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The Adjustment of Consumption to Changing Expectations about Future Income

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The paper analyzes the role of current income in providing new information about future income and thus signaling changes in permanent income. Using time-series analysis to quantify the revision in permanent income induced by an innovation in the current income process, a structural econometric model of consumption is developed. The rejection of the joint rational expectations–permanent income hypothesis is both statistically and quantitatively significant. The paper also shows that the test of the rational expectations–permanent income hypothesis proposed by Hall is based on the reduced form of this structural model and reconciles Sargent's consumption paper with Hall's.

Because the path of future income is uncertain, the individual's permanent income and, therefore, his consumption plans will be revised period by period as new information about future income becomes available. Under the assumption that individuals hold rational expectations of future income, the stochastic properties of these period-by-period revisions in permanent income can be exploited in order to formulate testable restrictions on the joint time-series behavior of consumption and other observable variables.

This paper first reviews the econometric tests of the permanent income–rational expectations hypothesis proposed by Hall (1978) and Sargent (1978), then develops and executes an alternative approach to testing the hypothesis. Section I formally defines permanent in-

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come and derives the basic implication of the permanent income–rational expectations hypothesis: If consumption is proportional to permanent income in each period, and permanent income, in turn, represents the best estimate, given currently available information, of the individual's lifetime resources, then current consumption should diverge from consumption in the previous period by the amount of the contemporaneous revision in permanent income.

The published papers of Hall (1978) and Sargent (1978) both analyze this predicted relationship between consumption, lagged consumption, and the revision in permanent income, conditional on lagged information sets.[1] The two papers reach conflicting empirical conclusions, Hall confirming and Sargent decisively rejecting the permanent-income hypothesis. Section II argues that an incorrect definition of permanent income used in Sargent's formulation of the restrictions is the source of the discrepancy and shows that Sargent's analysis, once the definition of permanent income is corrected, leads to the same parameter restrictions tested by Hall.

The restrictions tested by Hall and Sargent apply to regressions in which all of the right-hand-side variables are lagged at least 1 period with respect to the dependent variable, consumption. By analyzing the expectation conditional on lagged information sets, their approach makes use of the property that the revision in permanent income should be uncorrelated with lagged variables. The Hall-Sargent tests can be thought of as tests based on a reduced-form consumption equation, since a reduced form expresses the expectation of an endogenous variable, conditional on predetermined variables.

This paper takes an alternative approach to testing the permanent income–rational expectations hypothesis by explicitly specifying a structural model of consumption. The model developed in Section III analyzes the role of current income in providing new information about future income and thus signaling changes in permanent income. The model is intended to underscore the point that the observed sensitivity of consumption to current income should not be interpreted simply as the behavioral marginal propensity to consume

[1] Bilson and Glassman (1979) consider some extensions on the model which I develop in this paper; Bilson (1979) reports comparative estimates of the model applied to data for the United States, United Kingdom, and Germany. Barro (1978) has also addressed the question of whether certain variables affect consumption only through their role in predicting future income. Recent empirical work on consumption by Blinder (1981) incorporates explicit treatment of the expectations of future income in the permanent-income model. Blinder's analysis concentrates on measuring the response of consumer expenditure to temporary changes in taxes rather than testing the permanent-income hypothesis itself. Hayashi (1979) also tests the permanent-income hypothesis and finds that consumption is more sensitive to current income than predicted by the hypothesis.

out of current income. Because income is a fairly highly serially correlated process, the fluctuations in current income will be correlated with fluctuations in permanent income. Formally, the model uses an autoregressive–moving average (ARMA) representation of the income time series in order to quantify the magnitude of the revision in permanent income implied by the contemporaneous observation of current income. The forecast error, or innovation, in the income time-series process represents the new information contained in the current observation on income. Based on the realization of the current innovation in the income process, individuals revise their expectations about future income. The magnitude of the revisions in expectations of future income in response to the realization of the current innovation depends on the parameters of the ARMA representation of the income process. Therefore, the revision in permanent income in a given period will be proportional to the innovation in current income, with the factor of proportionality determined by the parameters of the income time-series process. Following this approach, one can specify a structural equation relating the change in consumption to the contemporaneous revision in permanent income (modeled using the income innovation) and the change in current income. The coefficient of the change in income is a measure of the behavioral marginal propensity to consume out of current income, since the role of current income in signaling changes in permanent income has been explicitly modeled. The permanent income–rational expectations hypothesis can then be tested by testing whether the marginal propensity to consume out of current income is significantly different from zero. Having developed a test of the permanent income–rational expectations hypothesis in the context of a simple structural econometric system, it is easy to show that the Hall-Sargent test is the equivalent test based on the reduced form of the system. If the structural system is just identified, the test based on the structural system and the Hall-Sargent test based on the reduced form yield numerically identical values of the test statistic for the permanent income–rational expectations hypothesis. While the structural approach to testing the hypothesis is not statistically more powerful than the reduced-form approach, it is considerably more illuminating, since it permits the recovery of the structural parameters of the system.

The empirical results indicate that the observed sensitivity of consumption to current income is greater than is warranted by the permanent income–life cycle hypothesis, even when the role of current income in signaling changes in permanent income is taken into account. The restrictions implied by the permanent income–rational expectations hypothesis can be rejected statistically at very high confidence levels. Further, the estimates of the marginal propensity to consume out of current income are quite large, indicating that the failure of the permanent-income hypothesis is quantitatively, as well as statistically, significant.

I

In its simplest form, the permanent-income hypothesis holds that individuals ought to gear their consumption to their lifetime resources rather than to their measured income in some arbitrary time period. To start with a particularly simple case, consider the consumption behavior of an infinitely lived individual in a world in which the return to real wealth is certain and constant over time. The total resources available to the individual for current and future consumption consist of current real wealth, w_t, plus current and future labor income, x_{t+s}, $s = 0, 1, 2, \ldots, \infty$. Labor income is assumed to be exogenously determined. The individual's lifetime resources can be represented in either stock or flow form—the stock form being net worth, or the sum of real wealth and the present discounted value of current and future labor income, and the flow form being permanent income, or the annuity value of net worth.

Since the path of future income is not known with certainty, the individual must make his consumption plans on the basis of some set of expectations about future income. Given the expectations about future income held in period t, the individual's permanent income can be expressed as

$$y_t^p = r\left[w_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s+1} E_t x_{t+s}\right]. \tag{1}$$

In this discrete time formulation, w_t represents real wealth at the beginning of period t; x_t is assumed to be paid at the end of the period; and the real rate of return, r, is assumed constant.

Permanent income can be thought of as the constant resource flow which, conditional on expections in period t, can be sustained for the remainder of the individual's time horizon. Thus, permanent income ought to have the property

$$E_t y_{t+1}^p = y_t^p. \tag{2}$$

To verify that permanent income, as specified in equation (1), has this property, note that real wealth will evolve according to

$$w_{t+1} = (1 + r)w_t + x_t - y_t^p \tag{3}$$

if the individual's consumption is identically equal to his permanent income in each period. Therefore, y^p_{t+1} will be related to y^p_t according to the equation:

$$y^p_{t+1} = r\left[(1 + r)w_t + x_t - y^p_t + \sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1} E_{t+1}x_{t+s+1}\right]. \tag{4}$$

Rewriting equation (4) to separate out the component based on information which becomes available after time t and rearranging gives:

$$\begin{aligned} y^p_{t+1} = r\Bigg\{(1 + r)\left[w_t + \sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1} E_t x_{t+s}\right] - y^p_t \\ + \sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1}(E_{t+1} - E_t)x_{t+s+1}\Bigg\}. \end{aligned} \tag{5}$$

Substituting in $(1/r)(y^p_t)$ for $w_t + \Sigma_{s=0}^{\infty}\,[1/(1 + r)]^{s+1}E_t x_{t+s}$ gives:

$$y^p_{t+1} = y^p_t + r\sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1}(E_{t+1} - E_t)x_{t+s+1}. \tag{6}$$

If the expectations of future income are rational, the expectation of next period's revision in expectation, $(E_{t+1} - E_t)x_{t+s+1}$, is zero. Thus, $E_t y^p_{t+1} = y^p_t$.

Allowing for a stochastic, or transitory, component of consumption, the consumption function for the representative individual becomes $c_t = y^p_t + u_t$, or

$$c_t = r\left[w_t + \sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1} E_t x_{t+s}\right] + u_t, \tag{7}$$

where the error term u_t represents the transitory component of consumption. Solving for c_{t+1} in terms of c_t, given the wealth constraint $w_{t+1} = (1 + r)w_t + x_t - c_t$, gives:

$$c_{t+1} = c_t + r\sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1}(E_{t+1} - E_t)x_{t+s+1} - (1 + r)u_t + u_{t+1}. \tag{8}$$

Note that the coefficient of u_t is not -1 but $-(1 + r)$. Strictly speaking, permanent income and consumption will evolve as random walks only if the transitory consumption term is identically zero, $u_t \equiv 0$. Since permanent income represents the constant flow which can be sustained without changing net worth (in the infinite horizon case), consumption in excess of y^p in a given period reduces next period's net worth by u_t and the annuity value of net worth, y^p_{t+1}, by ru_t:

$$y^p_{t+1} = y^p_t + r\sum_{s=0}^{\infty}\left(\frac{1}{1 + r}\right)^{s+1}(E_{t+1} - E_t)x_{t+s+1} - ru_t. \tag{9}$$

The remainder of the paper discusses two approaches to testing the permanent income–rational expectations hypothesis. Both approaches take as their starting point the predicted relationship expressed in equation (8) between consumption, lagged consumption, and the revision in the expected lifetime budget constraint attributable to new information available since last period.

II. A Reconciliation of the Hall and Sargent Papers

Sargent substitutes average discounted expected future income, $(1 - \alpha) \sum_{j=0}^{\infty} \alpha^j E_t y_{t+j}$, where α represents the discount factor, $1/(1 + r)$, into the consumption function $c_t = B y_t^p + u_t$, to yield (p. 687):

$$c_t = B(1 - \alpha) \sum_{j=0}^{\infty} \alpha^j E_t y_{t+j} + u_t. \tag{10}$$

This section argues that average future income is not an appropriate definition of permanent income and that once the definition of permanent income is corrected, Sargent's analysis leads to the same parameter restrictions as those tested by Hall. Briefly retracing Sargent's derivation of the restrictions, average future income for date t is expressed as a weighted average of current income and average future income for date $t + 1$:

$$c_t = B\left[(1 - \alpha) y_t + (1 - \alpha)\alpha \sum_{j=0}^{\infty} \alpha^j E_t y_{t+j+1}\right] + u_t. \tag{11}$$

Since consumption in period $t + 1$ will be proportional to average expected future income in period $t + 1$, according to Sargent's specification, this equation can be written equivalently as

$$c_t = B\left[(1 - \alpha) y_t + \alpha \left(\frac{E_t c_{t+1} - E_t u_{t+1}}{B}\right)\right] + u_t \tag{12}$$

or

$$c_t = \alpha E_t c_{t+1} + B(1 - \alpha) y_t + u_t - \alpha E_t u_{t+1} \tag{13}$$

(Sargent's eq. [18]).

Equation (13), projected on information available at time $t - 1$, is the restriction imposed across the autoregressive representations of consumption and income:

$$E_{t-1} c_{t+1} = \frac{1}{\alpha} E_{t-1} c_t - B \frac{(1 - \alpha)}{\alpha} E_{t-1} y_t - \frac{1}{\alpha} E_{t-1} u_t + E_{t-1} u_{t+1}. \tag{14}$$

Rather than restrict the marginal propensity to consume (MPC) out of permanent income to be one, as in Section I, Sargent uses the more general consumption function $c_t = B y_t^p + u_t$. The basic point—that discounted future income is not the appropriate definition of permanent income—is not affected by relaxing the restriction that $B = 1$. In order to accommodate Sargent's more general consumption function, the analysis of Section I will be generalized to allow for an unrestricted MPC.

Based on the definition of permanent income as the annuity value of net worth, equation (1), and the consumption function with unrestricted MPC, c_{t+1} will be related to c_t as follows:

$$c_{t+1} = [1 + r(1 - B)]c_t + rB \sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^{s+1} (E_{t+1} - E_t)x_{t+s+1} - (1 + r)u_t + u_{t+1}. \quad (15)$$

If $B \neq 1$, consumption will be a random walk with drift instead of a simple random walk. Taking the expectation in period $t - 1$ gives:

$$E_{t-1}c_{t+1} = [1 + r(1 - B)]E_{t-1}c_t - (1 + r)E_{t-1}u_t + E_{t-1}u_{t+1}. \quad (16)$$

In comparing equation (16) with Sargent's equation for $E_{t-1}c_{t+1}$, (14), remember that $1/\alpha = 1 + r$.

The presence of the term $B(1 - \alpha)E_{t-1}y_t$ in Sargent's equation for $E_{t-1}c_{t+1}$ is an artifact of his definition of permanent income as average future income. Sargent's income series, y_t, is disposable income and therefore includes future nonlabor income as well as future labor income. However, the present discounted value of the future path of nonlabor income is not equal to current nonhuman wealth. Consider the definition of permanent income as the annuity value of net worth:

$$y_t^p = r\left[w_t + \sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^{s+1} E_t x_{t+s}\right], \quad (17)$$

where x_{t+s} is future labor earnings. The nonhuman wealth term, w_t, could be expressed as the present value of a future stream of earnings rather than as a current stock. Nonhuman wealth w_t, if maintained at that level forever, would yield a stream of nonlabor income of rw_t each period. Thus,

$$w_t = \sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^{s+1} rw_t. \quad (18)$$

Note, however, that the nonlabor income in the present value term is rw_t, the future nonlabor income the individual would receive each period *if* nonhuman wealth were maintained at w_t. Expanding equation (18) to isolate actual nonlabor income in each future period, rw_{t+s}, gives

$$w_t = \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s+1} r(w_t - w_{t+s}) + \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s+1} r w_{t+s}. \tag{19}$$

Equation (19) distinguishes between two conceptually different future revenue flows associated with w_t. The second term is simply the present value of future nonlabor earnings, which of course depends on the path of nonhuman wealth over time. The first term reflects receipts from the sale of the wealth's principal and payments to increase the principal. By defining permanent income as discounted future income, Sargent includes the second term but omits the first. Since the individual will systematically accumulate and decumulate nonhuman wealth over time in order to smooth his planned consumption path, the first term in equation (19) will not be identically zero or even constant over time. It is the omission of the term $\Sigma_{s=0}^{\infty}[1/(1+r)]^{s+1}r(w_t - w_{t+s})$ from his definition of permanent income which is responsible for the discrepancy between Sargent's version of the restrictions implied by the permanent-income hypothesis (eq. [14]) and the corrected version of the restrictions (eq. [16]).

Sargent's procedure is to impose the restrictions across the parameters of the vector autoregression of (c_t, y_t) implied by equation (14). The unrestricted autoregression of (c_t, y_t) is:

$$E_{t-1}c_t = c_1 y_{t-1} + c_2 y_{t-2} + \ldots + c_n y_{t-n} + d_1 c_{t-1} + d_2 c_{t-2} + \ldots + d_n c_{t-n} \tag{20}$$

and

$$E_{t-1}y_t = a_1 y_{t-1} + a_2 y_{t-2} + \ldots + a_n y_{t-n} + b_1 c_{t-1} + b_2 c_{t-2} + \ldots + b_n c_{t-n}. \tag{21}$$

Since $E_{t-1}c_{t+1} = c_1 E_{t-1}y_t + c_2 y_{t-1} + \ldots + c_n y_{t-n+1} + d_1 E_{t-1}c_t + d_2 c_{t-1} + \ldots + d_n c_{t-n+1}$, substituting out for $E_{t-1}y_t$ and $E_{t-1}c_t$ yields an equation for $E_{t-1}c_{t+1}$ in terms of y_{t-i} and c_{t-i}, $i = 1, n$, and the a_i, b_i, c_i, d_i parameters. Substituting these autoregressive representations for $E_{t-1}c_t$, $E_{t-1}y_t$, and $E_{t-1}c_{t+1}$ into equation (14) gives a set of restrictions across the a_i, b_i, c_i, d_i parameters. These restrictions are tested by estimating equations (20) and (21) with and without the cross-equation restrictions imposed and computing a likelihood ratio statistic.

Following Sargent's procedure, but substituting the autoregressive representations of $E_{t-1}c_t$ and $E_{t-1}c_{t+1}$ into equation (16), the "corrected" version of equation (14), provides the following set of restrictions on the parameters of the (c_t, y_t) vector autoregression:

$$\begin{aligned}
&c_1a_1 + c_2 + d_1c_1 = [1 + r(1 - B)]c_1 \\
&\ldots \\
&c_1a_{n-1} + c_n + d_1c_{n-1} = [1 + r(1 - B)]c_{n-1} \\
&c_1a_n + d_1c_n = [1 + r(1 - B)]c_n \\
&c_1b_1 + d_1^2 + d_2 = [1 + r(1 - B)]d_1 \\
&\ldots \\
&c_1b_{n-1} + d_1d_{n-1} + d_n = [1 + r(1 - B)]d_{n-1} \\
&c_1b_n + d_1d_n = [1 + r(1 - B)]d_n.
\end{aligned} \tag{22}$$

This modified set of restrictions could be tested, following Sargent's procedure, with a likelihood ratio test. However, rather than test the restrictions in their present form, I will argue that the time-series implications of the rational-expectations and permanent-income hypotheses can be stated and tested in a simpler form.

The restrictions in equation (22) are based on the projection of equation (15) on information known at time $t - 1$. Projecting equation (15) on information known at time t instead gives

$$E_tc_{t+1} = [1 + r(1 - B)]c_t - (1 + r)u_t + E_tu_{t+1}, \tag{23}$$

which is the time-series restriction tested in Hall's paper. Hall assumes that transitory consumption, u_t, is identically zero. Therefore, as formulated by Hall, the testable hypothesis implied by (23) is that other variables dated $t - 1$ or earlier have no additional explanatory value in predicting c_t if c_{t-1} is included in the regression. That is, equation (23) implies that

$$\begin{aligned}
&d_1 = [1 + r(1 - B)], \\
&d_i = 0, i = 2, \ldots, n \\
&c_i = 0, i = 1, \ldots, n
\end{aligned} \tag{24}$$

in the consumption autoregression (eq. [20]). The coefficient of lagged consumption, d_1, is estimated unrestricted. Presumably d_1 will be close to one.

The parameter values given in (24) satisfy the modified version of Sargent's cross-equation restrictions (eq. [22]): If the parameters of the consumption autoregression (c_i, d_i) are set equal to the values specified in (24), the equation restrictions stated in (22) all reduce to the form $0 \cdot a = 0$ or $0 \cdot b = 0$, so that the restrictions are satisfied for arbitrary values of the parameters of the income process (a_i, b_i). That is, imposing the restrictions $d_i = 0, i = 2, \ldots, n$; and $c_i = 0, i = 1, \ldots, n$, and estimating the income autoregression unconstrained would satisfy the restrictions in equation (22).

Note that Sargent's procedure yields restrictions in the form of a set of equations—one equation for each parameter of the consumption autoregression. In Hall's paper, however, the parameters of the consumption autoregression are constrained to particular values—lagged variables other than the first lagged value of consumption are constrained to enter with coefficients of zero. While correcting the definition of permanent income makes the two sets of restrictions consistent, in the sense that the set of parameter values in (24) satisfies the equation restrictions in (22), there remains the issue of why the two sets of restrictions are not identical in form.

The set of equation restrictions in (22) is derived by modeling $E_{t-1}c_t$ as a general (unrestricted) autoregressive process, then solving for the restrictions across the parameters of the consumption and income autoregressions which are implied by equation (16). However, since equation (23) is also an implication of the rational-expectations and permanent-income hypotheses, no additional assumptions are required to restrict the consumption autoregression to a first-order process. If the parameters of the consumption autoregression are assigned the values implied by equation (23), the equation restrictions in (22) all reduce to the form $0 = 0$. That is, once the restrictions are imposed within the consumption autoregression, no further restrictions are implied by the set of restrictions in (22). Thus the difference in form of the two sets of restrictions, (22) and (24), is attributable to the fact that, in deriving the restrictions in (22), $E_{t-1}c_t$ is modeled as an unnecessarily general autoregressive process.

At this point, the conflict between the Hall and Sargent papers concerning the restrictions on the systematic part of the consumption autoregression has been resolved in favor of Hall's formulation. The two authors also differ in their execution of the test, Sargent estimating the autoregressions with quasi-differenced data and Hall estimating the autoregression with the data in levels.

Referring to equation (15), note that the disturbance in the regression of c_t on c_{t-1} and other lagged variables will include a term which represents the revision in permanent income in period t as well as a moving average of transitory consumption. Using ξ_t to represent the revision in permanent income,

$$\xi_t = rB \sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^{s+1} (E_t - E_{t-1})x_{t+s}, \tag{25}$$

the consumption autoregression, under the null hypothesis, is:

$$c_t = [1 + r(1 - B)]c_{t-1} + \xi_t - (1 + r)u_{t-1} + u_t. \tag{26}$$

Hall assumes that transitory consumption, u_t, is identically zero. In his analytical discussion, Sargent assumes u_t is an AR1 process, $u_t = \rho u_{t-1} + \omega_t$, and quasi-differences (26):

$$c_t - \rho c_{t-1} = [1 + r(1 - B)](c_{t-1} - \rho c_{t-2}) + \xi_t - \rho\xi_{t-1} + \omega_t - (1 + r)\omega_{t-1}. \tag{27}$$

In the econometric specification of the test, however, Sargent assumes that the disturbance in the consumption autoregression is uncorrelated with the regressors (p. 689). The disturbance will be uncorrelated with the regressors only if transitory consumption is identically zero, since ω_t is correlated with c_{t-1}. Quasi-differencing the data introduces a moving average of the revision in permanent income term, ξ_t. Since Sargent in effect assumes away transitory consumption by asserting that the error is uncorrelated with regressors, it seems more sensible to leave the regression in levels and thus preserve the white-noise property of the ξ_t error term.

The results of tests of the corrected version of the restrictions are presented in table 1. Sargent's Y_1 income variable was constructed as Y_1 = GNP − capital consumption − state, local, and federal tax receipts + transfers; his consumption variable C was total personal consumption expenditures. Both series are seasonally adjusted and measured in constant (1972) dollars. For comparison, table 1 also includes similar tests using Hall's income and consumption variables, which were per capita personal disposable income and per capita expenditures on services and nondurable goods. Hall's data were also seasonally adjusted and stated in 1972 dollars.[2] Following Sargent's specification, all regressions included a linear time trend, denoted t.

Sargent conducted his tests with the data in first differences, having assumed the autoregressive parameter in the transitory consumption process to be unity. Using differenced data, the test statistic for the corrected restrictions was 19.492, which indicates rejection of the hypothesis at the 99 percent confidence level but not at the 99.5 percent level. Testing the incorrect restrictions, Sargent reported (p. 696) a marginal confidence level of 1.000 when B, the MPC out of permanent income, was constrained to equal .90. The marginal confidence level of the test was only .697 when B was unrestricted; however, the resulting point estimate was "highly implausible" (6.06×10^7).

For the reasons stated above, I think that it is more appropriate to run the regressions in levels than in differences. With the data in

[2] While the data set used to rerun Hall's regressions was constructed according to his basic definitions of the variables, it is not an exact reproduction of his data set. The data set used in table 1 differs from Hall's because it incorporates subsequent NIPA revisions of the latest observations and because a different price deflator was used to deflate the income series.

TABLE 1

Regression	Likelihood Ratio Statistic	Critical Value of $\chi^2(k)$	D-W	SER
	Sargent's Data			
Sargent's sample (1948:II–1972:IV):				
1. Differenced data:				
Restricted ($\Delta c_t = \alpha + d_1 \Delta c_{t-1} + \gamma t$)	...	...	1.95	4.04
Unrestricted (Δc_{t-i}, i = 2–4 and Δy_{t-i}, i = 1–4 included)	19.492	$\chi^2_{99.}(7) = 18.48$	1.95	3.81
2. Data in levels:				
Restricted ($c_t = \alpha + d_1 c_{t-1} + \gamma t$)	...	...	2.00	4.04
Unrestricted (c_{t-i}, i = 2–4 and y_{t-i}, i = 1–4 included)	17.054	$\chi^2_{97.5}(7) = 16.01$	1.95	3.85
Hall's sample (1948:I–1977:1, data in levels):				
Restricted ($c_t = \alpha + d_1 c_{t-1} + \gamma t$)	...	...	1.63	4.82
Unrestricted (c_{t-i}, i = 2–4 and y_{t-i}, i = 1–4 included)	17.326	$\chi^2_{97.5}(7) = 16.01$	1.95	4.62
Unrestricted (c_{t-i}, i = 2–4 included)	9.944	$\chi^2_{97.5}(3) = 9.35$	1.97	4.68
Unrestricted (y_{t-i}, i = 1–4 included)	15.712	$\chi^2_{99.5}(4) = 14.86$	2.01	4.59
	Hall's Data and Sample (1948:I–1977:I)			
Restricted ($c_t = \alpha + d_1 c_{t-1} + \gamma t$)	...	...	1.68	14.27
Unrestricted (c_{t-i}, i = 2–4 and y_{t-i}, i = 1–4 included)	12.04	$\chi^2_{90.}(7) = 12.02$	1.96	13.99
Unrestricted (c_{t-i}, i = 2–4 included)	5.004	$\chi^2_{90.}(3) = 6.25$	1.95	14.15
Unrestricted (y_{t-i}, i = 1–4 included)	10.044	$\chi^2_{95.}(4) = 9.49$	2.03	13.91

levels, the test statistic drops to 17.054, which is significant at the 97.5 percent level. The difference in sample periods does not affect the results much.

The results are sensitive to the definition of consumption, however. Using Hall's data, which omit expenditures on durable goods from the consumption variable, the test statistic drops to 12.04, which is barely significant at the 90 percent level. Thus the difference in the consumption variables used by the two authors accounts for a major part of the gap between Hall's results and those reported by Sargent. Correcting the restrictions and omitting the autoregressive transformation of the data does have a noticeable effect, though, decreasing the marginal confidence level from the 1.000 reported by Sargent to 97.5 percent.

III. The Role of Current Income in Signaling Changes in Permanent Income

The model in this section is based on the notion that, in forming their expectations of future income, rational economic agents will exploit the fact that income is a stochastic process which exhibits a high degree of serial correlation. Based on the observation that the current realization of income is greater than anticipated, agents will revise their expectations of future income, and therefore their permanent income, upward. Further, the magnitude of the revision in permanent income associated with the realization of a disturbance in current income will depend on the parameters of the time-series representation of current income. In this section, the quantitative relationship between an innovation in current income and the revision in permanent income is derived for a general ARMA representation of the income process.

This section uses the innovation in the income process to model the period-by-period revisions in permanent income and develop a simple structural econometric model of consumption. The implications of the permanent income–rational expectations hypothesis are tested in the context of the structural model. The Hall-Sargent test of the permanent income–rational expectations hypothesis is then shown to be a test based on the reduced form of the structural model developed here.

The Income Process

More formally, the time-series behavior of aggregate income is modeled as an ARMA process which is driven by a series of independent shocks or "innovations" in the process.[3] Thus,

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_p y_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \ldots + \phi_q \epsilon_{t-q}, \quad (28)$$

or $y_t = [1 + M(L)]/[1 - A(L)]\epsilon_t$ where L is the lag, or backshift, operator; $A(L) = \rho_1 L + \rho_2 L^2 + \ldots + \rho_p L^p$; $M(L) = \phi_1 L + \phi_2 L^2 + \ldots + \phi_q L^q$; and ϵ is a white-noise disturbance. The income variable is expressed in deviations from trend.

An ARMA model of order (p, q) can be equivalently characterized as a pure moving average model of infinite order. That is, the time-series representation

$$y_t = \epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \psi_3 \epsilon_{t-3} + \ldots \quad (29)$$

is numerically equivalent to the model

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_p y_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \ldots + \phi_q \epsilon_{t-q} \quad (30)$$

for $\psi_s = \phi_s + \Sigma_{j=1}^{s} \rho_j \psi_{s-j}$.

Expressed in the pure moving average form, the time-series model summarizes the relationship between the forecasted value of the variable and the realization of the disturbance term:

$$\frac{dy_{t+s}}{d\epsilon_t} = \frac{dy_t}{d\epsilon_{t-s}} = \psi_s. \quad (31)$$

Since the disturbance terms are white noise by construction, the expectation of all future values of ϵ is zero. When the current forecast error ϵ_t is realized, the predicted value of income at each date in the future is revised proportionally to ϵ_t:

$$E_t y_{t+s} - E_{t-1} y_{t+s} = \psi_s \epsilon_t. \quad (32)$$

The present value of the revision, at time t, in the anticipated path of future income is

$$\sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s (E_t y_{t+s} - E_{t-1} y_{t+s}) = \left[\sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s \psi_s\right] \epsilon_t. \quad (33)$$

[3] It should be emphasized that the paper uses the ARMA framework to quantify the extent to which income is serially correlated only because the framework is a simple way of describing the time-series behavior of income on the aggregate level. The treatment of aggregate income as a stable ARMA process is not meant to imply that individuals view income as pure time series in forming expectations about their future income. On the individual level all that is assumed is that consumers will exploit the serial correlation that characterizes their individual income streams. For instance, a student who is unable to find a summer job realizes that summer jobs will probably be scarce next year as well; the production worker who is currently laid off realizes that he may not be rehired for some weeks; the stockbroker whose commissions have declined in volume may expect his income to remain below normal for 4–6 quarters.

In the infinite horizon case, the value of $\Sigma_{s=0}^{\infty} [1/(1 + r)]^s \psi_s$ can be calculated directly from the parameters ϕ_s, ρ_j of the ARMA (p, q) representation of the process. It is shown in Appendix I that

$$\sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^s \psi_s = \frac{1 + \sum_{s=1}^{q} \left(\frac{1}{1 + r}\right)^s \phi_s}{1 - \sum_{j=1}^{p} \left(\frac{1}{1 + r}\right)^j \rho_j}. \tag{34}$$

Treatment of Unanticipated Capital Gains

In the simplest version of the model discussed in Section I, the return to real wealth, r, was assumed to be constant. This assumption implies that there are no unanticipated capital gains or losses on nonhuman wealth. In the absence of unanticipated capital gains, future labor income is the only component of permanent income which is not perfectly predictable. In this case the period-by-period revisions in permanent income would be modeled as proportional to the innovations in labor income alone.

As an empirical matter, however, unanticipated capital gains and losses on nonhuman wealth probably constitute a significant fraction of the revisions in permanent income which this model is trying to capture. The following discussion describes the construction of an approximate measure of unanticipated capital gains on nonhuman wealth.

Conceptually, the unanticipated capital gain on nonhuman wealth could be defined as the present value of the revision in the expected earnings associated with the current asset holdings:

$$\text{capital gain in period } t + 1 = \sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^{s+1} (E_{t+1} - E_t) r_{t+s} w_t, \tag{35}$$

where r_{t+s} is the rate of return to capital in period $t + s$. The actual path of nonlabor income, $r_{t+s}w_{t+s}$, will reflect both the "exogenous" movements in the return to capital, r_{t+s}, which we want to capture, and the endogenous changes in earnings flows which result from the individual's decisions to accumulate or decumulate nonhuman wealth, w_{t+s}. I assume that changes in the rate of return to capital, r_{t+s}, are quantitatively more important than the endogenous changes in w_{t+s} in determining the time-series properties of the observed path of nonlabor income. With this assumption, unanticipated capital gains are approximated as the present value of the revision in expected nonlabor income.

Repeating the derivation of the consumption equation as in Section I, the unanticipated capital gains in real wealth enter the equation symmetrically with the present value of the revision in expected labor income:

$$c_t = c_{t-1} + r\left[\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s+1}(E_t - E_{t-1})(x_{t+s} + y^k_{t+s})\right] - (1+r)u_{t-1} + u_t, \tag{36}$$

where x_{t+s} is future labor income and y^k_{t+s} is future nonlabor income.

Substituting into this equation the time-series characterization of the revision in the predicted path of future income yields:

$$c_t = c_{t-1} + r\left[\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s+1}\psi_s\right]\epsilon_t - (1+r)u_{t-1} + u_t, \tag{37}$$

where ψ_s are the parameters of the moving average representation, and ϵ_t the innovations, of the disposable income series, $y_t = x_t + y^k_t$. To simplify the notation, let $\Phi = \Sigma_{s=0}^{\infty}[1/(1+r)]^{s+1}\psi_s$. Using this notation, the present value of the revision, at time t, in the anticipated path of future income is denoted $\Phi\epsilon_t$; the revision in permanent income is denoted by $r\Phi\epsilon_t$. Equation (37) can be rewritten as:

$$c_t = c_{t-1} + r\Phi\epsilon_t - (1+r)u_{t-1} + u_t. \tag{38}$$

Finally, it is necessary to consider the effect on aggregate consumption of a trend in per capita income. While an individual's permanent income can be defined so that its movement over time is trendless, the presence of a trend in per capita income will induce a trend in aggregate permanent income and therefore in aggregate consumption. If per capita income has a positive trend, due to increasing productivity, for example, later generations will have greater lifetime wealth than earlier ones. As the older generations die and are replaced by the younger generations, aggregate consumption will trend upward at the same rate of growth as per capita income. No change in the expectation of future income is involved in this trend movement of aggregate consumption, nor does any particular individual change his level of consumption. Since the model is intended to explain revisions in planned consumption which are caused by changes in the expectations about future income, equation (38) applies to the movement of consumption around a trend attributable to the trend in per capita income.

A basic empirical implication of the model developed in Section I is that, even if the behavioral marginal propensity to consume out of current income is zero, consumption should respond to innovations in current income because these innovations provide new information about future income and therefore induce revisions in permanent income. Expressing the income forecast error in terms of current and lagged income yields the following system of equations:

$$y_t = \mu_1 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_p y_{t-p} + \epsilon_{1t}, \tag{39}$$

$$\Delta c_t = \mu_2 + k\Phi(y_t - \mu_1 - \rho_1 y_{t-1} - \ldots - \rho_p y_{t-p})$$

$$+ \beta_0 \Delta y_t + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \ldots + \beta_{p-1} \Delta y_{t-(p-1)} + \epsilon_{2t}, \tag{40}$$

where

$$\Phi = \left[\frac{1}{1 - \frac{\rho_1}{1+r} - \frac{\rho_2}{(1+r)^2} - \ldots - \frac{\rho_p}{(1+r)^p}} \right],$$

r is the quarterly interest rate, and k represents the annuity rate. (If the individual's time horizon is finite, the annuity rate will not be equal to the interest rate, as in the infinite horizon case.)

In the unrestricted version of the model, the first difference of consumption responds to current and lagged changes in income as well as the innovation in the income process. In equation (40) the β_i coefficients are measures of the "excess sensitivity" of consumption to current income, that is, sensitivity in excess of the response attributable to the new information contained in current income. The empirical work discussed in this section provides estimates of the β_i parameters and tests the implication of the permanent income–rational expectations hypothesis that consumption exhibits no excess sensitivity to current income, $\beta_i = 0$.

The permanent income–rational expectations hypothesis could be tested by estimating the system in its structural representation, equations (39) and (40), by full information maximum likelihood (FIML) both with and without the restriction $\beta_i = 0$ and forming a likelihood ratio statistic.

As discussed below, the unrestricted version of the system is just identified. Because the system is just identified, the maximum likelihood estimates of the structural parameters can be recovered from the estimated coefficients of the reduced form. Working with the reduced form has several advantages: First, estimation of the reduced form is computationally simpler than estimating the structural representation by FIML; second, analysis of the reduced form clarifies the exact relationship between the likelihood ratio test of $\beta_i = 0$ proposed here and the F-test proposed by Hall. It will be shown that Hall's test is a test based on the reduced form of the consumption equation (40). Further, it will be shown that because the structural system (eqq. [39] and [40]) is just identified, the test statistic from Hall's test, based on the reduced form of the consumption equation alone, is numerically identical to the test statistic for the hypothesis $\beta_i = 0$ in the two-equation structural system. For the purpose of determining the statistical significance of a departure from the permanent income–rational expectations hypothesis, both Hall's test and the test proposed here yield identical answers. For the purposes of assessing whether or not an observed departure from the hypothesis is of quantitative significance, however, the test proposed here has the advantage of permitting the recovery of the point estimates of the structural parameters, the β_i, and their individual standard errors, whereas Hall's procedure estimates only reduced-form coefficients.

Identification of the Model

Before discussing the identification of the system, I will make explicit my assumptions concerning the properties of the income-forecast error, ϵ_1, and the disturbance in the consumption equation, ϵ_2.

Conceptually, one would like to estimate the 1-period-ahead forecast of income, conditional on all available information, $E(y_t|I_{t-1})$. Since the estimated prediction equation (39) includes only a very limited subset of the available information, the predictive value of all the lagged values of variables observed by the individual, but not explicitly incorporated in the regression, will be reflected in the error term, ϵ_1. Using the notation $R(L)x_{t-1}$ to represent the discrepancy between the 1-period-ahead forecast of income conditional on the complete information set I_{t-1} and the income forecast conditional on the limited information set actually included in the regression, $R(L)x_{t-1} = E(y_t|I_{t-1}) - E(y_t|y_{t-1}, \ldots, y_{t-p})$, the disturbance term ϵ_1 in equation (39) will be the sum of the "true" innovation $\tilde{\epsilon}_{1t}$ and the $R(L)x_{t-1}$ term,

$$\epsilon_{1t} = \tilde{\epsilon}_{1t} + R(L)x_{t-1}. \tag{41}$$

The term $R(L)x_{t-1}$ can be thought of as measurement error due to the omission of elements of the available information set from the forecasting equation. The $R(L)x_{t-1}$ term is assumed to be serially uncorrelated and uncorrelated with the explanatory variables included in the income-prediction equation. By definition, the "true" innovation $\tilde{\epsilon}_t$ is uncorrelated with the $R(L)x_{t-1}$ term as well as the included explanatory variables.

Since the "true" income innovation, $\tilde{\epsilon}_{1t} = y_t - \rho_1 y_{t-1} - \ldots - \rho_p y_{t-p} - R(L)x_{t-1}$, is the explanatory variable in the consumption equation, the consumption disturbance ϵ_{2t} will contain the term $-k\Phi R(L)x_{t-1}$, where $k\Phi$ is the coefficient of the true innovation.

The systematic part of the consumption equation models the response of consumption to the new information contained in the current observation on income. Obviously there are many sources of new information about future income in addition to the current observation on income itself. These omitted current variables which induce individuals to revise their expectations about future income will also contribute to the disturbance in the consumption equation. Let the variable θ_t represent the revision, in period t, in permanent income induced by other (nonincome) new information about future income. The "omitted news" disturbance θ_t is assumed to be serially uncorrelated and uncorrelated with lagged variables.

Because consumption is one component of current income, the omitted news term will have a nonzero correlation with the true innovation in income, $\tilde{\epsilon}_{1t}$. If, in period t, current variables other than income induce individuals to revise their forecasts of future income upward, θ_t will be positive. The positive realization of θ_t increases current consumption and therefore increases current income. This implies that the omitted news term θ_t is positively correlated with the true income innovation, $\tilde{\epsilon}_{1t}$.

In addition to the measurement error term, $-k\Phi R(L)x_{t-1}$, and the omitted news term, θ_t, the consumption equation will, in principle, also include a moving average of transitory consumption, $\eta_t - (1 + k)\eta_{t-1}$. For the purpose of estimating the model, however, the transitory consumption component of the error term will be ignored. Even though an individual's transitory consumption in a given quarter may be nonnegligible in relation to his permanent consumption, the per capita value of transitory consumption will be small relative to permanent per capita consumption if the individual realizations of transitory consumption are largely independent across the population. I think that it is reasonable to assume that the quantitative contribution of the transitory consumption term to the overall error ϵ_{2t} is small compared to the contribution of the measurement error term, $-k\Phi R(L)x_{t-1}$, and the omitted news error term, θ_t.

This discussion leads to the following characterization of the structural disturbances:

$$\epsilon_{1t} = R(L)x_{t-1} + \tilde{\epsilon}_{1t} \tag{42}$$

and

$$\epsilon_{2t} = -k\Phi R(L)x_{t-1} + \theta_t. \tag{43}$$

Based on these assumptions, ϵ_1 and ϵ_2 are contemporaneously correlated,

$$\text{cov}\,(\epsilon_{1t}, \epsilon_{2t}) = -k\Phi\ \text{var}\ [R(L)x_{t-1}] + \text{cov}\,(\theta_t, \tilde{\epsilon}_{1t}), \tag{44}$$

but not serially correlated.

In order to simplify the discussion, identification of the system will be analyzed for a first-order AR income process. The generalization to a longer AR process is straightforward.

The unrestricted system is:

$$y_t = \mu_1 + \rho y_{t-1} + \epsilon_{1t} \tag{45}$$

and

$$\Delta c_t = \mu_2 + k\Phi\epsilon_{1t} + \beta\Delta y_t + \epsilon_{2t}, \tag{46}$$

where $\Phi = 1/\{1 - [\rho/(1 + r)]\}$. The value of the parameter k, which represents the marginal propensity to consume out of wealth, is imposed a priori. The parameter Φ, which represents the change in wealth associated with an innovation in income, is a function of the interest rate and the autoregressive parameters of the income process. The value of the interest rate is imposed a priori, so the identification of ρ identifies Φ as well.

Since $\epsilon_{1t} = y_t - \rho y_{t-1}$, the consumption equation can be rewritten as:

$$\Delta c_t = \mu_2 + (k\Phi + \beta)\epsilon_{1t} + \beta(\rho - 1)y_{t-1} + \epsilon_{2t}. \tag{47}$$

The covariances of the structural disturbances will be denoted as follows: var $(\epsilon_{1t}) = \sigma_1^2$; var $(\epsilon_{2t}) = \sigma_2^2$; and cov $(\epsilon_{1t}, \epsilon_{2t}) = \sigma_{12}$.

The parameters of the income process, μ_1, ρ, and σ_1^2, are identified by the income data alone:

$$\text{cov}\,(y_t, y_{t-1}) = \rho\ \text{var}\,(y_t), \tag{48}$$

$$\text{var}\,(y_t) = \rho^2\ \text{var}\,(y_t) + \sigma_1^2, \tag{49}$$

and

$$\mu_1 = (1 - \rho)\mu_y, \tag{50}$$

where μ_y is the mean of y.

Using the fact that ϵ_{1t} is by construction orthogonal to y_{t-1} and ϵ_{2t} is assumed uncorrelated with y_{t-1},

$$\text{cov}\,(\Delta c_t, y_{t-1}) = \beta(\rho - 1)\ \text{var}\, y_t, \tag{51}$$

$$\text{cov}\,(\Delta c_t, y_t) = \beta(\rho - 1)\ \text{cov}\,(y_t, y_{t-1}) + (k\Phi + \beta)\sigma_1^2 + \sigma_{12}, \tag{52}$$

$$\text{var}\,(\Delta c_t) = \beta^2(\rho - 1)^2\ \text{var}\,(y_t) + (k\Phi + \beta)^2\sigma_1^2 + 2(k\Phi + \beta)\sigma_{12} + \sigma_2^2, \tag{53}$$

and

$$\mu_2 = \mu_{\Delta c_t} + \beta(1 - \rho)\mu_{y_t}, \tag{54}$$

where $\mu_{\Delta c_t}$ is the mean of Δc_t.

The model is just identified—equations (48) through (54) constitute a system of seven equations in seven unknown parameters. Because the model is just identified, indirect least squares—that is, estimating the model as a reduced form, then transforming the reduced-form coefficients to recover the structural parameters—is equivalent to FIML.

Consider the following reduced form:

$$y_t = \mu_1 + \rho y_{t-1} + \nu_{1t} \tag{55}$$

$$\Delta c_t = \mu_2 + \beta(\rho - 1)y_{t-1} + \nu_{2t}, \tag{56}$$

where $\nu_{1t} = \epsilon_{1t}$ and $\nu_{2t} = (k\Phi + \beta)\epsilon_{1t} + \epsilon_{2t}$. The covariance matrix of the reduced form, written in terms of the structural covariances, is:

$$\Omega = \begin{bmatrix} \sigma_1^2 & (k\Phi + \beta)\sigma_1^2 + \sigma_{12} \\ (k\Phi + \beta)\sigma_1^2 + \sigma_{12} & (k\Phi + \beta)^2\sigma_1^2 + 2(k\Phi + \beta)\sigma_{12} + \sigma_2^2 \end{bmatrix}. \tag{57}$$

Note that the maximum likelihood estimates of the structural coefficients—μ_1, μ_2, ρ, β—are independent of the value of $k\Phi$, the parameter which measures the predicted response of consumption to the innovation in current income. That is, the parameters μ_1, μ_2, ρ, and β are identified by equations (48), (49), (50), and (54), none of which contains $k\Phi$.

Examination of equations (53) and (54), or, equivalently, the reduced-form covariance matrix, equation (57), indicates that a priori restriction on the value of $k\Phi$ makes it possible to recover the structural covariances σ_{12} and σ_2^2 from the estimated covariances of the reduced form:

$$\text{cov}\,(\nu_{1t}, \nu_{2t}) = (k\Phi + \beta)\sigma_1^2 + \sigma_{12} \tag{58}$$

and

$$\text{var}\,(\nu_{2t}) = (k\Phi + \beta)^2\sigma_1^2 + 2(k\Phi + \beta)\sigma_{12} + \sigma_2^2. \tag{59}$$

Since indirect least squares and FIML are equivalent in the just-identified case, the likelihood ratio statistic for the test of the restriction $\beta = 0$ computed by estimating the structural form, equations (45) and (46), by FIML will be numerically identical to the likelihood ratio test statistic for $\beta = 0$ computed by estimating the reduced form, equations (55) and (56), by multivariate least squares.[4]

Comparison of the Test of $\beta_i = 0$ with Hall's Test

In the equation system actually estimated, income was modeled as an eighth-order pure autoregressive process:

$$y_t = \mu_1 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_8 y_{t-8} + \epsilon_{1t}, \tag{60}$$

$$\begin{aligned} \Delta c_t = \mu_2 + k\Phi(y_t - \mu_1 - \rho_1 y_{t-1} - \rho_2 y_{t-2} - \ldots - \rho_8 y_{t-8}) \\ + \beta_0 \Delta y_t + \beta_1 \Delta y_{t-1} + \ldots + \beta_7 \Delta y_{t-7} + \epsilon_{2t}. \end{aligned} \tag{61}$$

Like the simpler system discussed above, this system is just identified. If a less general alternative hypothesis were specified—for example, if only the contemporaneous and three lagged first differences in income were included in the unrestricted model—the system would be overidentified. The particular specification of the alternative hypothesis was deliberately chosen in order to achieve just identification. Because the model is just identified, the maximum-likelihood estimates of the structural parameters can be obtained by estimating the reduced form:

$$y_t = \mu_1 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_8 y_{t-8} + \nu_{1t} \tag{62}$$

and

$$\begin{aligned} \Delta c_t = (\mu_2 + \beta_0\mu_1) + \beta_0[(\rho_1 - 1)y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_8 y_{t-8}] \\ + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \ldots + \beta_7 \Delta y_{t-7} + \nu_{2t}, \end{aligned} \tag{63}$$

where $\nu_{1t} = \epsilon_{1t}$ and $\nu_{2t} = (k\Phi + \beta_0)\epsilon_{1t} + \epsilon_{2t}$. Using the notation π_i to denote the reduced-form coefficient of y_{t-i},

$$\Delta c_t = \tilde{\mu}_2 + \pi_1 y_{t-1} + \pi_2 y_{t-2} + \ldots + \pi_8 y_{t-8} + \nu_{2t}. \tag{64}$$

Equation (63) makes very explicit the relationship between the reduced-form coefficients π_i and the underlying structural parameters:

[4] Eq. (51) suggests a simple instrumental variables estimator of the excess sensitivity parameter $\hat{\beta}$. Noting that $(\rho - 1)$ var y_t = cov $(\Delta y_t, y_{t-1})$, eq. (51) can be rewritten as

$$\text{cov}\,(\Delta c_t, y_{t-1}) = \beta\,\text{cov}\,(\Delta y_t, y_{t-1}). \tag{51$'$}$$

Thus $\hat{\beta}$ is identified as the ratio [cov $(\Delta c_t, y_{t-1})$]/[cov $(\Delta y_t, y_{t-1})$]. Writing the consumption equation with the income innovation in the error term, $\Delta c_t = \mu_2 + \beta\Delta y_t + (k\Phi\epsilon_{1t} + \epsilon_{2t})$, one could estimate β by using y_{t-1} as an instrument for Δy_t. Lagged income, y_{t-1}, is clearly a valid instrument for Δy_t, since it is correlated with Δy_t but uncorrelated with the composite error term, $(k\Phi\epsilon_{1t} + \epsilon_{2t})$. Using y_{t-1} as an instrument for Δy_t, the instrumental variables estimate of β is [cov $(\Delta c_t, y_{t-1})$]/[cov $(\Delta y_t, y_{t-1})$], which is exactly the expression for β obtained from eq. (51).

$$
\begin{aligned}
\tilde{\mu}_2 &= \mu_2 + \beta_0\mu_1 \\
\pi_1 &= \beta_0(\rho_1 - 1) + \beta_1 \\
\pi_2 &= \beta_0\rho_2 - \beta_1 + \beta_2 \\
\pi_3 &= \beta_0\rho_3 - \beta_2 + \beta_3 \\
&\;\;\vdots \\
\pi_7 &= \beta_0\rho_7 - \beta_6 + \beta_7 \\
\pi_8 &= \beta_0\rho_8 - \beta_7.
\end{aligned}
\tag{65}
$$

Regardless of the time-series process describing income, the restriction that consumption exhibits no excess sensitivity to current income, $\beta_0 = \beta_1 = \ldots = \beta_7 = 0$, implies that the reduced-form coefficients π_i are all equal to zero.

The reduced form expresses the conditional expectation of the endogenous variable, Δc_t, given the predetermined variables, which in this case are the lagged values of income. Thus, the empirical content of the restriction that $\beta_0 = \beta_1 = \ldots = \beta_7 = 0$ is that the conditional expectation of Δc_t given lagged values of income is zero. This implication of the permanent income–rational expectations hypothesis is precisely the hypothesis tested by Hall. Quoting from Hall's paper (Hall 1978, p. 976), "The tests of the stochastic implications of the life cycle–permanent income hypothesis carried out in this paper all have the form of estimating a conditional expectation, $E(c_t \mid c_{t-1}, x_{t-1})$, where x_{t-1} is a vector of data known in period $t - 1$, and then testing the hypothesis that the conditional expectation is actually not a function of x_{t-1}."

The analytical approach taken by this paper was to formulate a general structural model in which consumption responded to both innovations in permanent income and changes in current income. The restrictions imposed by the permanent income–rational expectations hypothesis on the structural parameters, $\beta_i = 0$, were shown to imply the restriction that the reduced-form parameters π_i were equal to zero. Hall's approach to formulating a test of the hypothesis avoided the econometric specification of a structural consumption equation. Eschewing structural estimation altogether, Hall analyzed the restrictions imposed by the permanent income–rational expectations hypothesis on the conditional expectation of Δc_t given lagged variables. Thus Hall arrived at the restrictions imposed on the reduced-form consumption equation by an analytical shortcut; rather than derive the reduced-form restrictions from an explicit structural econometric model, he exploited the interpretation of a reduced form as stating a conditional expectation and considered the implications of the permanent income–rational expectations hypothesis for the conditional expectation of consumption.

Although both Hall's analysis and the structural model developed in this paper lead to the same restrictions on the reduced form of the consumption equation, the test procedure used here differs from Hall's in that the restriction is tested by computing the likelihood ratio statistic for the two-equation reduced form (eqq. [62] and [63]), while Hall's test statistic is computed by estimating the consumption equation (eq. [64]) alone. As proved in Appendix II, however, the two tests yield numerically identical values of the likelihood ratio statistic for the hypothesis that the conditional expectation of consumption given lagged income is zero.[5]

The basic advantage of the structurally derived test over Hall's test is that it permits the recovery of the point estimates of the structural parameters measuring the excess sensitivity of consumption to current income, the β_i, and their individual standard errors. With Hall's test it is difficult to interpret the quantitative importance of a departure from the predicted behavior of consumption since only the reduced-form coefficients are estimated.

Data

The test of the restriction $\beta_i = 0$ is a test of the joint hypothesis that (*a*) consumers exploit the time-series properties of income in forming their estimates of permanent income and (*b*) the adjustment of consumption to a revision in permanent income is fully achieved within the quarter that the new information becomes available. In order to limit the consumption concept to a component which can be adjusted rapidly and smoothly to changes in permanent income, the consumption concept used as the dependent variable was expenditures on nondurable goods. Even if one went to the trouble of converting the National Income and Product Accounts (NIPA) data on expenditures on durable goods into a series of imputed service flows, the existence of substantial transaction costs of adjusting stocks of durable goods suggests that the consumption of services of durable goods would exhibit lagged adjustment to changes in permanent income. The

[5] I am indebted to Hall for suggesting that I estimate the model by indirect least squares instead of FIML. The observation that the test based on the two-equation reduced form and the test based on the reduced form of the consumption equation alone will yield numerically identical values of the likelihood ratio statistic is also due to Hall.

NIPA data on consumption of services are subject to a similar objection, since services are defined to include the imputed service flow from housing. Nondurable goods represent about 45 percent of total personal consumption expenditures.

Both the consumption series and the income series, which was disposable personal income, were seasonally adjusted and expressed in real per capita terms. After allowing for the construction of the lagged variables, the sample period used in the estimation was 1949:III–1979:I.

The income process was modeled as an eighth-order autoregressive process around an exponential trend. In order to simplify the estimation, the income data series was first detrended by its estimated exponential trend for the sample period 1947:I–1979:I of .00565259 per quarter, and the model was estimated with the detrended series. As mentioned above, the presence of a trend in per capita income will give rise to a trend in per capita consumption. Because the consumption variable is expenditures on nondurable goods rather than total consumption, the trend in nondurable goods consumption will not be equal to the trend in income unless the income elasticity of nondurables consumption is unity. For this reason, the consumption series was detrended by its own estimated trend, which was .0032616 per quarter. Both the income and consumption series were expressed in units of (constant) dollars per capita.

Empirical Results

The reduced form was written out in terms of the structural parameters and estimated by multivariate least squares. The unconstrained system estimated was:

$$y_t = \mu_1 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_8 y_{t-8} + \nu_{1t} \tag{66}$$

and

$$\begin{aligned}\Delta c_t = \tilde{\mu}_2 + \beta_0[(\rho_1 - 1)y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_8 y_{t-8}] + \beta_1 \Delta y_{t-1} \\ + \beta_2 \Delta y_{t-2} + \ldots + \beta_7 \Delta y_{t-7} + \nu_{2t}.\end{aligned} \tag{67}$$

The point estimates and their standard errors are reported in table 2A.

Imposing the restriction $\beta_0 = \beta_1 = \ldots = \beta_7 = 0$, the constrained system consisting of the income equation (66) and the consumption equation

$$\Delta c_t = \tilde{\mu}_2 + \nu_{2t} \tag{68}$$

was also estimated by multivariate least squares. The point estimates

TABLE 2

A. Point Estimates and Standard Errors for Equations (66) and (67)

μ_1	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8
92.54	.964	.069	.074	−.360	.112	.157	−.008	−.053
(64.26)	(.092)	(.126)	(.125)	(.122)	(.123)	(.124)	(.122)	(.085)
$\tilde{\mu}_2$	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
32.71	.355	.071	.049	−.116	.114	−.073	.050	−.012
(24.95)	(.275)	(.036)	(.038)	(.050)	(.071)	(.046)	(.036)	(.035)

B. Point Estimates and Standard Errors for Equation (68)

μ_1	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	$\tilde{\mu}_2$
55.62	.898	.067	.231	−.476	.280	−.045	.065	−.045	.068
(57.72)	(.082)	(.113)	(.112)	(.110)	(.111)	(.111)	(.110)	(.076)	(.862)

C. Reduced-Form Coefficients and Standard Errors for Equation (69)

$\tilde{\mu}_2$	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8
32.71	.058	.002	−.138	.103	−.148	.179	−.065	−.007
(25.95)	(.037)	(.051)	(.050)	(.049)	(.050)	(.050)	(.049)	(.034)

and standard errors, based on the constrained estimation, are reported in table 2B.

Using the notation L_c and L_u to denote the log likelihood of the constrained and unconstrained systems, respectively, the likelihood ratio statistic $\lambda = -2(L_c - L_u)$ for the hypothesis $\beta_0 = \beta_1 = \ldots = \beta_7 = 0$ is distributed $\chi^2(8)$. The value of the likelihood ratio test statistic, λ, was 27.024. Since the critical value of the $\chi^2(8)$ distribution is 21.96 for the 0.5 percent significance level, a test statistic of 27.0 represents a very decisive rejection of the hypothesis.

For comparison, Hall's version of the test using the reduced form of the consumption equation alone,

$$\Delta c_t = \tilde{\mu}_2 + \pi_1 y_{t-1} + \pi_2 y_{t-2} + \ldots + \pi_8 y_{t-8} + \nu_{1t}, \tag{69}$$

was also estimated. The reduced-form coefficients and their standard errors are reported in table 2C.[6] The likelihood ratio statistic for the hypothesis that $\beta_0 = \beta_1 = \ldots = \beta_7 = 0$, it is necessary to compute the computer work confirms the assertion that the test of $\beta_0 = \beta_1 = \ldots =$

[6] The estimated constant term in the constrained version of eq. (69) was .068, with a standard error of .866.

TABLE 3

STANDARD ERRORS OF REGRESSION AND DURBIN-WATSON STATISTICS

	SYSTEM ESTIMATED					
	Two-Equation Reduced Form				Single-Equation Reduced Form	
	Unrestricted		Restricted		Unrestricted	Restricted
	y	Δc	y	Δc	Δc	Δc
SER	\$22.50	\$8.73	\$23.04	\$9.45	\$8.73	\$9.45
D-W	2.00	2.15	1.90	2.13	2.15	2.13

NOTE.—y and Δc = dependent variables.

$\beta_7 = 0$ in the context of the two-equation reduced form (eqq. [66] and [67]) and the test of $\pi_1 = \pi_2 = \ldots = \pi_8 = 0$ in the context of the reduced-form consumption equation alone, equation (69), yield numerically identical values of the likelihood ratio statistic.

The standard errors of regression (SER) and Durbin-Watson statistics are reported in table 3. The SER of each equation was computed by dividing the sum of squared residuals by the degrees of freedom rather than the number of observations. The standard errors have the interpretation of the dollars per capita prediction error of each equation. For comparison, the standard deviations of y and Δc were \$71.57 and \$9.45, respectively.

The discussion of the identification of the system showed that the maximum-likelihood estimates of the β_i parameters were independent of the value of $k\Phi$, the parameter which measures the predicted response of consumption to an innovation in current income. While the identification of the parameter $k\Phi$ is not required to test the hypothesis that $\beta_0 = \beta_1 = \ldots = \beta_7 = 0$, it is necessary to compute the value of $k\Phi$ in order to recover the parameters of the covariance matrix of structural disturbances.

By substituting the estimated parameters of the income process into equation (34), one can compute the value of Φ, which gives the capitalized value of the revision in expected future income or the change in wealth associated with an innovation in current income. For the estimated parameters of the income process, and an annual interest rate of 5 percent, an innovation in current income implies a change in wealth of 17.80 times the income innovation.[7] Assuming an

[7] In order to simplify the estimation, the income process was modeled as an 8-quarter pure AR process. Formal tests of more general mixed ARMA models of the income process identified the (3, 0, 4) specification (three AR parameters and four MA parameters) and the (1, 0, 4) specification as the most parsimonious mixed ARMA

average horizon of 20 years, the annuity rate is 0.0198 per quarter.[8] The change in permanent income associated with an innovation in current income is (0.0198)(17.80), or 0.35 times the income innovation. Since the consumption variable used to estimate the system was nondurables consumption rather than total consumption, the parameter $k\Phi$ will be the product of the marginal propensity to consume nondurables out of permanent income and the change in permanent income induced by an innovation in current income. With nondurable consumption accounting for 47.5 percent of total consumption, $k\Phi = (0.475)(0.0198)(17.80) = 0.1675$.

The estimated covariance matrix of the disturbances of the unrestricted reduced form is:

$$\hat{\Omega} = \begin{bmatrix} 467.808 & 79.8921 \\ 79.8921 & 70.5175 \end{bmatrix}.$$

Having identified the value of $k\Phi$ as 0.1675 and the value of β_0 as .355, the covariance matrix of the structural disturbances can be recovered using equation (57). The estimated covariance matrix of the structural disturbances is:

$$\hat{\Sigma} = \begin{bmatrix} 467.808 & -164.538 \\ -164.538 & 114.745 \end{bmatrix}.$$

Note that the covariance between the structural disturbances of the two equations, cov (ϵ_1, ϵ_2), is negative and very large relative to the variance of the disturbance in the consumption equation, ϵ_2. In the discussion of the properties of the structural disturbances, it was pointed out that the omission of elements of the available information set from the income-forecasting equation implies that the income residual is subject to measurement error and that the presence of measurement error gives rise to a negative covariance between the structural disturbances. The large negative estimate of cov (ϵ_1, ϵ_2) suggests that the measurement error introduced by using the income-forecast error as a measure of the "true" income innovation cannot be dismissed as negligible. The finding that the measurement error is quantitatively important indicates that attempts to estimate $k\Phi$

representations of the income process. The value of Φ implied by the (3, 0, 4) model was 16.41; by the (1, 0, 4) model, 15.45. Since the ARMA models identified through formal time-series analysis of the income series yield values of Φ very similar to the value of Φ computed from the 8-quarter AR process, there is no indication that the assumed pure AR process is inadequate in capturing the time-series properties of income.

[8] The quarterly annuity rate was calculated with the following formula: annuity rate $= r/(1 - e^{-rT})$. The quarterly interest rate, r, was assumed to be 0.0125; the horizon of the representative individual, T, was assumed to be 80 quarters.

directly by doing regressions of the change in consumption on the income residuals will be subject to severe downward bias.

The discrepancy between the predicted and observed behavior of consumption is quantitatively large as well as statistically significant. Since the dependent variable is consumption of nondurable goods rather than total consumption, the β_i coefficients represent the excess sensitivity of nondurables consumption alone. Considering that nondurable goods consumption represents 45 percent of total consumption expenditures, the point estimates of β_0, β_1, and β_2 of .355, .071, and .049, respectively, indicate a strong "excess" response of consumption to current income. Unfortunately, the estimate of the parameter β_0 is imprecise, as indicated by its fairly large standard error of .275.

Because of the large sampling error of the estimate of the crucial parameter β_0, these tests stop short of providing conclusive evidence that the permanent income–rational expectations hypothesis fails in a quantitatively significant way. Still, the large point estimate β_0, together with the statistical rejection at the 0.5 percent level of the zero restrictions on the β_i coefficients, constitute significant, if not conclusive, evidence against the hypothesis.

In Hall's paper, tests of the predictive value of lagged income did not reveal strong evidence against the permanent-income hypothesis.[9] Hall concluded, "The results of this paper have the strong implication that beyond the next few quarters consumption should be treated as an exogenous variable. . . . With respect to the analysis of stabilization policy, the findings of this paper [support] the view that policy affects consumption only as much as it affects permanent income" (p. 986).

Why is it that the tests reported here reveal substantial evidence against the permanent-income hypothesis, whereas Hall's did not? As shown above, the test based on the two-equation reduced form is not intrinsically more powerful than Hall's test. The structurally derived reduced-form test provides stronger results for two reasons, one obvious and the other more subtle. First, by permitting the recovery of the structural parameter estimates measuring the excess sensitivity of consumption to current income, the β_i, the structurally derived reduced-form test makes it possible to assess the quantitative significance of a departure from the null hypothesis.

Of the regressions reported by Hall, the regression of current consumption (of nondurable goods and services) on lagged consumption and four lagged values of income is closest to the reduced

[9] Hall was able to statistically reject the permanent-income hypothesis based on tests of the predictive value of a stock price index.

form estimated in this paper. On page 983 Hall reports the following coefficient estimates and standard errors.

$$\underset{}{c_t} = \underset{(11)}{-23} + \underset{(.047)}{1.076c_{t-1}} + \underset{(.043)}{.049y_{t-1}} - \underset{(.052)}{.051y_{t-2}} - \underset{(.051)}{.023y_{t-3}} - \underset{(.037)}{.024y_{t-4}}. \tag{70}$$

Testing the hypothesis that the coefficients of all four lagged values of income were zero, Hall got an F-statistic of 2.0, which was high enough to reject the hypothesis at the 10 percent level, although not the 5 percent level. Noting that the first lagged value of income had a small positive coefficient and that the sum of the coefficients was small and negative, Hall concluded, "There is a statistically marginal and numerically small relation between consumption and very recent levels of disposable income" (p. 984).

The statement that the relationship between consumption and disposable income is "numerically small" refers to the magnitude of the reduced-form coefficients, the π_i, not the structural parameters β_i. Hall is explicit on the point that his regressions did not provide estimates of structural parameters: "No claim is made that the true structural relation between consumption and its determinants is revealed by this approach" (p. 977).

The obvious advantage of the structurally derived reduced-form test is that it permits the estimation of the structural parameters. In equation system (54), the reduced-form coefficients are written out in terms of the underlying structural parameters. Equation system (68) indicates that even if consumption responds strongly to current and lagged changes in income, that is, the β_i are large and positive, the reduced-form parameters will be small. Consider in particular the reduced-form coefficient of y_{t-1}:

$$\pi_1 = \beta_0(\rho_1 - 1) + \beta_1. \tag{71}$$

When nondurables consumption alone is used to form the dependent variable, the estimate of β_0 is .355 and the estimate of β_1 is .071. However, since the first autoregressive parameter in the income process, ρ_1, is estimated to be .964, the value of π_1 is only .058. When the consumption concept is defined as nondurable goods and services, β_0 and β_1 are estimated to be .461 and .128, respectively. The resulting reduced-form coefficient π_1 in this case is .112. In addition to the 4-quarter distributed lag, Hall reported the results for a test using a 12-quarter Almon lag. In both cases the sum of the reduced-form parameters was small and negative. Using equation system (65), note that

$$\sum_{i=1}^{8} \pi_i = \beta_0 \left(\sum_{i=1}^{8} \rho_i - 1 \right). \tag{72}$$

The sum of the autoregressive parameters of the income process is about .95 for quarterly data. Therefore, even if β_0 has a healthy positive value, the sum of the reduced-form parameters will be small and negative.

While the test based on the reduced form of the consumption equation alone gives the same test statistic as the test based on the two-equation reduced form, it does not by itself provide any information about the quantitative importance of a departure from the null hypothesis. Observing that the reduced-form coefficients are small does not imply that the structural coefficients are small. To the contrary, large positive values of the β_i coefficients are associated with reduced-form parameters which are small and which sum to a negative value.

The second, more subtle advantage of the structural approach is as follows: Because it requires the explicit specification of a structural model, the structural approach provides some guidance concerning the specification of the reduced form.

While Hall's approach of analyzing the restrictions on the conditional expectation of consumption led to the correct conclusion that the reduced-form coefficients of lagged variables should be zero, it did not suggest any criteria for selecting a particular collection of variables to be included in the reduced form. By studying the structural specification, one learns that, under the alternative hypothesis ($\beta_0 > 0$), any lagged value of income which has a nonzero coefficient in the income autoregression will in general have a nonzero coefficient in the consumption reduced form.

Further, if one believes that the income process is best modeled as an AR process around an exponential trend, then the consumption reduced form should be fit to the detrended income series rather than the raw income series. Alternatively, one could leave the income series in its original (not detrended) form and include an exponential trend term, $e^{\gamma t}$, in the consumption reduced form. In Hall's original tests, the income series was not detrended and the reduced form did not include a trend term. If the income process does include a trend, Hall's tests are misspecified under the alternative hypothesis. One would expect that the failure to allow for the trend in income, which causes the reduced form to be misspecified under the alternative hypothesis, would reduce the power of the test. As an empirical matter, the treatment of the income trend does have a noticeable effect on the value of the likelihood-ratio statistic.[10] In all of the tests reported above, the income series have been detrended.

[10] In a comment on Hall's paper, Startz (1979) pointed out that the inclusion of an exponential trend in the regression tended to increase the value of the test statistic.

Having shown that the test based on the bivariate reduced form and Hall's test based on the single-equation reduced form yield numerically identical values of the likelihood ratio statistic for comparable specifications, it is necessary to determine why the rational expectations–permanent income hypothesis is rejected at the 0.5 percent level in the tests reported here, while the hypothesis could not be rejected at the 5 percent level in Hall's original tests. In particular, one would like to know whether the difference in the test statistic is attributable to differences in the data sets, or to differences in the specification of the alternative hypothesis, in terms of the number of lagged values of income included and the treatment of the trend in income.

Hall used a broader concept of consumption—expenditures on nondurable goods and services—than the consumption concept, nondurable goods, used in this paper. Data on real per capita consumption of nondurables and services were constructed to provide a direct comparison to Hall's original results. While the data set used to rerun Hall's regressions was constructed according to his basic definitions of the variables, it is not an exact reproduction of his data set. The data set used here differs from Hall's because it incorporates subsequent NIPA revisions of the most recent observations and because a different price deflator was used to deflate the income series. In the regression of current consumption on lagged consumption and four lagged values of income, Hall obtained an F-statistic for the exclusion of the lagged values of income of 2.0, which is somewhat below the critical value of 2.45 for the 5 percent level. For the same sample period and specification, the test statistic was 2.75 using the reconstructed data set. Thus, minor differences in the construction of the data raise the significance level of the test statistic from somewhat below to somewhat above the 5 percent level. Consideration of the structural model indicates that the reduced form should include as many lagged values of income as the income autoregression and that some allowance for a trend in income should be made, either by detrending the income series or by including a trend term in the consumption reduced form. Using the original (not detrended) income series but increasing the number of lagged values of income included in the consumption reduced form from four to eight increased the value of the likelihood-ratio statistic from 10.95 to 22.616. This raises the significance level of the test from the 5 percent level to the 0.5 percent level. Detrending the income series also has a substantial effect on the test statistic. For the specification with four lagged values of income, the test statistic was 16.492 for the detrended income series (which is significant at the 0.5 percent level) as opposed to 10.95 for the raw income series. For the specification with eight lagged values of income, detrending the income series raises the test statistic from 22.616 to 27.108. Thus, the insight concerning the appropriate specification of the reduced form which is obtained by explicitly formulating the underlying structural model was responsible for raising the significance level of the test from a marginal rejection to a decisive rejection.

IV. Conclusions

This paper develops a simple structural econometric model of consumption. In the general specification of the model, consumption responds to the changes in permanent income signaled by innovations in the current income process and to changes in current income itself. The response of consumption to current income beyond that attributable to the role of current income in signaling changes in permanent income is termed the "excess sensitivity" of consumption to current income. The econometric model is then used to estimate the excess sensitivity of consumption to current income and to test the implication of the permanent-income hypothesis that the excess sensitivity of consumption is zero.

The tests reveal substantial evidence against the permanent-income hypothesis. Using either nondurables consumption or consumption of nondurables and services as the dependent variable, the hypothesis that consumption exhibits no excess sensitivity to current income can be rejected at the 0.5 percent level. Using expenditures on nondurable goods as the consumption variable, the point estimate of the excess sensitivity of consumption to the contemporaneous change in income is .355. Keeping in mind that nondurable goods represent only a fraction of total personal consumption expenditures—about 45 percent—a point estimate of .355 represents a large departure from the permanent-income hypothesis.

The paper also shows that the test of the permanent income–rational expectations hypothesis proposed by Hall (1978) can be thought of as a test based on the reduced form of the structural model developed here. Rather than develop a structural econometric model and derive the associated restrictions on the reduced-form parameters, Hall's approach was to analyze the implications of the permanent income–rational expectations hypothesis for the conditional expectation of consumption given lagged variables. Since a reduced form has the interpretation of expressing a conditional expectation, Hall's analysis led directly to testable restrictions on the parameters of a reduced-form consumption equation. When the structural model is just identified, both tests yield numerically identical values of the test statistic. While the structural approach does not provide a statistically more powerful test, in general, than the reduced-form approach, it does have the advantage of providing estimates of the structural parameters of the model, including estimates of the "excess sensitivity" of consumption to current income.

Appendix I

For the general model

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_p y_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \ldots + \phi_q \epsilon_{t-q} \quad \text{(A1)}$$

$$\sum_{s=0}^{\infty} \frac{dy_{t+s}}{d\epsilon_t} = \sum_{i=0}^{\infty} \psi_i, \quad \text{(A2)}$$

where the ψ_i are the infinite series of moving average parameters in the pure moving average representation of the model: $y_t = \epsilon_t + \psi_1\epsilon_{t-1} + \psi_2\epsilon_{t-2} + \ldots$. Solving for ψ_i in terms of the estimated ρ_i and ϕ_i,

$$\psi_0 = \phi_0 = 1 \quad \text{(A3)}$$

$$\frac{dy_t}{d\epsilon_{t-i}} = \frac{\partial y_t}{\partial \epsilon_{t-i}} + \sum_{j=1}^{i} \frac{\partial y_t}{\partial y_{t-j}} \frac{dy_{t-j}}{d\epsilon_{t-i}} \quad \text{for } i = 1, \infty \quad \text{(A4)}$$

$$\psi_i = \phi_i + \sum_{j=1}^{i} \rho_j \psi_{i-j} \quad \text{(A5)}$$

where $\rho_j = 0$ for $j > p$, $\phi_i = 0$ for $i > q$,

$$\sum_{i=1}^{\infty} \psi_i = \sum_{i=1}^{\infty} \phi_i + \sum_{i=1}^{\infty} \sum_{j=1}^{i} \rho_j \psi_{i-j} \quad \text{(A6)}$$

$$\text{adding } \psi_0 = \phi_0, \quad \text{(A7)}$$

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \phi_i + \sum_{i=1}^{\infty} \sum_{j=1}^{i} \rho_j \psi_{i-j} \quad \text{(A8)}$$

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \phi_i + \sum_{j=1}^{\infty} \rho_j \sum_{i=j}^{\infty} \psi_{i-j} \quad \text{(A9)}$$

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \phi_i + \sum_{j=1}^{\infty} \rho_j \sum_{i=0}^{\infty} \psi_i \quad \text{(A10)}$$

$$\left(1 - \sum_{j=1}^{\infty} \rho_j\right) \sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \phi_i \quad \text{(A11)}$$

$$\sum_{i=0}^{\infty} \psi_i = \frac{\sum_{i=0}^{\infty} \phi_i}{1 - \sum_{j=1}^{\infty} \rho_j} = \frac{1 + \sum_{i=1}^{q} \phi_i}{1 - \sum_{j=1}^{p} \rho_j}. \quad \text{(A12)}$$

Appendix II

Proof that the likelihood ratio statistic for the test of $\gamma = 0$ in the equation

$$\Delta c_t = \mu + \gamma y_{t-1} + \nu_t \tag{A13}$$

is numerically identical to the likelihood ratio statistic for the test of $\beta = 0$ in the bivariate reduced form:

$$\underset{(T \times 2)}{[y_t \quad \Delta c_t]} = \underset{(T \times 2)}{[1 \quad y_{t-1}]} \underset{(2 \times 2)}{\begin{bmatrix} \mu_1 & \mu_2 \\ \rho & \beta(\rho - 1) \end{bmatrix}} + \underset{(T \times 2)}{[\nu_{1t} \quad \nu_{2t}]}. \tag{A14}$$

The GLS transformation of equation (A14) is obtained by postmultiplying by the lower triangular matrix P such that $PP' = \Omega^{-1}$, where Ω is the covariance matrix of the reduced form:

$$P = \begin{bmatrix} P_{11} & 0 \\ P_{12} & P_{22} \end{bmatrix}.$$

It will be convenient to normalize P such that the lower-right-hand element is equal to one. Define

$$\tilde{P} = \begin{bmatrix} \tilde{P}_{11} & 0 \\ \tilde{P}_{12} & 1 \end{bmatrix}$$

where $\tilde{P}_{11} = (P_{11}/P_{22})$ and $\tilde{P}_{12} = (P_{12}/P_{22})$. Transforming (A14) by $\tilde{P}$ yields:

$$\tilde{P}_{11}y_t + \tilde{P}_{12}\Delta c_t = (\tilde{P}_{11}\mu_1 + \tilde{P}_{12}\mu_2) + y_{t-1}[\tilde{P}_{11}\rho + \tilde{P}_{12}\beta(\rho - 1)] + \tilde{\nu}_{1t} \tag{A15}$$

and

$$\Delta c_t = \mu_2 + y_{t-1}\beta(\rho - 1) + \nu_{2t} \tag{A16}$$

where $\tilde{\nu}_{1t} = \tilde{P}_{11}\nu_{1t} + \tilde{P}_{12}\nu_{2t}$.

Let L^1 denote the log likelihood of the transformed income equation (A15), L^2 denote the log likelihood of the consumption equation (A16), and $L^{1,2}$ denote the log likelihood of the bivariate system. Since cov $(\tilde{\nu}_{1t}, \nu_{2t}) = 0$, the value of the log likelihood function of the bivariate system is the sum of the values of the log likelihood of the two individual equations: $L^{1,2} = L^1 + L^2$. Imposing the constraint that $\beta = 0$ reduces the value of L^2, $L_c^2 < L_u^2$. Imposing the restriction that $\beta = 0$ does not restrict the log likelihood function of the transformed income equation, since μ_1 and ρ are unrestricted parameters, $L_c^1 = L_u^1$. Therefore, all of the decrease in the likelihood function of the bivariate system is attributable to the decrease in the likelihood function of the consumption equation alone: $L_u^{1,2} - L_c^{1,2} = L_u^2 - L_c^2$.

References

Abel, Andrew, and Mishkin, Frederic S. "On the Econometric Testing of Rationality and Market Efficiency." Report 7933, Univ. Chicago, Center Math. Studies Bus. and Econ., August 1979.

Ando, Albert, and Modigliani, Franco. "The 'Life Cycle' Hypothesis of Saving: Aggregate Implications and Tests." *A.E.R.* 53 (March 1963): 55–84.

Barro, Robert J. "Permanent Income and Expected Future Income in a Time Series Study of Consumer Expenditure." Manuscript. Rochester, N.Y.: Univ. Rochester, April 1978.

Bilson, John F. O. "The Rational Expectations Approach to the Consumption Function: A Multi-Country Study." Paper presented at the International Seminar on Macroeconomics, Paris, September 1979.

Bilson, John F. O., and Glassman, James E. "A Consumption Function with Rational Forecasts of Permanent Income." Manuscript. Chicago: Univ. Chicago, February 1979.

Blinder, Alan S. "Temporary Income Taxes and Consumer Spending." *J.P.E.* 89, no. 1 (February 1981): 26–53.

Friedman, Milton. *A Theory of the Consumption Function.* Princeton, N.J.: Princeton Univ. Press (for Nat. Bur. Econ. Res.), 1957.

Hall, Robert E. "Stochastic Implications of the Life Cycle–Permanent Income Hypothesis: Theory and Evidence." *J.P.E.* 86, no. 6 (December 1978): 971–87.

Hayashi, Fumio. "Estimation of Permanent Income Consumption Functions under Rational Expectations." Manuscript. Evanston, Ill.: Northwestern Univ., April 1979.

Muth, John. "Optimal Properties of Exponentially Weighted Forecasts." *J. American Statis. Assoc.* 55 (June 1960): 299–306.

Sargent, Thomas J. "Rational Expectations, Econometric Exogeneity, and Consumption." *J.P.E.* 86, no. 4 (August 1978): 673–700.

Startz, Richard. "Comment on 'Stochastic Implications of the Life Cycle–Permanent Income Hypothesis.'" Unpublished paper. Philadelphia: Univ. Pennsylvania, Wharton School, February 1979.