. 3)
Scope elasticity of product-introd. cost	$\alpha(\sigma-1)$	2.69	Estimate (Table 3, col. 3)
Pareto shape parameter of total sales	$\tilde{\theta}$	1.21	Exports distribution
Elasticity of substitution betw. varieties	$\sigma$	7.61, 5.13	
Pareto shape parameter of productivity	$\theta$	8.00, 5.00	$\theta = (\sigma-1)\tilde{\theta}$

model in the cross-section of firms. We simulate data from our estimated model and generate statistics exactly as we did using the original data for Facts 2 and 3 discussed in Section 2.

The upper panel of Figure 5 shows the actual (left-hand side) and simulated (right-hand side) exporter scope distributions, and the middle panel the actual and simulated mean sales distributions. Our estimate of  $\delta + \alpha(\sigma-1) = 1.28$  implies overall diseconomies of scope so that firms introduce more products only if they are much more productive. Despite the relatively low estimate of  $\tilde{\theta}=1.21$  (large firm heterogeneity) the model is still able to predict a large part of the increase in scope with overall firm size.

The lower panel of Figure 5 shows the model predictions for mean exporter scope and exporter scale as functions of the overall firm size percentile. The model tracks reasonably well the distribution of mean exporter scope. This success comes despite the fact that the parameters  $\delta$  and  $\alpha(\sigma-1)$  are estimated to match the within-firm heterogeneity of sales. A reason for the close prediction is that averaging over all upper percentiles in cumulative graphs assigns little weight to narrow-scope exporters. The estimated model can also generate the increase in the exporter scale with firm size but falls short of delivering the exact magnitude of that increase.

Overall, we find the model's quantitative performance satisfactory. The fact that parameter estimates to fit the within-firm sales heterogeneity (Figure 1) also deliver a good approximation to the relationships in Figures 2 and 3 reassures us of our estimates. Having queried the predictions of the calibrated model in several dimensions we proceed to perform counterfactual experiments of a hypothetical trade liberalization with respect to local entry costs.



Source: SECEX 2000, manufacturing firms and their manufactured products to the United States.  
 Note: Products at HS 6-digit level. Left panels repeat Figure 3. Parameters for simulations as reported in Table 4.

**Figure 5: Exporter Scope and Exporter Scale and Their Model Predictions for the USA**

## 6 Counterfactuals

We conduct a counterfactual simulation to quantify the implied impact of our estimates (Table 4) for changes to bilateral trade when market-specific entry costs drop. Brazil being close to the median country in exports per capita, we consider our parameter estimates informative for global trade.

To perform counterfactual experiments we add three ingredients to the model following Eaton, Kortum, and Kramarz (2010). (i) We introduce intermediate inputs as in Eaton and Kortum (2002). In particular, we assume that the production of each product uses a Cobb-Douglas aggregate of labor and a composite of all other manufacturing products with cost  $P_d$ . The labor share in manufacturing production is  $\beta$ , and the share of intermediate inputs  $1 - \beta$ . Therefore the total input cost is  $w_d = W_d^\beta P_d^{1-\beta}$ , where we now think of  $w_d$  as the input cost and  $W_d$  as the wage. (ii) There is a non-manufacturing sector in each country as in Alvarez and Lucas (2007) that combines manufactures with labor, in a Cobb-Douglas fashion, where manufactures have a share  $\gamma$  in GDP. The price of final output in country  $d$  is proportional to  $P_d^\gamma W_d^{1-\gamma}$ . We state the resulting equations in Appendix A. (iii) We allow for a manufacturing trade deficit  $D_d$ , and for an overall trade deficit  $D_d^T$  in goods and services. Both deficits are set to their observed levels in 2000.

We compute the share of manufacturing in GDP for each country using data on GDP, manufacturing production and trade (as described in Appendix D.3). We set the labor share in manufacturing production to  $\beta = .330$ , the sample average for countries with available information (Appendix D.3). To compute the impact of a counterfactual change in entry costs, we use the Dekle, Eaton, and Kortum (2007) methodology (details in Appendix B). The merit of this method is that it requires no information on the initial level of technology, iceberg trade costs, and entry costs. Instead, we can compute the changes in all equilibrium variables using as a simple input the percentage change in the underlying parameter of interest (entry cost parameters in our case).

We conduct two experiments with entry costs. The first experiment is a 25-percent drop in the entry costs of exporting the first product  $\hat{f}_{sd}(1) = .75$  for  $s \neq d$ . We interpret this experiment as representing a decline in firm-level exporting cost such as one-time costs for information acquisition, search costs for commercial representatives abroad, expenses for trade fairs, the setup of accredited testing facilities, one-time costs of product re-design and building up export logistics, and perhaps brand marketing costs. In general, this experiment captures declines in the beachhead costs of exporting (the cost of exporter entry subsumed in the local entry costs of the first product).

The second experiment is a 25-percent drop in entry costs of exporting  $\hat{f}_{sd}(g) = .75$  for all prod-

ucts  $g = 1, 2, \dots$  and  $s \neq d$ . We view this exercise as a counterfactual decrease of non-variable trade cost barriers that apply to individual products repeatedly, such as technical barriers to trade.<sup>33</sup> Examples of such trade barriers are individual product re-designs to meet local technical requirements or energy-saving regulations, costs to satisfy performance requirements, costs of compliance with voluntary safety rules or mandatory sanitary regulations including administrative and legal fees, product-specific fixed costs of labelling and re-packaging, access costs to additional wholesale representatives and retail shelf space, or labelling and marking costs for particular exporting markets.

Table 5 summarizes the results for increases in real wages, which are proportional to relative increases in bilateral trade flows. We find for both experiments  $\hat{f}_{sd}(1) = .75$  (first product only) and  $\hat{f}_{sd}(g) = .75$  (all products) that declining fixed costs do not have a large impact on welfare. This finding is similar to simulation results for home-market entry by di Giovanni and Levchenko (2010). Our explanation differs, however. At the firm entry margin, we confirm the di Giovanni and Levchenko (2010) home-market simulation by which only small firms surpass the entry threshold as fixed costs decline; these firms create little additional trade. For multi-product firms, however, the effect could potentially differ if a reduction of fixed local entry costs induced the highly productive wide-scope firms to expand fast. But our estimates imply a strong decline in product sales with scope when wide-scope firms take up additional products. Bilateral trade therefore responds little to the extensive margin of exporting products at wide-scope firms. The results from the two experiments are similar to each other. This implies that a large share of the simulated increase in welfare is attributable to declines in the firm's entry cost for the first product. The explanation is again that local entry costs decline fast with scope so that sales of the low-ranked marginal products at wide-scope firms matter little for bilateral trade.

It is a common feature of trade models that the percentage change in trade is larger for countries with less initial trade. This is also the case in our simulations. Overall, the simulated real-wage gains from declining entry costs are smaller than the ones typically found for falling variable trade costs (e.g. Eaton, Kortum, and Kramarz 2010). Welfare gains from trade crucially depend on  $\tilde{\theta}$  and  $\theta$ . The elasticity of trade with respect to fixed costs  $-(\tilde{\theta}-1)$  is close to zero and therefore makes the aggregate

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<sup>33</sup>The perceived importance of technical barriers to trade for individual products is reflected in dispute settlement cases at the World Trade Organization. As of October 2010, out of the total 418 dispute settlement cases ever brought, 41 cases cite the Technical Barriers to Trade (TBT) and 37 cases the Sanitary and Phytosanitary Measures (SPS) agreements in their request for consultations. In numbers, these case counts are comparable to the 84 cases that cite anti-dumping (Article VI of GATT 1994) in the consultations request. The first dispute settlement case ever brought against the United States (DS2) cites the Technical Barriers to Trade agreement with regards to gasoline standards.

Table 5: Simulation of Real-Wage Increase in Percentage Points due to Decline of Entry Costs

25-percent decline in	$\theta = 8.00$		$\theta = 5.00$			$\theta = 8.00$		$\theta = 5.00$	
	$\hat{f}_{sd}(1)$	$\hat{f}_{sd}(g)$	$\hat{f}_{sd}(1)$	$\hat{f}_{sd}(g)$		$\hat{f}_{sd}(1)$	$\hat{f}_{sd}(g)$	$\hat{f}_{sd}(1)$	$\hat{f}_{sd}(g)$
Armenia	.08	.09	.12	.15	Kuwait	.15	.18	.24	.30
Australia	.08	.10	.13	.16	Kyrgyzstan	.11	.14	.18	.22
Austria	.21	.26	.34	.41	Latvia	.23	.28	.37	.44
Azerbaijan	.09	.11	.15	.18	Lithuania	.16	.20	.26	.32
BelgmLuxNthl	.37	.44	.58	.70	Malta	.44	.53	.70	.84
Bolivia	.07	.09	.11	.14	Mexico	.26	.31	.41	.49
Brazil	.04	.05	.06	.08	Morocco	.12	.15	.20	.24
Bulgaria	.18	.22	.29	.35	Norway	.10	.13	.17	.21
Canada	.24	.29	.38	.46	Oman	.14	.17	.24	.28
Chile	.10	.12	.17	.20	Poland	.12	.14	.19	.23
ChinaHKG	.13	.16	.21	.25	Portugal	.15	.18	.24	.29
Colombia	.05	.06	.08	.10	Romania	.15	.19	.25	.30
CostaRica	.09	.11	.15	.18	RussianFed	.15	.19	.24	.29
Cyprus	.16	.19	.27	.32	Senegal	.11	.13	.17	.21
Ethiopia	.07	.08	.11	.13	Slovenia	.27	.32	.43	.52
Finland	.18	.21	.28	.34	SouthKorea	.15	.18	.24	.29
FranceMonaco	.13	.15	.20	.25	Spain	.11	.13	.18	.22
Germany	.14	.17	.22	.27	SriLanka	.10	.13	.17	.20
Hungary	.42	.50	.67	.80	Sweden	.18	.21	.28	.34
IndMalSgThai	.48	.57	.75	.91	TrinidadTbg	.07	.08	.11	.13
India	.12	.14	.19	.23	Tunisia	.19	.23	.31	.37
Iran	.004	.005	.01	.02	Turkey	.08	.09	.12	.15
Ireland	.49	.60	.78	.95	UK	.11	.13	.17	.21
Israel	.12	.15	.19	.23	USA	.05	.06	.08	.09
Italy	.10	.12	.16	.19	Ukraine	.14	.17	.23	.28
Japan	.04	.04	.06	.07	Uruguay	.06	.08	.10	.12
Jordan	.12	.14	.19	.23	VietNam	.17	.20	.27	.33
Kazakhstan	.17	.20	.27	.32	Yemen	.04	.05	.07	.08
Kenya	.06	.08	.10	.13	RestOfWorld	.42	.50	.65	.79
					<i>World Avg.</i>	.16	.19	.25	.30

Notes: Own calculations based on parameters as reported in Table 4, real wage change in percentage points. Values of  $\theta = 8.00$  similar to Eaton and Kortum (2002) and  $\theta = 5.00$  similar to Eaton, Kortum, and Kramarz (2010). See Appendix D.3 for data construction. Following Dekle, Eaton, and Kortum (2007), we collapse (i) Hong Kong, Macão and mainland China, (ii) Belgium, Luxembourg and the Netherlands, and (iii) Indonesia, Malaysia, Singapore, and Thailand into single markets.

impact of changes in fixed costs small. If we use the lower value of  $\theta = 5$  for our simulations, while keeping  $\tilde{\theta} = 1.21$ , then the welfare gains roughly double for all countries in both experiments. The value of  $\theta = 5$  strongly alters the gains from changes in fixed costs. Reliable estimates of  $\theta$ , or equivalently of  $\sigma$  given  $\tilde{\theta}$ , are an arguably important aspect of confidence in trade simulations (see for example Helpman, Melitz, and Rubinstein 2008).

As a final check on the extensive margin of exporting products, we simulate a 25-percent drop in variable trade costs  $\hat{\tau}_{sd} = .75$  for all products. Real-wage gains from trade with respect to changes in variable trade costs are much larger and within the magnitudes reported by Eaton, Kortum, and Kramarz (2010). However, these magnitudes are affected only to a minor degree by changes in  $\theta$ , reminiscent of results in Atkeson and Burstein (2010) and Bergstrand, Egger, and Larch (2010). We decompose the gains from trade into the contributions at each margin, by holding the other two margins constant at a time. The extensive margin of firm entry contributes 31 percent of the welfare gains from reduced variable trade costs. The extensive margin of product entry from expanding exporter scope contributes only 5 percent. The bulk of welfare gains accrues at the remaining intensive margin of the exporter's average sales per product with 64 percent.

The importance of the within-firm extensive margin varies by exporter size. The model implies for the small and medium size exporters that the extensive margin of products is of paramount importance. In our simulations for iceberg cost declines, the extensive margin of products typically accounts for 20 percent of the total change in exports among the lower tercile of exporters and 24 percent among the middle tercile of exporters. In contrast, the contribution of the products margin to exports for the top tercile of exporters is only around 2 percent and dominated by the intensive margin of firm product sales. New products have a large impact on the sales of small exporters but are of negligible importance to large exporters. At these dominant exporters, mainly changes in the intensive sales margin drive their growth.

## 7 Concluding Remarks

We have used three-dimensional panel data of Brazilian exporters, their products and their destination markets to assess the importance of the extensive margin of adding products for exporters. The level of detail of the Brazilian data and the multi-product model that we develop allow us to follow a reductionist approach, relating bilateral trade in the aggregate back to the product adoption decision

at individual firms. Building up from a firm's choices of destination markets and local product lines, our model leads to relationships that are consistent with the disaggregated trade data but also with previous theories of trade such as Chaney (2008), Eaton, Kortum, and Kramarz (2010), and Arkolakis (2010), which consider more aggregate trade flows. Estimation of this micro-founded model allows us to quantify the responsiveness of bilateral trade to fixed entry cost components. Simulations of our model imply that adding products is a salient margin of adjustment for small exporters but comparably less relevant for large exporters at a destination.

Our approach leaves unexplored recently available information on unit prices and time series information. Such additional information may prove valuable in understanding more precisely the patterns of product entry and exporter expansion.

# Appendix

## A Proofs

### A.1 Proof of Proposition 2

The optimal choice of  $G_{sd}(\phi)$  is the largest  $G_{sd} \in \{0, 1, \dots\}$  such that inequality (9),

$$\left( \frac{P_d}{\tilde{\sigma} \tau_{sd} w_s} \frac{\phi}{h(G_{sd})} \right)^{\sigma-1} \frac{T_d}{\sigma} \geq f_{sd}(G_{sd}),$$

is weakly satisfied, as shown in the text. In discrete product space,  $G_{sd}$  is an integer. For simplicity, define  $\tilde{G}_{sd}$  as the continuous variable that solves

$$z_{sd}(\tilde{G}_{sd}) = \left( \frac{P_d}{\tilde{\sigma} \tau_{sd} w_s} \phi \right)^{\sigma-1} \frac{T_d}{\sigma},$$

where  $z_{sd}(G) \equiv f_{sd}(G) h(G)^{\sigma-1}$ .  $z_{sd}(G)$  strictly increases in  $G$  by Case C1. So  $z_{sd}(G)$  is invertible. Hence  $\tilde{G}_{sd} = \tilde{G}_{sd}((\phi/\phi_{sd}^*)^{\sigma-1})$  can also be expressed as the inverse function

$$\tilde{G}_{sd} = z_{sd}^{-1} \left( f_{sd}(1) (\phi/\phi_{sd}^*)^{\sigma-1} \right). \quad (\text{A.1})$$

Note that  $\tilde{G}_{sd} = G_{sd}$  for  $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}$ , and so forth.

Case C1 is equivalent to

$$\frac{\partial \ln z_{sd}(G)}{\partial \ln G} > 1 \iff \frac{Z(G)}{Z'(G)G} < 1 \iff \frac{[Z^{-1}(G)]' Z(G)}{G} < 1,$$

where the last step follows by the inverse function theorem since  $z_{sd}(G)$  strictly increases in  $G$ . As a shorthand, define the argument  $x \equiv f_{sd}(1) (\phi/\phi_{sd}^*)^{\sigma-1}$ . By equation (A.1),  $\tilde{G}_{sd} = z_{sd}^{-1}(x)$ , so Case C1 is also equivalent to

$$\frac{\partial \ln z_{sd}(G)}{\partial \ln G} > 1 \iff \frac{\tilde{G}'(x)x}{\tilde{G}(x)} < 1 \iff \tilde{G}(x) - \tilde{G}'(x)x > 0. \quad (\text{A.2})$$

We want to show that, if Case C1 holds, exporter scale  $a_{sd}(\phi)$  strictly increases in  $\phi$  when evaluated at the points  $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}$ , and so forth. So by (14) and with the shorthand  $x = f_{sd}(1) (\phi/\phi_{sd}^*)^{\sigma-1}$ , consider

$$a_{sd}(x) = \sigma x H(\tilde{G}(x))^{-(\sigma-1)} / \tilde{G}(x).$$

The term  $H(\tilde{G}(x))^{-(\sigma-1)}$  strictly increases in  $\tilde{G}(x)$  at  $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}$ , and so forth. Since we are only interested in sufficiency of the restriction and since  $H(\cdot)^{-(\sigma-1)}$  strictly increases at the evaluation points, it suffices to show that the result is true for  $x/\tilde{G}(x)$ .  $x/\tilde{G}(x)$  is differentiable at the points  $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}$ , and so forth, so exporter scale  $a_{sd}(\cdot)$  strictly increases if

$$\left(x/\tilde{G}(x)\right)' > 0 \iff \frac{\tilde{G}(x) - \tilde{G}'(x)x}{[\tilde{G}(x)]^2} > 0 \iff \tilde{G}(x) - \tilde{G}'(x)x > 0.$$

The last step is an implication of Case C1 as shown in (A.2). This establishes the result.

## A.2 Proof of Proposition 3

Average sales of firms in  $s$  shipping to  $d$  are

$$\bar{T}_{sd} = \int_{\phi_{sd}^*} t_{sd}(\phi) \cdot \mu_{sd}(\phi) \, d\phi = \int_{\phi_{sd}^*} \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*}\right)^{\sigma-1} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} \cdot \theta \frac{(\phi_{sd}^*)^\theta}{(\phi)^{\theta+1}} \, d\phi$$

by optimal total exports (13) and the definition of a firm's product efficiency index (5). Consider the term  $\int_{\phi_{sd}^*} \phi^{\sigma-1-(\theta+1)} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} \, d\phi$ . Rewrite the term as a piecewise integral

$$\begin{aligned} \int_{\phi_{sd}^*} \sum_{g=1}^{G_{sd}(\phi)} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} \, d\phi &= \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \sum_{g=1}^1 \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} \, d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} \sum_{g=1}^2 \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} \, d\phi + \dots \\ &= \frac{1}{h(1)^{\sigma-1}} \int_{\phi_{sd}^*}^{\infty} \phi^{\sigma-1-(\theta+1)} \, d\phi + \frac{1}{h(2)^{\sigma-1}} \int_{\phi_{sd}^{*,2}}^{\infty} \phi^{\sigma-1-(\theta+1)} \, d\phi + \dots \\ &= \frac{1}{h(1)^{\sigma-1}} \frac{(\phi_{sd}^*)^{(\sigma-1)-\theta}}{\theta - (\sigma-1)} + \frac{1}{h(2)^{\sigma-1}} \frac{(\phi_{sd}^{*,2})^{(\sigma-1)-\theta}}{\theta - (\sigma-1)} + \dots \end{aligned}$$

for  $\theta > \sigma - 1$ . Using the definitions of  $\phi_{sd}^*, \phi_{sd}^{*,2}$  etc. from (11), we have

$$\int_{\phi_{sd}^*} \sum_{g=1}^{G_{sd}(\phi)} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} \, d\phi = \frac{1}{\theta - (\sigma-1)} \left(\frac{f_{sd}(1)}{(\phi_{sd}^*)^{\sigma-1}}\right)^{\tilde{\theta}-1} \sum_{G=1}^{\infty} \frac{f_{sd}(G)^{-(\tilde{\theta}-1)}}{h(G)^\theta}. \quad (\text{A.3})$$

Therefore  $\bar{T}_{sd} = [\tilde{\theta} \sigma / (\tilde{\theta} - 1)] f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$ , proving the first equality in (18).

The expression is finite by Assumption 3.

Average fixed costs paid by firms in  $s$  selling to  $d$  are

$$\begin{aligned} \bar{F}_{sd} &= \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} F_{sd}(1) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^{\theta+1}} \, d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} F_{sd}(2) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^{\theta+1}} \, d\phi + \\ &= F_{sd}(1) (\phi_{sd}^*)^\theta \left[ (\phi_{sd}^*)^{-\theta} - (\phi_{sd}^{*,2})^{-\theta} \right] + F_{sd}(2) (\phi_{sd}^*)^\theta \left[ (\phi_{sd}^{*,2})^{-\theta} - (\phi_{sd}^{*,3})^{-\theta} \right] + \dots \end{aligned}$$

Using the definition  $F_{sd}(G_{sd}) = \sum_{g=1}^{G_{sd}} f_{sd}(g)$  and collecting terms with a common  $\phi_{sd}^{*,G}$  we can rewrite the above expression as

$$\bar{F}_{sd} = f_{sd}(1) + (\phi_{sd}^{*,2})^{-\theta} (\phi_{sd}^*)^{\theta} f_{sd}(2) + (\phi_{sd}^{*,3})^{-\theta} (\phi_{sd}^*)^{\theta} f_{sd}(3) + \dots$$

Using the definition of  $\phi_{sd}^{*,G}$  from equation (11) in the above equation we get

$$\begin{aligned} \bar{F}_{sd} &= f_{sd}(1) + \left( \frac{f_{sd}(2)^{1/(\sigma-1)} h(2)}{f_{sd}(1)^{1/(\sigma-1)} h(1)} \right)^{-\theta} f_{sd}(2) + \dots \\ &= \left[ f_{sd}(1) + f_{sd}(1)^{\tilde{\theta}} (f_{sd}(2)^{1/(\sigma-1)} h(2))^{-\theta} f_{sd}(2) + \dots \right] \\ &= f_{sd}(1)^{\tilde{\theta}} \left[ f_{sd}(1)^{-(\tilde{\theta}-1)} + f_{sd}(2)^{-(\tilde{\theta}-1)} h(2)^{-\theta} + \dots \right] \\ &= f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} \\ &= \frac{\tilde{\theta}-1}{\tilde{\theta} \sigma} \bar{T}_{sd}. \end{aligned}$$

This proves the second equality in (18). So  $\bar{F}_{sd}/\bar{T}_{sd}$  is a destination invariant constant.

### A.3 Mean sales per product

Figure 1 (Fact 1) depicts average sales of the  $g$ -th product for firms with an exporter scope of exactly  $G_{sd}$  products in market  $d$ . To compute this statistic, we integrate the sales of the  $g$ -th product  $p_{sdg}(\phi)x_{sdg}(\phi)$  over the probability density of firms with  $\phi$  such that  $\phi_{sd}^{*,G} \leq \phi \leq \phi_{sd}^{*,G+1}$ . We thus have

$$\begin{aligned} &\int_{\phi_{sd}^{*,G}}^{\phi_{sd}^{*,G+1}} p_{sdg}(\phi)x_{sdg}(\phi)\theta \frac{[(\phi_{sd}^{*,G})^{-\theta} - (\phi_{sd}^{*,G+1})^{-\theta}]^{-1}}{\phi^{\theta+1}} d\phi = \\ &= \sigma f_{sd}(1) G_{sd}^{\delta+\alpha(\sigma-1)} \int_{\phi_{sd}^{*,G}}^{\phi_{sd}^{*,G+1}} \left( \frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma-1} g^{-\alpha(\sigma-1)} \theta \left( \frac{[(\phi_{sd}^{*,G})^{-\theta} - (\phi_{sd}^{*,G+1})^{-\theta}]^{-1}}{\phi^{\theta+1}} \right) d\phi \\ &= -f_{sd}(1) G_{sd}^{\delta+\alpha(\sigma-1)} g^{-\alpha(\sigma-1)} \frac{\sigma\theta}{\theta-(\sigma-1)} \frac{(\phi_{sd}^{*,G+1})^{(\sigma-1)-\theta} - (\phi_{sd}^{*,G})^{\sigma-1-\theta}}{(\phi_{sd}^{*,G})^{(\sigma-1)-\theta} - (\phi_{sd}^{*,G})^{\sigma-1} (\phi_{sd}^{*,G+1})^{-\theta}} \\ &= \frac{\sigma\theta}{\theta-(\sigma-1)} f_{sd}(1) G_{sd}^{\delta+\alpha(\sigma-1)} \frac{1 - \left(\frac{G_{sd}+1}{G_{sd}}\right)^{[1-\tilde{\theta}][\delta+\alpha(\sigma-1)]}}{1 - \left(\frac{G_{sd}+1}{G_{sd}}\right)^{-\tilde{\theta}[\delta+\alpha(\sigma-1)]}} g^{-\alpha(\sigma-1)}, \end{aligned}$$

where we used the definition from equation (10) in the last equality.

### A.4 Share of wages and profits

We show here that the share of wages and profits in total income is constant (source country invariant). Note that the share of net profits from bilateral sales is the share of gross variable profits in total sales

$1/\sigma$ , less the fixed costs paid, divided by total sales  $(\tilde{\theta}-1)/\tilde{\theta}\sigma$ . Thus, using the result of Proposition 3,  $\pi_{sd}L_d/T_{sd} = 1/\sigma - (\tilde{\theta}-1)/(\tilde{\theta}\sigma) = 1/(\tilde{\theta}\sigma) = 1/(\tilde{\theta}\tilde{\sigma})$ . Total profits for country  $s$  are  $\pi_s L_s = \sum_k \lambda_{sk} T_k / (\tilde{\theta}\sigma)$ , where  $\sum_k \lambda_{sk} T_k$  is the country's total income by (22). So profit income and wage income can be expressed as constant shares of total income as in the main text, equation (23).

## A.5 Mean log total sales

Using a firm's total sales (13), taking logs of both sides of the expression, and averaging over all firms with exporter scope of at least  $G_{sd}$ , we can express mean log sales as

$$\int_{\phi_{sd}^{*,G}}^{\infty} \ln t_{sd}(\phi) \mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G}) d\phi = \ln \sigma f_{sd}(1) + (\sigma-1) \int_{\phi_{sd}^{*,G}}^{\infty} \ln \left( \frac{\phi}{\phi_{sd}^{*,G}} \right) \mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G}) d\phi \\ + \int_{\phi_{sd}^{*,G}}^{\infty} \ln H(G_{sd}(\phi))^{-(\sigma-1)} \mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G}) d\phi$$

at or above the cutoff productivity  $\phi_{sd}^{*,G}$ , where  $\mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G})$  is the conditional probability density for firms with productivity at or above the cutoff. For a Pareto distribution of productivity, the first integral on the right-hand side simplifies to

$$(\sigma-1) \int_{\phi_{sd}^{*,G}}^{\infty} \ln \left( \frac{\phi}{\phi_{sd}^{*,G}} \right) \mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G}) d\phi = \frac{\sigma-1}{\theta} \left( 1 + \theta \ln \frac{\phi_{sd}^{*,G}}{\phi_{sd}^{*,G}} \right)$$

and the ratio  $\phi_{sd}^{*,G}/\phi_{sd}^*$  is related to the percentile of the firm with  $1 - \Pr_{sd}^{*,G} = (\phi_{sd}^{*,G}/\phi_{sd}^*)^\theta$ .

Using these two results we can express our estimation equation as

$$\int_{\phi_{sd}^{*,G}}^{\infty} \ln t_{sd}(\phi) \mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G}) d\phi = \ln \sigma f_{sd}(1) + \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \ln \left( 1 - \Pr_{sd}^{*,G} \right) \\ + \int_{\phi_{sd}^{*,G}}^{\infty} \ln H(G_{sd}(\phi))^{-(\sigma-1)} \mu_{sd}(\phi|\phi \geq \phi_{sd}^{*,G}) d\phi.$$

For estimation, we group exporters at or above a sales percentile in each destination and consider the sales and the scope in the market of each exporter in the group. From scope we infer the term  $H(G_{sd}(\phi))^{-(\sigma-1)}$  using our parameter estimate for  $\alpha(\sigma-1) = 2.69$ . We construct the empirical analogs of the two integral terms for each group by taking the means over the exporters' log sales and the constructed  $\log H(G_{sd}(\phi))^{-(\sigma-1)}$ .

## B Counterfactuals and Calibration

For the counterfactuals we set  $w_s = W_s^\beta P_s^{1-\beta}$  and, introducing auxiliary notation, replace  $f_{sd}(1)\bar{F}_{sd}$  with  $W_d^{-(\tilde{\theta}-1)}\tilde{F}_{sd}$  since fixed costs are homogeneous of degree one in the import country's wage (and thus  $f_{sd}(1)\bar{F}_{sd}$  is homogeneous of degree  $1-\tilde{\theta}$ ). We consider technological parameters and labor endowments as time invariant. Using expression (19) for current trade shares  $\lambda_{sd}$ , we can express counterfactual trade shares as

$$\lambda'_{sd} = \frac{\lambda_{sd} \left( \hat{W}_s^\beta \hat{P}_s^{1-\beta} \right)^{-\theta} \hat{\tau}_{sd}^{-\theta} \widehat{F}_{sd}}{\sum_k \lambda_{kd} \left( \hat{W}_k^\beta \hat{P}_k^{1-\beta} \right)^{-\theta} \hat{\tau}_{kd}^{-\theta} \widehat{F}_{kd}} \quad (\text{B.4})$$

The price index (3) can be re-written as

$$\begin{aligned} P_d^{1-\sigma} &= \sum_k \int_{\phi_{kd}^*} M_{kd} \left[ \sum_{g=1}^{G_{kd}(\phi)} \left( \tilde{\sigma} \frac{W_k^\beta P_k^{1-\beta}}{\phi/h(g)} \tau_{kd} \right)^{1-\sigma} \right] \frac{\theta (\phi_{kd}^*)^\theta}{\phi^{\theta+1}} \mathbf{d}\phi \\ &= \sum_k \left( W_k^\beta P_k^{1-\beta} \tilde{\sigma} \tau_{kd} \right)^{1-\sigma} J_k(b_k)^\theta \int_{\phi_{kd}^*} \sum_{v=1}^{G_{kd}(\phi)} \frac{\phi^{\sigma-\theta-2}}{h(g)^{\sigma-1}} \mathbf{d}\phi. \end{aligned}$$

By Proposition 3, we can replace the integral term so that

$$P_d^{-\theta} = (T_d)^{\tilde{\theta}-1} \frac{(\sigma)^{-(\tilde{\theta}-1)} (\tilde{\sigma})^{-\theta}}{1-1/\tilde{\theta}} \sum_k J_k b_k^\theta \left( W_k^\beta P_k^{1-\beta} \right)^{-\theta} \tau_{kd}^{-\theta} W_d^{-(\tilde{\theta}-1)} \tilde{F}_{kd}, \quad (\text{B.5})$$

which can be restated as<sup>34</sup>

$$\hat{P}_d = \left[ \sum_k \lambda_{kd} \left( \hat{W}_k^\beta \hat{P}_k^{1-\beta} \right)^{-\theta} \hat{\tau}_{kd}^{-\theta} \widehat{F}_{kd} \right]^{-1/\theta} \left( \frac{\hat{T}_d}{\hat{W}_d} \right)^{1/\theta-1/(\sigma-1)}. \quad (\text{B.6})$$

<sup>34</sup>We can use expression (B.5) together with equation (19) to obtain

$$P_d^{-\theta} = (T_d)^{\tilde{\theta}-1} \frac{(\sigma)^{-(\tilde{\theta}-1)} (\tilde{\sigma})^{-\theta}}{1-1/\tilde{\theta}} \frac{J_d b_d^\theta \left( W_d^\beta P_d^{1-\beta} \right)^{-\theta} W_d^{-(\tilde{\theta}-1)} \tilde{F}_{dd}}{\lambda_{dd}}.$$

Thus changes in real wage are

$$\left( \frac{\widehat{W}_d}{\widehat{P}_d} \right) = \left( \widehat{\lambda}_{dd} \right)^{-1/\theta} \frac{\left( \widehat{T}_d / \widehat{W}_d \right)^{\frac{1-\tilde{\theta}}{\theta}}}{\left( \widehat{F}_{dd} \right)^{-1/\theta}}.$$

We consider  $\widehat{F}_{dd} = 1$  in our counterfactual exercise, so this expression differs for domestic entry costs from a similar one in Arkolakis, Costinot, and Rodríguez-Clare (2010) inasmuch as changes in the ratio  $\widehat{T}_d/\widehat{W}_d$  reflect changes in the ratio of total absorption to wages (which is not one due to non-zero deficits).

The final equation for our counterfactuals follows Eaton, Kortum, and Kramarz (2010, Appendix E). We allow the share  $\gamma_d$  of manufacturing value added in GDP to be country specific. Total manufacturing absorption is

$$T_d = \underbrace{\gamma_d \cdot (Y_d^T + D_d^T)}_{\text{final demand (total labor income, profits)}} + (1-\beta) \cdot \underbrace{\frac{\sigma-1}{\sigma} Y_d}_{\text{demand for intermediates by manufacturing sector}},$$

where  $Y_d^T$  is total GDP of country  $d$ , including labor income and profits,  $D_d^T$  is the current account deficit and  $Y_d$  output of the manufacturing sector. Notice that manufacturing spending equals  $T_d = Y_d + D_d$ , where  $D_d$  is the trade deficit in the manufacturing sector. We can therefore solve for  $T_d$  and  $Y_d$  and obtain

$$\begin{aligned} T_d &= \frac{\gamma_d (Y_d^T + D_d^T) - (1-\beta)(1-1/\sigma)D_d}{1/\sigma + \beta(1-1/\sigma)}, \\ Y_d &= \frac{\gamma_d (Y_d^T + D_d^T) - D_d}{1/\sigma + \beta(1-1/\sigma)}. \end{aligned} \quad (\text{B.7})$$

To summarize, using the Dekle, Eaton, and Kortum (2007) algorithm, we can compute how given changes in the fixed costs  $\widehat{F}_{kd}$  lead to  $\widehat{\lambda}_{sd}$ ,  $\widehat{P}_d$ ,  $\widehat{W}_d$ . Denoting future variables with a prime, we find  $T'_d$ ,  $Y'_d$  by inspecting equations (B.4), (B.5) and imposing the current account balance condition

$$Y'_s L_s = \sum_{k=1}^N \lambda'_{sk} T'_k, \quad (\text{B.8})$$

where we use relationship (B.7) and  $T'_d = Y'_d + D_d$ .

We obtain individual  $\gamma_d$  by country. To do so, we solve equation (B.7) for  $\gamma_d$  and compute

$$\gamma_d = \frac{T_d - (1-\beta)(1-1/\sigma)(T_d - D_d)}{Y_d^T + D_d^T}. \quad (\text{B.9})$$

for every country, using data on  $T_d$ ,  $D_d$ ,  $Y_d^T$  and  $D_d^T$ .

## C Nested Utility

We can generalize the model to consumer preferences

$$\left( \sum_{s=1}^N \int_{\omega \in \Omega_{sd}} \left[ \sum_{g=1}^{G_{sd}(\omega)} x_{sdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1} \frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \varepsilon > 1, \sigma > 1, \varepsilon \neq \sigma.$$

In this case we redefine the product efficiency index as:

$$H(G_{sd}) \equiv \left( \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\varepsilon-1)} \right)^{-\frac{1}{\varepsilon-1}}. \quad (\text{C.10})$$

With this new definition, the expressions for firm product sales (12) and for aggregate bilateral trade (18) in Proposition 3 remain unaltered. Cases C1 and C2 take a generalized form but the expressions are similar. For remaining details on the generalized model see our online Technical Appendix.

With this generalization, a firm's individual products can be less substitutable among themselves than with outside products (if  $\varepsilon < \sigma$ ) or more substitutable ( $\varepsilon > \sigma$ ). In the latter case, a firm's additional products cannibalize sales of its infra-marginal products. The cannibalization effect is symmetric for all products, so relative sales of a firm's existing products are not affected by the introduction of additional products. This constancy of relative sales in our model does not carry over to models with CES-preferences and a countable number of firms such as Feenstra and Ma (2008) or to models with non-CES preferences such as Mayer, Melitz, and Ottaviano (2011) and Dhingra (2010).

## D Data

### D.1 Exporter-product-destination data

We identify an exporter's sector from the firm's reported *CNAE* four-digit industry (for 654 industries across all sectors of the economy) in the administrative RAIS records (*Relação Anual de Informações Sociais*) at the Brazilian labor ministry.<sup>35</sup> The level of detail in *CNAE* is comparable to the *NAICS 2007* five-digit level. To map from the HS 6-digit codes to ISIC revision 2 at the two-digit level we use an extended SITC-to-ISIC concordance, augmenting an OECD concordance for select manufacturing industries to all industries.<sup>36</sup>

As Table D.1 shows in columns 5 and 6, our Brazilian manufacturer sample includes 10,215 firms with shipments of 3,717 manufacturing products at the 6-digit Harmonized System level to 170

<sup>35</sup>By the WTF and WDI data for all industries and countries, Brazil ranks at the 48th percentile (top 100th out of 192) in terms of exports per capita in 2000. In 2000, Brazil's total exports are at the 88th percentile worldwide (top 27th out of 205).

<sup>36</sup>Our SITC-to-ISIC concordance is available at URL [econ.ucsd.edu/muendler/resource](http://econ.ucsd.edu/muendler/resource).

Table D.1: Sample Characteristics by Destination

to destination ( $d$ )	From Brazil				
	USA (1)	Argentina (2)	OECD <sup>a</sup> (3)	non-OECD <sup>a</sup> (4)	Agg. World <sup>a</sup> (5)
# of Firms ( $M$ )	3,083	4,590	5,041	8,664	10,215
# of Destinations ( $N$ )	1	1	23	147	170
# of HS-6 products ( $G$ )	2,144	2,814	2,772	3,537	3,717
# of Observations	10,775	21,623	36,359	126,211	162,570
Destination share in Tot. exp.	.257	.144	.559	.441	1
Firm shares in Total exports					
Single-prod. firms	.123	.086	.142	.069	.090
Multi-prod. firms' top prod.	.662	.555	.625	.573	.597
Multi-prod. firms' other prod.	.215	.359	.233	.359	.313
Median Total exp. ( $T_d(m)$ )	.120	.068	.137	.066	.089
Median Exp. scope ( $G_d(m)$ )	1	2	2	2	2
Median Exp. scale ( $a_d(m)$ )	.068	.031	.070	.028	.037
Mean Total exports ( $\bar{t}_d$ )	3.170	1.192	4.217	1.932	3.720
Mean Exp. scope ( $\bar{G}_d$ )	3.495	4.711	3.933	5.176	5.278
Mean Exp. scale ( $\bar{a}_d$ )	.907	.253	1.072	.373	.705

<sup>a</sup>Each aggregate region (world, OECD, non-OECD) treated as a single destination, collapsing product shipments to different countries into single product shipment.

Sources: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. Exports in US\$ million fob. Firms' exporter scale ( $\bar{a}_d$  in US\$ million fob) is the scope-weighted arithmetic mean of exporter scales. OECD includes all OECD members in 1990. The United States is Brazil's top export destination in 2000, Argentina second to top.

A source country's total exports  $T_d$  are decomposed into  $T_d = M_d \bar{G}_d \bar{a}_d$ , where  $M_d$  is the number of exporters to destination  $d$ ,  $\bar{G}_d \equiv \sum_{\phi=1}^{M_d} G_d(\phi)/M_d$  is the exporters' mean exporter scope, and  $\bar{a}_d \equiv \bar{t}_d/\bar{G}_d$  is their products' exporter scale. Equivalently,  $\bar{a}_d$  is the weighted arithmetic mean of  $a_d(\phi)$  over all  $\phi$ , with weights  $G_d(\phi)$ :  $\bar{a}_d = \sum_{\phi=1}^{M_d} G_d(\phi) a_d(\phi) / [\sum_{\phi=1}^{M_d} G_d(\phi)] = \bar{t}_d/\bar{G}_d$ . As the decomposition shows, scope weighting is necessary for the mean scope and the exporter scale to yield total exports when multiplied.

destinations, and a total of 162,570 exporter-destination-product observations.<sup>37</sup> Exporters shipping multiple products dominate. They ship more than 90 percent of all exports from Brazil, and their global top-selling product accounts for 60 percent of Brazilian exports worldwide. We report the top exporting products of Brazilian firms in the online appendix.<sup>38</sup>

To calculate summary medians and means of these variables for regional aggregates and the world as a whole in Table D.1 (columns 3 to 6), we treat each aggregate as if it were a single destination and collapse all product shipments to different countries within the aggregate into a single product

<sup>37</sup>We remove export records with zero value from the Brazilian data, which include shipments of commercial samples but also potential reporting errors, and lose 408 of initially 162,978 exporter-destination-product observations. Our results on exporter scope do not materially change when including or excluding zero-shipment products from the product count.

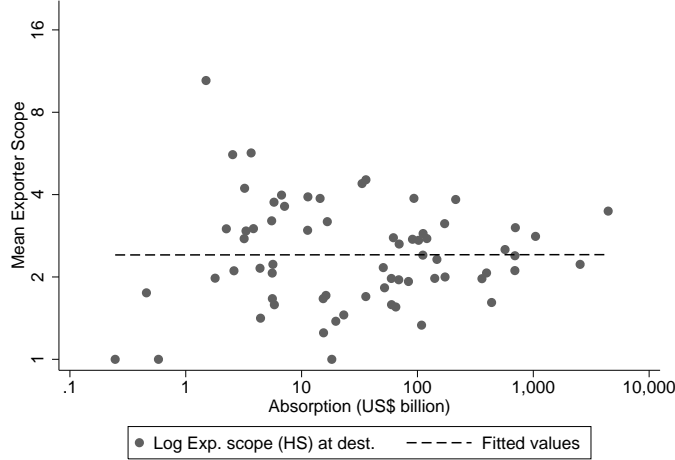
<sup>38</sup>The top-5 selling products of Brazilian exporters at the 6-digit level are: 1. Airplanes heavier than 2 tons, 2. Chemical woodpulp, 3. Soybean oilcake, 4. Passenger vehicles with engines above 1,500 cc, 5. Transmissions.

shipment. In most data treatments in the text, in contrast, we analyze these variables country by country, consistent with our main hypothesis that distribution-side determinants of trade matter repeatedly destination by destination.

The median exporter is a relatively small exporter, with sales to the rest of the world totalling around US\$ 89,000. The mean exporter, in contrast, sells around US\$ 3.7 million abroad, more than 40 times as much as the median exporter. Exporter scope and exporter scale exhibit similarly stark differences between mean and median. The median Brazilian manufacturer sells two products worldwide, but the mean scope per firm is 5.3 products. The median Brazilian exporter has a product scale of around US\$ 37,000 per product, but the exporter scale per exporter is US\$ 705,000, or around 20 times as high as that for the median firm.

The importance of the top-selling product at multi-product exporters and the mean-median ratios repeat across destinations. To investigate the robustness across countries, we select Brazil's top two export destinations (United States and Argentina), as well as the OECD and non-OECD aggregates. Our theory emphasizes the importance of exporting behavior within destinations. Within single countries, the mean manufacturer's exports exceed the median manufacturer's exports by similarly large factors as in the aggregate, between 14 (in Argentina, column 2) and 26 (in the United States, column 1). In the OECD aggregate (column 3), exports of the mean firm exceed the exports of the median firm by a factor of about 30. Interestingly, the same mean-median ratio of about 30 prevails in the non-OECD aggregate.

We further investigate the striking similarity of firm scope choices across destinations by relating the mean number of products to destination market size. Figure D.1 shows a scatter plot of the log mean exporter scope  $\bar{G}_{sd}$  against the log of total absorption at the destination  $T_d$ . The depicted fitted line, from an ordinary least squares regression, has a slope that is not significantly different from zero at conventional levels. In other words, most of the variation in firms' exports to markets of different size is due to variation in the firms' mean scale per product. At the firm level, the Brazilian data exhibit market-presence patterns that resemble those in the French and U.S. firm-destination data. Similar to Eaton, Kortum, and Kramarz (2004), for instance, the elasticity of the number of firms with respect to the number of export destinations is about -2.5, just as for French exporters.



Source: SECEX 2000 manufacturing firms and their manufactured products at the HS 6-digit level, destinations linked to WTF (Feenstra, Lipsey, Deng, Ma, and Mo 2005) and *Unido* Industrial Statistics (UNIDO 2005).

Figure D.1: Mean Exporter Scope and Absorption by Destination

## D.2 Estimation of Figure 1 (Fact 1)

To generate data such as those in Figure 1 (Fact 1), we aggregate individual product sales by averaging over all products of a given rank and all firms with a given exporter scope in a destination.

As shown in Appendix A.3, our model implies that the regression equation for average product sales in Figure 1 is

$$\ln \bar{a}_{sd} = \ln \frac{\theta\sigma}{\theta - (\sigma - 1)} f_{sd}(1) + [\delta + \alpha(\sigma - 1)] \ln G_{sd}(\phi) - [\alpha(\sigma - 1)] \ln g \\ + \ln \left[ 1 - [G_{sd}/(G_{sd} + 1)]^{(\tilde{\theta} - 1)(\delta + \alpha(\sigma - 1))} \right] - \ln \left[ 1 - [G_{sd}/(G_{sd} + 1)]^{\tilde{\theta}(\delta + \alpha(\sigma - 1))} \right]$$

for  $g \leq G_{sd}$ , where  $\bar{a}_{sd} \equiv \int_{\phi_{sd}^{*,G}}^{\phi_{sd}^{*,G+1}} p_{sdg}(\phi) x_{sdg}(\phi) [\theta/\phi^{\theta+1}] [1/(\phi_{sd}^{*,G})^{-\theta} - (\phi_{sd}^{*,G+1})^{-\theta}] d\phi$  is average product sales for a product of rank  $g$  over all firms with exporter scope  $G_{sd}$ .

We fit this regression equation with non-linear least squares under three choices of the free parameter:  $\tilde{\theta} = 1.06$  (as used by Luttmer (2007) and di Giovanni and Levchenko (2010) and reported by Axtell (2001));  $\tilde{\theta} = 1.21$  (the mean of the country-level coefficient estimates from a regression of log mean export sales on the log exporter percentile, Table 4); and  $\tilde{\theta} = 1.49$  (as reported by Arkolakis (2010)). We find for  $\delta + \alpha(\sigma - 1)$  estimates of 1.60 and 1.61 under the three parametrizations and for  $\alpha(\sigma - 1)$  an estimate of 2.55 under all three parametrizations. These estimates closely resemble those in Table 3. Not surprisingly, the estimate for  $\alpha(\sigma - 1)$  is close to the slopes between 2.6 and 2.8 that were reported in Section 2.2 for Figure 1.

### D.3 Data for simulations

For bilateral trade and trade balances in manufacturing products, we use World Trade Flow (WTF) data in U.S. dollars for the year 2000 (Feenstra, Lipsey, Deng, Ma, and Mo 2005). To mitigate the effect of entrepôt trade, we follow Dekle, Eaton, and Kortum (2007) and collapse (i) Hong Kong, Macão and mainland China, (ii) Belgium, Luxembourg and the Netherlands, and (iii) Indonesia, Malaysia, Singapore, and Thailand into single entities. In 2000, import information for India is missing from WTF. We obtain information for India in 2000 from UN Comtrade. We keep only manufactured products from the WTF data, using a concordance from the OECD at the SITC revision-2 4-digit level to determine manufactured products, and exclude agricultural and mining merchandise. By our construction, the world's trade balance is zero.

For information on GDP, manufacturing value added and the overall trade balances in goods and services in 2000 we use the World Bank's World Development Indicators 2009 (WDI). India included, our initial WTF sample has 132 countries that can be matched to the WDI data, and we collapse bilateral trade for the rest of the world by trade partner into a 133rd observation. We compute GDP and manufacturing value added for the rest of the world as the WDI reported world total less the sample total of our 132 matched countries. We set the overall trade balances in goods and services for the rest of the world so that the world total is zero.

We obtain  $\beta$  from the UNIDO Industrial Statistics Database at the 3-digit ISIC level (revision 2), which offers both manufacturing value added and manufacturing gross production for 51 of our sample countries and the rest of the world. Averaging the ratio of manufacturing value added to manufacturing output in 2000 over these countries yields  $\beta = .330$ . This worldwide  $\beta$  estimate enters our computation of  $\gamma_d$  by (B.9).

We finally need information on manufacturing absorption. Following Eaton, Kortum, and Kramarz (2004), we infer manufacturing absorption as manufacturing output (from the UNIDO Industrial Statistics Database 2005) plus the trade deficit (from WTF). The UNIDO data for manufacturing output are considerably less complete than either WTF or WDI. We obtain manufacturing output for Brazil from the Brazilian statistical agency IBGE (2010). Our final country sample for which we have manufacturing absorption contains 57 countries. By the model in Appendix B,  $\gamma_d$  is given by (B.9). We use our WTF-WDI-UNIDO data to calculate  $\gamma_d$  by country. For the rest of the world, we set  $\gamma_d$  to the sample average over our 57 countries (average  $\gamma = .244$ ) and back out manufacturing absorption for the rest of the world from (B.9).

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