When the Gambler's Fallacy becomes the Hot Hand Fallacy: An Experiment with Experts and Novices

Justin M. Rao*†
UC San Diego

October 20, 2009

Abstract

The "gambler's fallacy" (GF) and "hot hand fallacy" (HHF) are two related inference mistakes – GF is a belief in too little serial correlation, while HHF is a belief in too much. Theoretical models have shown that both can be explained by an underlying belief in the "law of small numbers." This study uses a subjective probability sequence (professional basketball shooting) to see if GF and HHF can both exist within a single agent. Subjects were experienced basketball fans or novices. Across subjects, there is statistically significant GF after short streaks and HHF after long streaks. On the individual level, a transition from GF to HHF is most common. Consistent GF or HHF was rare and the expert sample generally exhibited bias of a lower magnitude as compared to the novice sample.

*jmrao@ucsd.edu

†I would like to thank James Andreoni for continued guidance and support and Nageeb Ali, David Eil, Uri Gneezy, Craig McKenzie, Joel Sobel, and Charles Sprenger for helpful comments and discussion.

1 Introduction

The gambler's fallacy (GF) is a belief that a streak (consecutive occurrences of an outcome) of a given length is more likely to end than dictated by Bayesian inference, i.e. a underestimation of the degree of autocorrelation. The hot hand fallacy (HHF) is the inverse belief – a streak is believed to continue with irrationally high probability. There is considerable evidence for both these inference mistakes, yet they appear to be in direct opposition to each other. In an effort resolve this apparent inconsistency, researchers in both psychology and economics have argued that both biases are grounded in a belief in the "law of small numbers" (Tversky and Kahneman 1971, Rabin and Vayanos 2009).

A common root cause generates an interesting testable implication: if an agent believes in the law of small numbers then he should show evidence of both GF and HHF. The biases are not in competition, rather they complement each other and which one will be operational is situationally dependent. I test this hypothesis using a subjective probability sequence: shots in a professional basketball game. Beliefs were inferred from the subjects' predictive bets. The majority of subjects exhibited GF for certain past realizations and HHF for other sets of past outcomes – the transition between the two biases was a predictable function of streak length. The results show that GF and HHF can coexist and add credence to the argument that they are based in the same underlying reasoning mistake. A consequence is that it is not very informative to identify a person as GF-type or HHF-type. Rather the importance should be placed on the cognitive process, which is argued (following other authors) to be a use of the "representative heuristic," and the environmental factors that determine the operational bias.

GF was first identified in the laboratory in the "probability matching" experiments of the 1950's, although the etymology indicates the notion predates laboratory study (Estes 1950). Field evidence of GF includes betting in parimutuel lotteries (Terrell 1994, Clotfelter and Cook 1993) and roulette wagers in casinos (Croson and Sundali 2005). There is also strong evidence in the literature for the HHF. Mutual funds have been shown to have independent returns from year to year (Carhart 1997) yet investors have a preference for "hot" funds (Sirri and Tufano 1998). Pro basketball shooting skill has been shown to be serially uncorrelated (Gilovitch, Villone and Tversky 1985, Rao 2009a) but National Basketball Association (NBA) players take sub-optimally difficult shots following a string of makes (Rao 2009a) and spectators show a strong belief that skill level is affected by past shot outcomes (Gilovitch et al. 1985).

Tversky and Kahneman (1971) first argued that both GF and HHF could be explained by the representativeness heuristic – the belief that small samples should match their parent population (it is also known as the “law of small numbers”) . Rabin (2002) and Rabin and Vayanos (2009) formalize this argument mathematically. The intuition behind the Rabin model is that belief in “law of small numbers” generates a belief in frequent outcome alternation (GF). This is because consecutive occurrences of a given outcome (streaks) are unrepresentative of the parent population so they are expected to end. However, long streaks violate this expectation of alternation, to reconcile the observed outcomes the success rate of the underlying Data Generating Process (DGP) is revised. This pattern of revision leads to a belief in positive autocorrelation because strings of successes lead to upward revisions in the success rate, which helps explain the “surprising” (to the agent) lack of alternation observed in the sequence.

The model predicts that there should be a transition from GF to HHF when there is uncertainty about the underlying DGP. Sequences generated with a subjective probability measure (e.g. mutual fund performance, shooting accuracy) are better candidates for model revision than those generated from objective probabilities (e.g. spins of a roulette wheel). As such, we may not observe the GF to HHF transition in the objective cases. Consistent with this reasoning, Croson and Sundali (2005) find that after streaks of 5 or more roulette outcomes (colors) casino gambler’s showed significant GF (for streaks of 4 or less no biases were evident) but not HHF. More generally the field evidence which for HHF typically involves subjective processes such as mutual fund returns while evidence supporting GF involves objectively random processes such as draws from urns in a laboratory. This pattern also finds support in the laboratory (Burns and Corpus 2004).

In this experiment, subjects made outcome predictions of shots taken in a NBA game. The game tape was edited so that a “freeze-frame” displayed just as the ball left the shooter’s hand. During this interval, subjects placed a bet on the outcome of the shot, with payoffs weighted by the shot’s objective difficulty in a method described in detail in section 2. The payoffs were designed to mimic returns to stocks on an efficient exchange – subjects were incentivized to bet “hit” when they felt the shooter was more likely than usual to make that particular shot (i.e. conditional on distance, defense, etc.) and bet “miss” if they felt the shooter was less likely to make the shot than their performance on that type of shot would dictate.

Basketball shooting is a sequence traditionally thought to lead to the HHF (Gilovitch et al. 1985), yet I find significant GF for short streaks and HHF for longer streaks, especially among the subjects who reported that they did not watch or play basketball (novice group). Subjects who reported watching more than 30 basketball games per season (expert group) exhibited the same patterns but with lower magnitude and lacking statistical significance. However both groups display a statistically significant relationship between the two biases.

While most subjects displayed GF for short streaks and HHF for long streaks, there was a significant minority with precisely the opposite pattern. Indeed subjects who consistently displayed either GF or HHF across streak lengths were rare. The result provides strong support for the common root cause hypothesis, shows the interdependence between the two sets of beliefs and highlights the importance of context in discussing behavioral biases.

2 Experimental Design

The experiment was conducted at the University of California San Diego economics laboratory. Upon entry to the subject database, potential participants were asked background questions about the amount of basketball they typically watched. Two groups of subjects were invited to participate in the study: those who reported watching more than 30 games per season and those that reported watching 0.[1] These groups constituted the "expert" and "novice" sub-samples respectively. 70 subjects (39 experts and 31 novices) participated across six 1-hour sessions, earning $14.33 on average.

Subjects viewed game tape from the 2007/08 Los Angeles Laker's season. The tape was edited so that for some shots, referred to as "payoff shots," a freeze-frame was displayed for 10 seconds just as the ball was leaving the shooter's hand. During this interval subjects bet either "hit" or "miss" and chose to wager either 1 or 2 experimental points (EPs), worth $0.25 each. Two EPs were endowed for every payoff shot. If a bet was not placed within the 10-second interval it was not eligible for payment and was not used in the data analysis. After the freeze-frame, the tape rolled and the outcome of the shot was displayed.

The first 3 quarters of the Feb 3, 2008 game vs. the Washington Wizards and first half of the Feb 8, 2008 game vs. the Orlando Magic were used in the study.[2] Subjects were told in advance that 2 games were being used and were given a five minute break between games. The tape only included the Lakers' shots so more shots could be displayed in the time frame. It was played without sound so that the behavior of subject's could not be influenced by the commentator's opinions.[3] For all shots, the shooter's name was prominently displayed on screen so that subjects could easily identify the player. For payoff shots the shooter's name remained on screen for the entire freeze-interval. There were 37 payoff shots out of a total of 107 shots. The tape duration was 37.5 minutes.

The challenge of using a subjective probability sequence is formulating payoffs so that

[1] A post-questionnaire confirmed the answers to these questions and was used to place subjects in each group.

[2] The experimental game tape is available from the author, but can only be used for private viewing due to copyright restrictions.

[3] Basketball commentators frequently refer to players as "heating up" or "on fire."

beliefs concerning serial dependence can be inferred. NBA players take shots of widely varying difficulty. For example, 40% is considered a very good shooting percentage for a long-range shot while an open shot from 10 feet is made over 70% of the time. If payoffs were constant across shot difficulty, there would be little variation in bets based on perception of autocorrelation because shot difficulty (or perceptions thereof) would drive betting behavior.

The solution was to weight payoffs by difficulty. This was possible because the game tape used came from a larger sample, collected by the author and employed in other papers, of 60 games from the 2007/08 Laker's season consisting of over 5000 shots.[4] The data set was built through game logs and by actually watching the games to record all the relevant shot conditions. The shot conditions (distance, defensive pressure, shot type, time on the shot clock, physical location) were used to predict success rates ($\hat{p}$) for the shots used in the experiment. The data set and regressions used to formulate the success rates is described in more detail in Rao (2009a) and Rao (2009b).

Predicted success rate $\hat{p}$ is taken as the measure of shot difficulty. Accordingly, a 1 point bet on hit won $\frac{1}{\hat{p}}$ if correct and 0 otherwise. A single bet on miss earned $\frac{1}{1-\hat{p}}$ if correct and 0 otherwise.[5] Notice that this means that if one felt the model predictions (based on observable shot conditions) were correct, she would be indifferent between "hit" and "miss" as both offer the same expected value in this case. The experimenter carefully explained how payoffs were constructed and informed the subjects that it was in their financial interest to bet "hit" when they felt the player was more likely to make the shot than predicted by the model which used *only observable shot conditions* and bet "miss" when they felt the player was less likely to make the shot than predicted by the model. This point was reinforced by the notion that they goal was to "beat the model" and they had a financial incentive to do so. The shot conditions used were read to the subjects twice, specifically these did not include past shot outcomes, however the experimenter did not draw attention to this omission for fear of tainting the sample.[6]

Without the quantification of success probability, it would be difficult, if not impossible to study GF and HHF type beliefs with a subjective sequence such as professional basketball shots. Payoffs were designed to mimic those of purchasing or selling stock on an efficient exchange. Notice that in both cases, in the absence of private information both possible actions (buy or short, bet "hit" or "miss") offer the same expected value. In such an

[4]The data set was also used to support the result cited earlier that NBA players exhibit a belief in the hot hand through taking sub-optimally difficult shots following a string of hits.

[5]$\hat{p}$ was not displayed on screen. The purpose of the design was to pick up beliefs in autocorrelation not to see if say, subjects were correct/incorrect about the difficulty of certain shots. However, $\hat{p}$ was displayed on the instructional tape. This was done to add credibility to the design and to help explain payoffs.

[6]No subjects asked about the omission of past shot outcomes. All questions were answered privately.

environment beliefs about autocorrelation are likely to affect observed behavior.

Importantly, subject's were not told that the study was examining beliefs about serial dependence – the words "hot hand" or "gambler's fallacy" were not used in the instructions. An instructional video was shown with three example shots and displayed payoffs for each bet. It gave the subjects a feel for the length of the freeze-interval and added credibility to the design. Full instructions are in the appendix.

3 Results

The main method of analysis is to examine how betting behavior changes as a function of a player's past performance. The debate over whether the hot hand truly exists in professional basketball has raged since Gilovitch, Villone and Tversky (1985) famously proclaimed that it does not. The result however was criticized because the analysis did not control for shot difficulty. Rao (2009a), using the data which generated $\hat{p}$ for this study, shows that controlling for difficulty there is no evidence of increased ability after makes or decreased ability after misses. The paper not only finds no evidence for the hot hand it also bounds possible hot hand effects as being very minimal, if they exist at all.[7]

Table 1 examines how betting behavior changes with past performance. The dependent variable in columns 1-3 is bet outcome ("hit" or "miss") and the dependent variable in columns 4-6 is wager amount (1 or 2 EPs). The table supports the main claim of the paper GF to HHF transition. Subject fixed-effects and shot difficulty controls (cubic in $\hat{p}$) serves to reduce the influence of risk aversion.[8]

[7]The analysis cannot reject the existence of very rare departures from a stationary model. So if a player is thought to get "hot" a few times per season, this cannot be rejected by the data. What is rejected is a systematic dependence on past performance.

[8]Rao (2009a) finds that players take harder shots when they are on a make streak. If subject's had an unwillingness to bet hit for hard shots (risk aversion) this would work against finding hot hand effects. Including a flexible difficulty solves this problem.

Table 1: Fixed-Effect OLS Regressions, Effect of Streaks on Betting Behavior

Dependent Variable	1{Bet="Hit"}			Wager Amount		
Sample	All	Experts	Novices	All	Experts	Novices
Player made last shot	-0.0455	-0.0367	-0.0567	0.0150	0.0249	0.00262
	(0.033)	(0.043)	(0.052)	(0.036)	(0.049)	(0.052)
Player made last 2 shots	-0.0735**	-0.0320	-0.126**	-0.124***	-0.188***	-0.0437
	(0.033)	(0.043)	(0.051)	(0.035)	(0.048)	(0.051)
Player made last 3 shots	0.0506*	0.0178	0.0918**	0.0729**	0.137***	-0.00780
	(0.030)	(0.040)	(0.047)	(0.033)	(0.044)	(0.047)
Player missed last 2 shots	-0.103***	-0.134***	-0.0626	-0.0363	-0.121**	0.0707
	(0.037)	(0.048)	(0.057)	(0.039)	(0.054)	(0.057)
Cubic $\hat{p}$ control	X	X	X	X	X	X
Subjects	70	39	31	70	39	31
Obs	2520	1404	1116	2520	1404	1116

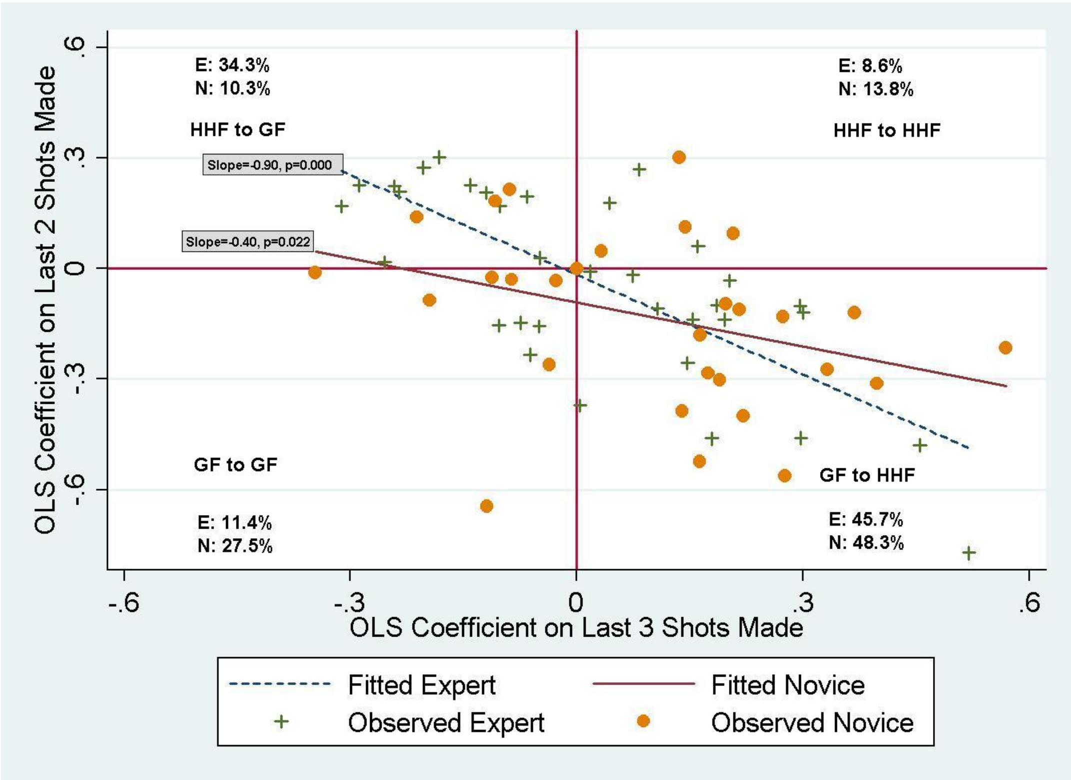
Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

We see that novice subjects were significantly more likely to bet "miss" following 2 made shots by the player (GF). The causal effect of the third shot made was a significant increase in the probability of betting "hit" (HHF). However it must be noted that since GF beliefs dominate for short sequences, the hot hand belief serves as a correction. In aggregate, expert subjects showed the same pattern with respect to string of hits, but with lower magnitudes and lacking statistical significance. They show significant HHF after 2 straight misses by the player; novice subjects do not exhibit this pattern. The effect of a single shot made or missed was indistinguishable from zero for both groups.

The idea behind giving subjects a choice in wager amount was to infer strength of belief. Table 1 shows that novice subjects did not change their betting behavior systematically with past outcomes. It was not, as one might suspect, simply that they were always wagering 1 EP; the mean for both groups was similar (1.54 experts, 1.52 for novices). Experts wagered significantly more after a player's string of 3 makes. Initially this is puzzling since column (2) shows they show little pattern in their shot predictions. The reason is given in Figure 1, which is explained in detail below. While on net the expert subjects did not show a significant propensity to bet "hit" after 3 makes, there is a group of subjects with significantly positive coefficients and a group with significantly negative coefficients. The resultant aggregate estimate is near zero and belies the fact that beliefs are indeed changing.

The regressions in columns (2) and (3) were run subject-by-subject to estimate individual coefficients. Figure 1 shows the relationship between the coefficient on "player made last 2 shots" and "player made last 3 shots." The percentages of subjects falling in each quadrant support result presented in Table 1 as the most common beliefs are GF after 2 makes and HHF after 3 makes (the lower right quadrant). The upper right quadrant, representing the classical hot hand belief, is virtually empty.



The figure also plots OLS linear fits, which reveal a statistically significant negative relationship between holding GF and HHF beliefs ($p = 0.0000$ for experts and $p = 0.022$ for novices). If the subject displayed GF for streak length 2 then HHF is predicted for streak length 3. If HHF is observed for streak length 2 then GF is predicted for streak length 3. The former transition is precisely the prediction of the Rabin (2002) model and this is the modal pattern. Furthermore, the relative magnitudes of the bias are positively related (another feature of the model). Contrary to the model's predictions a significant fraction of transitions are HHF to GF. That is, the relationship between the two inference mistakes is strong regardless of the initial bias. This an interesting finding as there do not appear to be GF-types or HHF-types, rather subjects flip from one bias to the other depending on the circumstance.

Had I found a null result (i.e. no effect of streaks) one could argue that keeping track of the many concurrent player streaks is too difficult a task for experimental subjects. In fact this criticism would be even stronger for novice subjects as they are unfamiliar with the players and the speed of professional basketball. However, even though these novice subjects were never told monitoring past player performance was important, their shot predictions indicate they naturally did so and responded in systematic ways.

In the post questionnaire, subjects were asked about their belief in the hot hand and gambler's fallacy. 32 (of 39) experts and 22 (of 31) novices reported that they believed a player is more likely than normal to make a shot following a made shot and streak of 3 makes (they reported similar beliefs after a single made shot). 19 experts and 14 novices reported they would bet black in roulette after a streak of reds (only 2 of each reported they would stay with red, the rest were indifferent).

78% subjects reported a belief in the hot hand for basketball while only 7% report a belief in a "gambler's hand" (15% said past outcomes did not matter). However, these reported beliefs do not show up in betting behavior. Appendix Table 1 presents regressions of the OLS coefficients from Figure 1 on the self-reported GF and HHF beliefs. For experts, the reported beliefs are totally uncorrelated with their bets. For novices, a reported HHF belief predicts a significantly higher coefficient on "last 2 made" and a lower coefficient on "last 3 made." The sign switch is not surprising in light of Figure 1. The results lend further credence to the argument that treating these biases as unchanging "types" and surveying people to determine their type is a flawed methodology.

In an interesting contrast to the beliefs of observers, basketball players themselves appear to only believe in the hot hand, if at all. Rao (2009a) shows that players who respond to their past performance (half the sample) do so by taking more difficult shots, passing the ball less and shooting more frequently. The response to streaks of 3 or 4 makes is generally stronger than the response to a streaks of 1 or 2 makes, but the direction is the same. That is, there is no evidence of switching into easier shots and shooting less frequently after short sequences of makes. This suggests a dichotomy between the beliefs of observers of a sequence and the practitioners themselves – interested readers are directed to Rao (2009a).[9]

One reason for the great interest in both GF and HHF is that it can help explain departures from rationality in the financial markets. The disposition effect, the tendency to lock in small gains in equity positions but not close out small losses, can be explained by GF beliefs.[10] HHF beliefs can potentially drive bubbles, as stocks get "hot" they increase

[9]Understanding the players' beliefs likely involves an account of the physiological feedback of a make versus a miss. A discussion of why players believe in the hot hand when there is no evidence it exists is beyond the scope of this paper.

[10]On an efficient exchange, selling at any point is rational in the absence of transactions costs. However, for many investors locking in small gains puts a drag on their net returns due to the accumulation of transactions costs.

further beyond levels driven by fundamentals.

In light of these observations, I examine betting behavior as a function of the success and failure of past bets in Table 2. Notice that the unit of observation has changed and is now beliefs about how their bet returns are correlated over time.[11] After a string of correct bets, a "personal hot hand" belief dictates sticking to the current bet and increasing the wager amount; a personal "gambler's hand" belief would be revealed through a pattern of switching the bet and reducing the bet number. For strings of past bet failures, switching indicates a "cold hand" belief and sticking with current bet can interpreted as "my luck is bound to change." Roughly, these correspond to HHF and GF respectively, although some authors posit that HHF refers a belief in positive autocorrelation following success only (i.e. a belief in the hot hand does not imply a belief in a cold hand).

Column (1) examines subjects' propensity to keep their bet unchanged as a function of past betting performance. The coefficients on the past success dummies are all negative, indicating switching after past success. Individual significance levels are confounded by the co-linearity of the regressors, but a Wald test rejects that they are jointly zero ($p = 0.049$, two-tailed test). There was significantly more switching after 3 incorrect bets (HHF) and significantly less after 4 incorrect bets (GF). This provides further evidence of the interconnectedness of the two biases and is our first example of a HHF to GF transition in aggregate. Column (2) shows that there is a slight pattern of decreasing wager amount after past success and increasing after past failures, however none of the coefficients are statistically significant and they have a small magnitude.[12]

[11]It is also natural to examine the effect of "team streaks." Regressions available from the author show that team streaks had little effect (slight evidence of GF). For this reason and due to space considerations, the analysis of team streaks is not included in the paper.

[12]In contrast, Croson and Sundali (2005) find that casino gambler's display personal hot hand effects with bets in craps and roulette.

Table 2: Fixed-Effect OLS Regressions, Effect of Past Bet Outcomes on Current Round Betting Behavior

Dependent Variable	1{Bet=Last Bet}	Wager Amount
1{Last bet correct}	-0.00567	-0.00519
	(0.0247)	(0.0237)
1{Last 2 bets correct}	-0.00291	-0.00469
	(0.0301)	(0.0289)
1{Last 3 bets correct}	-0.0391	-0.0553
	(0.0429)	(0.0412)
1{Last 4 bets correct}	-0.0729	-0.0122
	(0.0471)	(0.0452)
1{Last 2 bets incorrect}	0.0125	0.0247
	(0.0336)	(0.0323)
1{Last 3 bets incorrect}	-0.110*	0.0350
	(0.0602)	(0.0578)
1{Last 4 bets incorrect}	0.232**	-0.115
	(0.0930)	(0.0893)
Cubic $\hat{p}$ control	X	X
Subjects	39	31
Obs	1404	1116

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

It is interesting to note that in both columns, streaks of length 2 or shorter had very little effect – all coefficients are estimated to be within 0.005 of zero. Juxtaposed to the findings of Table 1 this result shows that the effect of streak length is situationally dependent. Rabin (2002) models a belief in the law of small numbers using an agent who perceives random outcomes as draws *without replacement* from an urn of size N. The size of the hypothetical urn determines how "unusual" the agent views streaks. With a large urn many streaks naturally occur, while with a small urn oscillation is expected to kick in quite quickly. In the language of the Rabin model, the urn appears bigger for inference of personal autocorrelation as compared to autocorrelation of the professional basketball players.

4 Conclusion

The hot hand fallacy and gambler's fallacy are seemingly opposing biases in the perception autocorrelation. HHF says that people over-estimate autocorrelation while GF says they underestimate it. Yet previous research has found strong evidence for both. To reconcile this apparent inconsistency, authors have argued that both biases have a common root cause in a belief in the law of small numbers. Rabin (2002) presents a formal model of this argument.

If the mistakes have a common cause, then not only should we expect both to manifest themselves in experimental and field data, but also they should coexist within a single agent. I find strong support for this (perhaps initially perplexing) assertion. Most subjects in the study display GF for short streaks and HHF for long streaks. While the GF to HHF transition predicted by the Rabin model is most common, there is a significant portion of subjects who display precisely the inverse pattern. Somewhat surprisingly, even though the sequence used (basketball shot outcomes) is thought to be the hot hand's domain, less than 10% of subjects showed consistent hot hand beliefs.

The results of this paper can be applied to decisions in financial markets and everyday choices alike. Indeed most probability sequences people encounter are likely more similar to the subjective world of basketball shooting than the cold objective world of roulette wheels, dice and urns that have been the standard vehicle to analyze inference mistakes. The payoffs used in the study duplicate those of returns on an efficient exchange. Using this realistic environment, the paper finds that GF and HHF can peacefully coexist within a single agent.

5 Appendix

5.1 Tables

Appendix Table 1: Correlation of Post-Questionaire
GF and HHF Questions with Observed Behavior

Dependent Variable	β Last 2 Made	β Last 3 Made	β Last 2 Made	β Last 3 Made
Sample	Expert	Novice	Expert	Novice
1{Reported GF Belief}	-0.0744	0.00113	0.0718	0.00114
	(0.0796)	(0.0814)	(0.0654)	(0.0665)
1{Reported HHF Belief}	0.000260	0.200**	-0.0222	-0.113
	(0.0600)	(0.0726)	(0.0524)	(0.0902)
Constant	0.00401	-0.269***	0.00106	0.174*
	(0.0543)	(0.0624)	(0.0451)	(0.0977)
Subjects	39	30	39	30

*** p<0.01, ** p<0.05, * p<0.1

Robust standard errors in parentheses

Appendix Table 2: Game Tape Shot Details
Feb 3, 2008 vs. Washington Wizards

Player	**Possession**	**Hit**	$\hat{p}$	**Payoff Shot**
Lamar Odom	1	0	0.29	0
Kobe Bryant	2	0	0.17	0
Kobe Bryant	3	1	0.37	0
Kobe Bryant	4	0	0.49	1
Ronny Turiaf	5	1	0.48	0
Derek Fisher	6	0	0.38	0
Vladimir Radmanovic	7	1	0.92	0
Kobe Bryant	8	1	0.63	0
Derek Fisher	9	1	0.56	1
Kobe Bryant	10	0	0.17	0
Lamar Odom	11	0	0.56	0
Ronny Turiaf	12	0	0.24	0
Kobe Bryant	13	1	0.63	0
Kobe Bryant	14	0	0.38	1
Vladimir Radmanovic	15	1	0.14	1
Kobe Bryant	16	0	0.15	0
Lamar Odom	17	1	0.46	1
Derek Fisher	18	0	0.52	0
Kobe Bryant	19	1	0.34	1
Kobe Bryant	20	1	0.22	1
Kobe Bryant	21	1	0.91	0
Sasha Vujacic	22	0	0.23	1
Jordan Farmar	23	0	0.08	0
Lamar Odom	24	0	0.25	0
Lamar Odom	25	0	0.29	0
Lamar Odom	27	1	0.5	1
Lamar Odom	28	1	0.53	0
Jordan Farmar	29	0	0.21	0
Luke Walton	30	1	0.4	0
Lamar Odom	31	1	0.29	1
Jordan Farmar	32	1	0.23	0
Sasha Vujacic	33	1	0.53	1
Vladimir Radmanovic	34	1	0.45	1
Kobe Bryant	35	1	0.38	1
Kobe Bryant	36	0	0.22	1
Ronny Turiaf	37	1	0.09	0
Kobe Bryant	38	1	0.56	0
Ronny Turiaf	39	0	0.36	0
Kobe Bryant	40	0	0.12	0
Derek Fisher	41	1	0.5	1
Ronny Turiaf	42	1	0.39	0
Vladimir Radmanovic	43	1	0.44	1
Ronny Turiaf	44	0	0.15	0
Kobe Bryant	45	1	0.31	0
Kobe Bryant	46	0	0.22	1
Vladimir Radmanovic	47	1	0.43	1
Ronny Turiaf	48	0	0.24	0
Ronny Turiaf	49	0	0.44	0
Ronny Turiaf	50	1	0.94	0
Ronny Turiaf	51	1	0.36	0
Vladimir Radmanovic	52	0	0.45	1
Derek Fisher	53	0	0.38	0
Lamar Odom	54	1	0.96	0
Vladimir Radmanovic	55	1	0.43	1
Vladimir Radmanovic	56	0	0.51	1
Ronny Turiaf	57	0	0.17	0
Lamar Odom	58	1	0.27	1
Sasha Vujacic	59	0	0.47	0
Jordan Farmar	61	0	0.3	1
Jordan Farmar	62	0	0.31	1
Sasha Vujacic	63	1	0.46	1

Appendix Table 2: Game Tape Shot Details
Feb 8, 2008 vs. Orlando Magic

Player	Possession	Hit	$\hat{p}$	Payoff Shot
Kobe Bryant	1	0	0.31	1
Kobe Bryant	2	1	0.33	0
Kobe Bryant	3	1	0.7	1
Pau Gasol	4	1	0.44	0
Pau Gasol	5	0	0.27	0
Kobe Bryant	6	0	0.38	1
Vladimir Radmanovic	7	1	0.94	0
Kobe Bryant	8	0	0.38	0
Kobe Bryant	9	0	0.58	0
Kobe Bryant	10	1	0.91	0
Kobe Bryant	11	0	0.14	0
Derek Fisher	12	0	0.36	0
Pau Gasol	13	1	0.74	0
Kobe Bryant	14	0	0.39	1
Derek Fisher	15	0	0.19	0
Vladimir Radmanovic	16	1	0.43	1
Kobe Bryant	17	1	0.91	0
Vladimir Radmanovic	18	1	0.42	1
Lamar Odom	19	1	0.19	0
Pau Gasol	20	1	0.52	0
Vladimir Radmanovic	21	0	0.44	1
Kobe Bryant	22	1	0.39	0
Jordan Farmar	23	0	0.38	0
Kobe Bryant	24	1	0.55	0
Kobe Bryant	25	0	0.39	1
Ronny Turiaf	26	0	0.36	0
Kobe Bryant	27	0	0.32	0
Kobe Bryant	28	0	0.13	0
Kobe Bryant	29	1	0.93	0
Sasha Vujacic	30	1	0.5	1
Jordan Farmar	31	0	0.31	0
Ronny Turiaf	32	0	0.49	0
Sasha Vujacic	33	0	0.23	0
Pau Gasol	34	1	0.56	0
Kobe Bryant	35	0	0.26	0
Lamar Odom	36	1	0.65	0
Sasha Vujacic	37	1	0.47	1
Kobe Bryant	38	0	0.38	1
Sasha Vujacic	39	0	0.52	0
Pau Gasol	40	0	0.41	1
Kobe Bryant	41	0	0.26	0
Kobe Bryant	42	0	0.39	0
Pau Gasol	43	0	0.25	0
Pau Gasol	44	1	0.25	0
Sasha Vujacic	45	1	0.47	0

5.2 Experimental Instructions

[Not For Publication]

Welcome

Thank your for participating in our experiment. We will begin shortly. Today's experiment will last about 1 hour. Although earnings will vary by subject, most subjects will earn between $10 and $25.

Informed Consent

Placed in front of you is an informed consent form to protect your rights as a subject. Please read and sign it. If you would like to choose not to participate in the study you are free to leave at this point. If you have any questions, we can address those now.

Anonymity

Your anonymity in this study is assured. Your name will never be collected or connected to any decision you make here today. Your email address was collected for invitation purposes only and will never be connected to your decisions. Furthermore, your earnings will be paid in a sealed envelope with your subject number so that even those running the study will not know your earnings. No other subject in the study will know any of your choices or performance in the study.

Rules

- Quiet please, please do not talk or communicate with other subjects.
- Please turn your cell phones off.
- If you have a question at any point, just raise your hand.
- Please put away any books, papers, computers, etc. that you have brought with you.

Your Earnings

The decisions you make today will determine your earnings. We will explain exactly how earnings will be calculated at the appropriate time. Your earnings will be paid in cash, placed in a sealed envelope with your subject number. Although earnings will vary by subject, most subjects will earn between $10 and $25.

Today's Experiment

In this experiment, you will be asked to predict the outcomes of shots taken in NBA basketball games. You were chosen for the experiment because you indicated on your entry into the subject database that you are a basketball fan. During the experiment you will be shown 5 quarters of basketball from the 2007/08 Lakers' season (i.e. last season). Specifically you will be watching the first 3 quarters of one game and the first half of another. The tape has been edited so only the Lakers' shots are shown. While the opponents shots are edited out, none of the Lakers' shots are skipped. The tape will be played without sound. The shooters' names are displayed on screen to help you identify the player shooting the ball. The tape duration is 36 minutes.

The game will be projected on the screen in front of you.

For a number of shots, called "payoff shots", the tape is freeze-framed just as the ball leaves the shooter's hand. The freeze-frame will remain on the screen for 10 seconds. During this time you will be asked to provide your prediction of the shot outcome. The exact method of your payment will be explained shortly – it will depend directly on the accuracy of your predictions. At the end of the instructions an example tape of three shots will be played to show you exactly how things work.

During the experiment you will make predictions for 43 payoff shots. For each payoff shot you will be endowed 2 experimental points (EPs), each valued at $0.20. For each payoff shot you are required to "bet" at least 1 and at most 2 of the endowed EPs. We will explain shortly the payoffs for these bets and how they are actually "placed". At the end of the experiment your EPs will be converted to cash and you will be paid in an envelope with your subject number.

The payoffs for your shot prediction bets are a bit complicated, so we will go through them slowly.

For the 2007/08 Lakers' season, a data set of 60 games (over 5000 shots) was compiled by watching the games and recording shot distance, location, defensive pressure, shot type (i.e. turnaround, fadeaway, hook) and if the shot was forced due to less than 5 seconds on the shot clock. Every effort was made to record all relevant shot conditions. These variables, , where used to predict the shooting percentage for every shot in the sample. The predictions were made by player, to allow for differences in ability and shot preference. Specifically the the model used to predict shot success was a popular binary dependent variable routine called "Probit."

The econometric model provides an estimated success rate for every shot taken on the edited game tape you will view. For each shot, we will call this estimated success rate $\hat{p}$ – this is a probability ranging from 0 (no chance of making) to 1 (made with certainty) and it can take any value in between. For example if $\hat{p}$=0.5, this means that the model predicts that 50% of the time the shot will be made, while 50% of the time the shot will be missed.

In front of you are 37 numbered “Bet Cards” each corresponding to a payoff shot. On them there are two words “hit” and “miss” and a blank by the word “Bets”. For each shot you circle the “hit” or “miss” to make your bet and write the number of bets you would like to place, 1 or 2, in the blank beside the word bets.

The monetary incentives in this study are designed so that you should bet “hit” when you feel the shot is more likely to go in *than predicted by the model* which uses *observable shot conditions alone* and “miss” when you feel the shot is *less likely to go in based on the model's predictions*. To those ends, the payouts are as follows: for every EP bet on “hit” you will earn $\frac{1}{\hat{p}}$ EPs if the shot goes in and 0 EPs if the shot is a miss; for every EP you bet on miss you will earn $\frac{1}{1-\hat{p}}$ EPs if the shot is in fact a miss and 0 EPs if the shot goes in. In a sense, your goal is to “beat the model” and these payoffs ensure that you have a financial incentive to try and do so.

Here is an example: suppose the payoff shot has a predicted success rate of 0.40 (40%). In this case a 1 point bet on “hit” earns 1/.4=2.5 points if the shot goes in and 0 points if the shot is a miss; a 1 point bet on “miss” earns 1/(1-.4)=1.67 points if the shot is a miss and 0 points if the shot goes in. Notice that in this example the outcome that the model predicts is less likely, hit, offers a higher payoff. If you felt that the model's predictions were exactly right then both bets have the same expected value: .4*(1/.4)+.6*0=1 for a one point bet on hit and .6*(1/.6)+.4*0=1 for a one point bet on miss. If you think “hit” is more likely than the model predicts (i.e. above 40%) then a bet on hit offers expected value in excess of 1 point whereas a bet on “miss” offers expected value less than 1. As such, in that hypothetical case you should bet on hit to make the most money on average. Conversely if you think the true success probability is less than the model's predictions for the shot, you are best served betting “miss.”

In front of you there is as stack of note cards, numbered 1-43 corresponding to the payoff shots that will be displayed in that order. When you a see freeze-frame, you must place your bet for that shot within the 10 second “freeze interval”. To do so, circle either “hit” or “miss” and write down the number of bets, 1 or 2, you would like to place on that shot. You must then place the card in the box placed in front of you. To be eligible for payment, the card must be in the box before the 10 second freeze frame ends and the shot outcome is revealed. Please place the cards face down in the box, as this will help us when we compute payments at the end. We will be passing through the aisles to ensure that all bets are placed within the allotted time.

In a few moments we will watch the sample game tape and answer questions. To recap:

1. For every payoff shot you will be endowed 2 EPs, you must bet at least 1 of these points.
2. If you bet on "hit" you will earn $\frac{1}{\hat{p}}$ points if the shot goes in and 0 points if the shot is a miss
3. If you bet on "miss" you will earn $\frac{1}{1-\hat{p}}$ points if the shot is a miss and 0 points if the shot is a hit
4. $\hat{p}$ is the predicted success rate for the shot based only on the shot's observable conditions, these were formulated used a data set of 5,000 shots
5. You must place your bets during the 10 second freeze frame window, after this interval the tape will roll and the outcome of the shot will be shown
6. The payoffs are designed so that you will make the most money on average by betting **"hit" when you think the shot is more likely to go in than predicted by the model** and **"miss" when you think the shot is less likely to go in than predicted by the model.**
7. Another way of putting this is that your job is to "beat the model."

The following video shows three sample payoff shots, after the shots the predicted success probabilities are shown and the amount of points that would have been earned for a bet on each outcome are shown.

[Video shown]

Are there any questions? The game tape will now begin, it will last 36 minutes. After this your payments will be computed and the experiment will conclude. Please try to pay close attention throughout the tape. The validity of our research depends on your honest efforts, as does your monetary payoff.

References

Burns, Bruce and Bryan Corpus, "Randomness and Induction from Streaks: Gamblers Fallacy vs. Hot hand," *Psychonomic Bulletin and Review*, 2004, *11*, 179–184.

Carhart, Mark M., "On Persistence in Mutual Fund Performance," *The Journal of Finance*, 1997, *52* (1), 57–82.

Clotfelter, C.T. and P.J. Cook, "The "Gambler's Fallacy" in Lottery Play," *Management Science*, 1993, pp. 1521–1525.

Croson, Rachael and James Sundali, "The Gambler's Fallacy and the Hot Hand: Empirical Data from Casinos," *Journal of Risk and Uncertainty*, May 2005, *30* (3), 195–209.

Estes, W.K., "Toward a Statistical Theory of Learning," *Psychological Review*, 1950, *57* (2), 94–107.

Gilovitch, Thomas, Robert Villone, and Amos Tversky, "The Hot Hand in Basketball: On the Misperception of Random Sequences," *Cognitive Psychology*, 1985, *17*, 295–314.

Rabin, Matthew, "Inference By Believers in the Law of Small Numbers," *Quarterly Journal of Economics*, 2002, *117* (3), 775–816.

——— **and Dimitri Vayanos**, "The Gambler's and Hot-hand Fallacies: Theory and Applications," 2009. SSRN Working Paper.

Rao, Justin M., "Experts' Perceptions of Autocorrelation: The Hot Hand Fallacy Among Professional Basketball Players," 2009a. UCSD.

———, ""He Got Game" Theory: Optimal Decision Making and the NBA," 2009b. UCSD.

Sirri, Erik R. and Peter Tufano, "Costly Search and Mutual Fund Flows," *The Journal of Finance*, 1998, *53* (5), 1589–1622.

Terrell, Dek, "A Test of the Gambler's Fallacy: Evidence from Pari-mutuel Games," *Journal of Risk and Uncertainty*, May 1994, *8* (3), 309–17.

Tversky, Amos and Daniel Kahneman, "Belief in the Law of Small Numbers," *Pyschonomic Bulletin*, 1971, *76*, 105–110.

Varian, H.R., "Differences of opinion in financial markets," in "Financial Risk: Theory, Evidence and Implications: Proceedings of the 11th Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis" 1989, pp. 3–37.