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PRIVATELY PROVIDED PUBLIC GOODS IN A LARGE ECONOMY: THE LIMITS OF ALTRUISM

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Private charity has often been modelled as a pure public good. The results reported in this paper, however, suggest that this model of altruism fails to confirm even the broadest empirical observations about charity. In particular, as the size of the economy grows, the fraction contributing to the public good diminishes to zero. This and other results imply that this approach leads to a very limited model with little, if any, predictive power. A truly descriptive model of privately provided public goods must be generalized to include other non-altruistic motives for giving.

1. Introduction

In the literature on private charity it is customary to model the charity as a pure public good in the Samuelsonian sense.[1] Individual utility is assumed to be a function of the consumption of private goods and of the total supply of the public good. Individuals are taken to gain no utility from their gift per se. Stated differently, preferences are assumed to be purely altruistic.

It is generally agreed, however, that giving is motivated by many things other than altruism. Guilt, sympathy, an ethic for duty, a taste for fairness, or a desire for recognition may all influence an individual's contribution to charity. The question is can the traditional model of altruistic giving be general enough to capture the important and interesting aspects of privately provided public goods. The results reported in this paper suggest that the traditional model fails to confirm even the broadest empirical observations about charity. A pure public goods approach to altruism is therefore severely limited.

Three facts characterize the charitable sector of the American economy. First, there is vast participation. According to two national surveys, over

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[1]See Olson (1965), Hochman and Rodgers (1969), Schwartz (1970), Becker (1974), Warr (1982), Young (1982), and Roberts (1984, 1987). Non-Nash variants of the Samuelsonian model have been considered by Cornes and Sandler (1984) and by Sugden (1985).

85% of all households make donations to charities. Over 50% of all tax returns include deductions for charitable giving. The United Way fund boasts millions of contributors. Second, both aggregate and individual gifts are large. In total, the charitable sector of the American economy accounts for about 2% of GNP. Average giving was over $200 per household in 1971, ranging from $70 for the lowest income quartile to $350 for the highest quartile. In addition, religious organizations collected about $10 billion in 1981, health organizations and hospitals raised over $7 billion, and civic orchestras received $150 million in donations.[2] These foundations and others depend all or in part on substantial amounts of voluntary donations. Third, when the government joins with the private sector in providing the public good, government donations *incompletely* crowd out private sector donations. Econometric studies indicate that a one dollar increase in government contributions to 'charitable activities' is associated with a decrease in private giving of only 5 to 28 cents.[3]

This paper shows, under general conditions, that the traditional model fails to explain either the extensive or the intensive nature of giving. Nor does it predict incomplete crowding out at the levels observed. The predictions of the traditional model are, in fact, exactly the opposite. In large economies free riding is not only natural, but it dominates. In the limit, only an infinitely small proportion of the economy contributes and, furthermore, average giving converges to zero. Finally, joint provision with the government can have virtually no impact on the total supply of the public good. Crowding out will be exactly (or *almost* exactly) dollar-for-dollar.

These results imply that an assumption of altruistic preferences leads to a very limited model with little, if any, predictive power. A truly descriptive model of privately provided public goods must be generalized to include other non-altruistic motives for giving.

2. Privately provided public goods in a large economy

This section examines how the equilibrium in the traditional model changes as the number of individuals in the economy, n, grows toward infinity. The analysis makes use of the fact that when people differ in their endowments of wealth, the population can be separated into those who contribute to the public good and those who do not. What is examined, then, is how the relative sizes of these two groups change as the economy becomes large. In sections 2.1 and 2.2 individuals are taken to have identical

[2]These figures are quoted from the 1975 National Survey of Philanthropy, Survey Research Center; Giving USA, the 1981 Annual report of the American Association of Fund Raising Council; and from the 1970–1971 Consumer Expenditure Survey, as reported by Reece and Zieschang (1985).

[3]See Abrams and Schmitz (1978, 1984), and Clotfelter (1985, pp. 78–79).

preferences, while in section 2.3 the results are extended to a world of heterogeneous tastes.

2.1. Identical preferences

Assume for simplicity that there is only one private good and one public good. Let x_i be consumption of the private good by person i and let g_i be i's gift to the public good. Individuals are endowed with exogenous wealth w_i. Assume that the total services of the public good can be measured in dollars and so $G=\sum_{i=1}^{n} g_i$ is the total provision of the public good. Utility is represented by a continuous and strictly quasi-concave function $U_i=U(x_i, G)$. This is a traditional, purely altruistic, model of privately provided public goods.

Individual donations functions can be found by solving the maximization problem

$$\max_{x_i, g_i} U(x_i, G)$$

$$\text{s.t. } x_i+g_i=w_i,$$

$$g_i \geqq 0.$$

Let $G_{-i}=\sum_{j\neq i} g_j$ be the gifts of everyone but person i. Alternatively, $G_{-i}=G-g_i$. Assuming that individuals behave as Nash utility maximizers, and so treat G_{-i} as constant, it follows that this maximization problem is equivalent to

$$\max_{x_i, G} U(x_i, G)$$

$$\text{s.t. } x_i+G=w_i+G_{-i}, \tag{1}$$

$$G \geqq G_{-i}.$$

Solving this yields a continuous demand function for the public good:

$$G=\max\{\gamma(w_i+G_{-i}), G_{-i}\} \quad i=1,2,\ldots,n. \tag{2}$$

If the inequality constraint in (1) is not binding, the choice of gifts by person i will be $G=\gamma(w_i+G_{-i})$, or equivalently, $g_i=\gamma(w_i+G_{-i})-G_{-i}$. The donation function $\gamma()$ is the Engel curve.

It is assumed that γ is differentiable, and that the derivative is positive and bounded away from 1: $0<\gamma'\leqq a<1$. This assumption is innocuous. It simply assures that both the public good and the private good are normal. Under these conditions, a unique Nash equilibrium can be shown to exist.[4]

Invert γ in (2) and add g_i to both sides:

$$\begin{aligned} g_i &= w_i - \gamma^{-1}(G) + G \\ &= w_i - \phi(G), \end{aligned}$$

where $\phi(G) = \gamma^{-1}(G) - G$. By normality, $0 < \phi'(G) < \infty$. This equation gives the equilibrium gift of an agent with wealth w_i, as long as i is wealthy enough to contribute to the public good. Let w^* be the level of wealth such that individuals with wealth above w^* will make donations, and individuals with wealth below w^* will not. From the above we see that this 'critical level' of wealth is simply $w^* = \phi(G)$. Note that w^* is the same for all i. Thus, the donations functions (2) can be written as[5]

$$g_i = \begin{cases} w_i - w^* & \text{if } w_i > w^*, \\ 0 & \text{if } w_i \leqq w^*. \end{cases}$$

This result implies that we can write

$$G = \sum_{i=1}^{n} g_i = \sum_{w_i > w^*} (w_i - w^*).$$

Since $G = \phi^{-1}(w^*)$,

$$\phi^{-1}(w^*) = \sum_{w > w^*} (w_i - w^*). \tag{3}$$

Given the distribution of wealth among the n individuals, we can now solve (3) for the critical wealth w^*. The fraction of the population giving to the public good is then determined by the number of individuals for whom $w_i > w^*$. What we are concerned with is what happens to this fraction as we let n grow large. That is, what is $\lim_{n\to\infty} w^*$?

Consider the function

$$H_n(s) = \frac{\phi^{-1}(s)}{n} = \frac{1}{n} \sum_{w > s} (w_i - s).$$

For a given vector $(w_1, w_2, \ldots, w_n)$, this has a solution, $s = w_n^*$. Now suppose

[4]For the proof, see Bergstrom, Blume and Varian (1986) or Andreoni (1985).

[5]This same result has been demonstrated by Arrow (1981, theorem 5), Bergstrom, Blume and Varian (1986), and shown graphically by Jeremias and Zardkoohi (1976).

we add to this vector of w's by making independent random draws from a continuous probability density function $f(w)$, $0 \leqq w \leqq \bar{w}$. This $f(w)$ then describes the underlying distribution of income in the economy. Hence, by the law of large numbers we know that as $n \to \infty$ the expression $H_n(s)$ will converge with probability one to $H(s)$, where

$$H(s) = \int_s^{\bar{w}} (w-s) f(w)\,\mathrm{d}w.$$

This integral equation can be solved for s. Call this solution w^{**}. Then $\lim_{n\to\infty} w_n^* = w^{**}$.

To obtain w^{**}, first determine the limiting value of H_n. By the assumption that private goods are strictly normal, ϕ^{-1} is finite.[6] Since w_n^* is bounded, $\lim_{n\to\infty} H_n = \lim_{n\to\infty} \phi^{-1}(w_n^*)/n = 0$. This leaves

$$H(w^{**}) = \int_{w^{**}}^{\bar{w}} (w - w^{**}) f(w)\,\mathrm{d}w = 0.$$

From this it is easy to prove that w^{**} must equal $\bar{w}$. Suppose not. Then there must exist a number λ such that $w^{**} < \lambda < \bar{w}$. It follows that in the limit we will observe $w > \lambda$ infinitely often. This implies that

$$0 < \int_{\lambda}^{\bar{w}} (w-\lambda) f(w)\,\mathrm{d}w \leqq \int_{w^{**}}^{\bar{w}} (w - w^{**}) f(w)\,\mathrm{d}w = 0,$$

a contradiction. So we can conclude $w_n^* \to \bar{w}$.

The fact that w_n^* converges to $\bar{w}$ has several important implications. First, as the economy grows, the proportion making positive donations to the public good, $1 - F(w_n^*)$, converges to zero – only the very richest will contribute. Second, as n grows, total giving will converge to a finite number strictly greater than zero, $\phi^{-1}(\bar{w})$. Total giving increases as n increases, but average giving diminishes to zero.[7] This then proves the following fundamental limit theorem about altruistic preferences:

Theorem 1. Given an economy of n individuals with identical altruistic preferences $U(x_i, G)$, and incomes distributed according to a continuous probability density function $f(w), 0 < w < \bar{w}$, then as n increases to infinity:

[6]The normality assumption assures that $\mathrm{d}\phi^{-1}/\mathrm{d}w$ is bounded away from infinity, so $0 < \partial\phi^{-1}/\partial w \leqq a/(1-a)$.

[7]How can total gifts be positive even though the proportion of givers converges to zero? To reconcile this one need only recognize that, by the normality assumption, the richest individual in the economy will *always* donate something to the public good, even if no one else does. Total gifts, therefore, will always be positive.

(*a*) *The proportion of the population contributing to the public good decreases to zero;*
(*b*) *Only the very richest members of the economy will contribute;*
(*c*) *Total donations to the public good increase to a finite value* $\phi^{-1}(\bar{w})$;
(*d*) *Average giving decreases to zero.*

Theorem 1 generalizes a point made by Chamberlin (1974) and McGuire (1974) which challenged an assertion of Olson (1965). They showed that even though free riding may be more prevalent in large economies, it does not directly follow that public goods will not be provided by large groups. Chamberlin and McGuire show that total giving converges to a finite positive number by assuming an economy of identical individuals with identical wealth endowments who, as a result, make identical positive gifts. Theorem 1 is more general in that wealth is not assumed to be identical, and so for every n a large fraction of the economy actually makes no donation. According to Theorem 1, however, as n grows large the set of contributors grows more and more homogeneous. The Chamberlin–McGuire example, therefore, is simply the limiting case.

2.2. Simulation

These results can be illustrated by taking actual parameter values for U_i and $f(w)$. Let utility be Cobb–Douglas: $U_i = x_i^{1-\alpha} G^{\alpha}$. α is then the slope of the donations functions. For $f(w)$ a density function fitted to the 1980 census data by McDonald (1984) is used. Wealth is allowed to range from 0 to 65, measured in thousands of 1980 dollars.[8] Given these parameters, we can solve (3) for the expected value of w_n^* under various assumptions on α and n. The results of the simulation are reported in table 1.

Evidence of the theorem is clear and strong. Expected free riding, $1-F(w^*)$, is high for all degrees of generosity, even for economies of size 2, the smallest economy in which public goods are possible. Moreover, comparing the results for $n=20$ and $n=200$ with those for $n=2$ we see that free riding quickly takes over as the norm. For $\alpha=0.5$ (a marginal propensity to donate of 0.50), for instance, the expected proportion of givers drops from 60% for $n=2$ to only 11% for $n=20$, and to only 3% for $n=200$. Even for extremely generous people, with a marginal propensity to donate of 0.9, the expected proportion of givers falls to only 57% (11 of 20), and 15.3% (31 of 200).

The figures in table 1 imply that, for large economies, the slope of the

[8]The Weibull distribution with a mean income of 23.065 was used for all of the simulations reported. Integrals were calculated numerically. The upper bound on wealth was chosen to correspond to $F(\bar{w}) \geqq 0.999$. The results are similar if upper bounds greater than 65 are used. Hard copies of the computer programs used in the simulations are available from the author upon request.

Table 1
Simulations of gifts to a public good.

	$n=2$				$n=20$				$n=200$			
α	w^*	$1-F(w^*)$	G	G/n	w^*	$1-F(w^*)$	G	G/n	w^*	$1-F(w^*)$	G	G/n
0.1	36	0.180	4.0	2.0	53	0.033	5.9	0.3	63	0.003	7.0	0.0
0.2	29	0.301	7.2	3.6	48	0.060	12.0	0.6	61	0.007	15.3	0.1
0.3	25	0.389	10.7	5.4	44	0.090	18.9	0.9	58	0.015	24.9	0.1
0.4	21	0.491	14.0	7.0	40	0.129	26.7	1.3	56	0.021	37.3	0.2
0.5	17	0.602	17.0	8.5	37	0.116	37.0	1.9	54	0.029	54.0	0.3
0.6	14	0.690	21.0	10.5	34	0.210	51.0	2.6	52	0.037	78.0	0.4
0.7	11	0.777	25.7	12.8	30	0.281	70.0	3.5	48	0.060	112.0	0.6
0.8	8	0.860	32.0	16.0	25	0.389	100.0	5.0	44	0.090	176.0	0.9
0.9	4	0.951	36.0	18.0	18	0.574	162.0	8.1	38	0.153	342.0	1.7

donations function matters very little – for even relatively small economies, $n=20$ for example, free riding is so powerful that private giving will most likely be provided by a minority of the population, and quite possibly a very small minority. Even so, it is interesting to ask what kind of theoretical world we are in if we have a realistic value of α in mind. Reece and Zieschang (1985) estimate linear donations functions for a cross section of individuals and estimate the income effect to be 0.0342.[9]

Table 2 lists the solutions to the expected value of (3) for $\alpha=0.0342$ under various assumptions on n. We see that a pure altruism model predicts that only about 1% of an economy of only 25 individuals can be expected to contribute. Total giving, furthermore, will converge to only \$2,300.

What has been established in this section is that, regardless of the assumptions about income distribution and the intensity of preferences for

Table 2
Simulations of gifts to a public good.

	$\alpha=0.0342$			
n	w^*	$1-F(w^*)$	G	G/n
2	45	0.081	1.59	0.797
5	52	0.037	1.84	0.368
10	56	0.021	1.98	0.198
25	60	0.010	2.13	0.085
50	62	0.005	2.20	0.044
100	63	0.003	2.23	0.022
200	64	0.002	2.26	0.011
500	65	0.000	2.30	0.005
1,000	65	0.000	2.30	0.002

[9]Similar results obtain if income effects estimated in other studies are used.

public goods, boundary solutions are the norm in theoretically altruistic economies. The larger the economy, the smaller the fraction of givers and the smaller the average individual gifts. Furthermore, the simulations indicate that public goods which serve as few as several dozen or as many as several thousand should be supplied by the contributions of only a handful of individuals. Obviously, these findings cannot be reconciled with the facts about giving.

2.3. Heterogeneous preferences

The analysis up to this point has relied on the assumption of identical preferences. All of the preceding results, however, are also true when the model is generalized to include individuals of different types. There is, moreover, one further conclusion. As the population increases to infinity, all of the giving shifts to individuals of a single type. This is demonstrated below.

Consider utility functions of the form

$$U_i = U(x_i, G; \theta),$$

where U is continuous, increasing and strictly quasi-concave in x_i and G, and is continuous in θ. The variable θ is an exogenous parameter that serves the dual purpose of making the utility functions distinct and of indexing the various 'types' of agents.[10] An interior solution to the consumer's optimization problem will, as before, lead to a donations function like the following:

$$G = \gamma(w_i + G_{-i}; \theta).$$

As in the previous case, invert γ, add g_i to both sides and solve to get

$$g_i = w_i - w^*(\theta).$$

Now there exists a 'critical level' of wealth for each θ, $w^*(\theta)$, such that an individual of type θ with wealth above $w^*(\theta)$ will make donations, and those with wealth below $w^*(\theta)$ will not. This now means we can write

$$G = \sum_{\theta} \sum_{w > w^*(\theta)} (w_i - w^*(\theta)).$$

Again define a function

[10]The parameter θ is taken to be a scalar for ease of notation. All of the following results apply when θ is defined as a vector. See footnote 11.

$$H_n(s(\theta))=\frac{1}{n}\sum_{\theta}\sum_{w>s(\theta)}(w_i-s(\theta)).$$

Let $w_n^*(\theta)$ be the solution to this equation. By arguments similar to the single type case,

$$\lim_{n\to\infty} H_n(s)=H(s)=\int_0^{\bar{\theta}}\int_{s(\theta)}^{\bar{w}}(w_i-s(\theta))f(w,\theta)\,\mathrm{d}w\,\mathrm{d}\theta,$$

where $f(w,\theta), 0\leqq w\leqq\bar{w}$ and $0\leqq\theta\leqq\bar{\theta}$, is the joint probability density function over w and θ. Again, call the solution to this equation $w^{**}(\theta)$. Then

$$H(w^{**}(\theta))=\int_0^{\bar{\theta}}\int_{w^{**}(\theta)}^{\bar{w}}(w_i-w^{**}(\theta))f(w,\theta)\,\mathrm{d}w\,\mathrm{d}\theta.$$

Let $f(w\,|\,\theta)$ be the density of w conditional on θ, and let $f(\theta)$ be the marginal density function of θ. By definition, $f(w,\theta)=f(w\,|\,\theta)f(\theta)$. Substitute this into the above:

$$H(w^{**}(\theta))=\int_0^{\bar{\theta}}\left\{\int_{w^{**}(\theta)}^{\bar{w}}(w_i-w^{**}(\theta))f(w\,|\,\theta)\,\mathrm{d}w\right\}f(\theta)\,\mathrm{d}\theta. \tag{4}$$

Just as in the last case, the left hand side of (4) must converge to zero. For the right hand side to converge to zero, it must be that for every θ

$$\int_{w^{**}(\theta)}^{\bar{w}}(w-w^{**}(\theta))f(w\,|\,\theta)\,\mathrm{d}w=0.$$

We saw earlier, however, that this implies that $w^{**}(\theta)=\bar{w}$. One can see immediately that all of the results of Theorem 1 also apply to this case.[11]

What happens to G? Since $\phi(G;\theta)$ is continuous and increasing in G, then there exists a function $G^*(\theta)$ such that $\phi(G^*(\theta);\theta)=\bar{w}$. In the limit for each θ one of two things can happen. First, some agents of type θ may be contributors to the public good. In this case G must converge to $G^*(\theta)$. Alternatively, no agents of type θ will be contributors. In this case G must converge to a value greater than $G^*(\theta)$. However, G cannot simultaneously converge to two or more distinct values. If $G^*(\theta_1)>G^*(\theta_2)$ then surely the agents of type θ_2 will not, in the limit, be contributors. The agents of type θ_1 will completely crowd them out. Extrapolating this to every value of θ leads to the conclusion: As $n\to\infty$, total giving will converge to $\hat{G}$, where

[11]One can also see that θ could have been defined as a vector of parameters. If θ were m-dimensional, then (4) would simply contain an additional integral for each additional dimension. All of the steps which follow (4), however, would be unchanged.

$$\hat{G} = \max_{\theta} \{G^*(\theta)\}$$

$$\text{s.t. } 0 \leqq \theta \leqq \bar{\theta}.$$

Assuming a unique maximal θ, then in the limit the most 'generous' type will crowd out all of the others. This then proves a more general version of Theorem 1:

Theorem 1.1 When the model of section 2.1 is generalized to include agents of different types, then Theorem 1 holds and, furthermore, as $n \to \infty$ the set of contributors to the public good will converge to a set containing individuals of a single type.

3. The invariance of total gifts

Over the past few years, several other puzzling implications of the pure altruism model have been discovered. Each of these individually adds to our reservations about the approach. In sections 3.1 and 3.2 these results are catalogued, reformulated and rederived in what the author hopes are more direct and intuitive ways. In section 3.3 the points made in 3.1 and 3.2 are merged. Together they are shown to have a sweeping implication: The total provision of the public good is invariant (or approximately invariant) with respect to changes in the contributions made by the government. This is at odds with the stylized fact that crowding out is incomplete. Moreover, this section shows that total provision is also invariant with respect to changes in the distribution of income and government subsidization.

3.1. The crowding out hypothesis

Peter Warr (1982) and Russell Roberts (1984) derived the now widely known result that government contributions to privately provided public goods, financed by lump sum taxes, will crowd out private contributions dollar-for-dollar.[12] When the government takes a dollar from a giver and puts it toward the public good, the person can restore his optimal vector of consumption by simply reducing his private gift by the dollar. In a related finding, Warr (1983) discovered that the total supply of the public good was also independent of the distribution of income among contributors. While Warr proves the redistribution result directly, it also follows simply from the crowding out result. This is because any redistribution can be reconstructed as a series of neutral tax increases and tax decreases. Hence, crowding out and the neutrality of income distribution can be considered together.

[12] See Bergstrom, Blume and Varian (1986) for a review of the evidence on crowding out.

While this crowding out prediction is clear and intuitive, one should be careful not to overstate its importance. The result is confined only to contributors, and moreover, will not hold if a tax or redistribution from a giver exceeds the size of the individual's gift. As has been discussed by Bergstrom, Blume and Varian (1986), these corner solutions restrict agents from completely 'undoing' the effects of the tax. The application of crowding out is therefore somewhat restricted.

It has been learned recently, however, that complete crowding out is not necessarily confined to the case of interior equilibria and lump sum taxation. In fact, as has been shown by Andreoni (1985) and Bernheim (1986), when the model is extended to include 'distortionary' subsidies to giving, then these too may have no real effect – even in the presence of corners.

Suppose that individuals are taxed at a flat rate, τ_i, and receive a subsidy to private giving, s_i, where $0 \leqq s_i < 1$. For each dollar they donate to the public good, individuals receive a reduction of s_i dollars in their tax bill. Letting t_i be i's net tax bill, then $t_i = \tau_i - s_i g_i$. Total net taxes collected by the government is then $T = \sum_{i=1}^{n} t_i$. While the following can be shown to hold under a number of conditions,[13] assume here that individuals take their tax and subsidy rates as exogenous, while they act as though the government donates total net taxes to the public good. The total supply of the public good is then the sum of private and public contributions, $G + T$. Individuals solve the following:

$$\begin{aligned} &\max_{x_i, g_i} U(x_i, G+T) \\ &\text{s.t. } x_i + g_i = w_i - \tau_i + s_i g_i, \\ &\qquad T = \sum_{i=1}^{n} \tau_i - s_i g_i, \\ &\qquad g \geqq 0. \end{aligned} \tag{5}$$

The equilibrium concept is Nash, hence individuals treat the gifts of others as exogenous. Let $y_i = g_i + t_i$. Then y_i represents i's total gift to the public good, including i's voluntary contribution and that made through the tax system. Letting the total supply of the public good be $Y = G + T$, then $Y = \sum_{i=1}^{n} y_i$. Likewise, let $Y_{-i} = \sum_{j \neq i} y_j$ be the total gift of everyone but i. Then, via the Nash assumption, solving (5) is equivalent to solving the following:

$$\max_{x_i, Y} U(x_i, Y)$$

[13]See Andreoni (1987b).

$$\text{s.t. } x_i + Y = w_i + Y_{-i}, \qquad (6)$$

$$Y \geqq Y_{-i} + \tau_i.$$

One can see immediately from (6) that neither taxes nor subsidies can have any effect on the equilibrium – changing τ_i or s_i will not affect this optimization problem. Both lump sum taxes and subsidies crowd out private giving dollar-for-dollar.

As before, this result has some qualifications.[14] If lump sum taxes or redistributions make the inequality constraint binding then the action will not be neutral. But notice that there is no such qualification for choosing s_i. If i is at an interior when s_i is changed, the agent will always be able to alter g_i to maintain y_i. Moreover, if i is at a corner, i will never wish to a move from the corner regardless of s_i. Stated differently, neither of the constraints in (6) is altered by the choice of any s_i in the range $[0, 1]$.

3.2. The paradox of public goods

The title of this section is derived from the verbal arguments of Margolis (1982) about a shortcoming of models of charity, which he saw as a paradox. A similar argument was presented by Sugden (1982). While this discussion is distinct, it taps into the same general phenomenon uncovered by Margolis and Sugden. The approach here, in contrast to section 2, is to take as given a large number of contributors, and then to derive the implications of this assumption on the equilibrium.

The observation to be demonstrated is the following. An exogenous increase in donations to a public good will, in the resultant equilibrium, have a virtually imperceptible effect on the total provision of the public good. This does not simply mean that the change will be imperceptible in a relative sense, but the change will be minute in an *absolute* sense. The paradox is that an exogenous change in gifts cannot have any (significant) impact on the total amount of giving.

Consider the simple pure altruism model without taxation. Preferences need not be identical. Suppose there is an exogenous increase in donations,

[14]The result of this section appears to depend on the assumption that individuals behave as though net tax revenue is donated to the public good. The crucial assumption, however, is that they recognize that the government budget constraint is binding. A different set of assumptions which have been applied in similar models is that people treat the government contribution as fixed and take tax and subsidy rates as exogenous, hence, in contrast to the above model, they behave as though the government's contribution is independent of the tax revenue. But this approach gives people the illusion of being able to choose over alternatives which generate unfinanced government deficits. Since this is a 'non-credible bribe', Andreoni (1987b) argues that the equilibrium attained under this later assumption is implausible.

Δ. For each individual, giving by others is now higher by Δ. When the contributors react to this increase, what will happen to total donations?

Totally differentiate i's donation function:

$$\mathrm{d}g_i = \gamma_i'(\mathrm{d}G_{-i} + \Delta) - \mathrm{d}G_{-i} - \Delta.$$

Substituting $\mathrm{d}G_{-i} = \mathrm{d}G - \mathrm{d}g_i$, and rearranging

$$\mathrm{d}g_i = -\frac{1-\gamma_i'}{\gamma_i'}(\mathrm{d}G + \Delta).$$

Sum across all i and solve for G:

$$\mathrm{d}G = \sum_{i=1}^{n} -\frac{1-\gamma_i'}{\gamma_i'}(\mathrm{d}G + \Delta)$$

$$= -\frac{\sum(1-\gamma_i')/\gamma_i'}{1+\sum(1-\gamma_i')/\gamma_i'}\Delta.$$

The change in total giving is, therefore,

$$\mathrm{d}G + \Delta = \frac{1}{1+\sum(1-\gamma_i')/\gamma_i'}\Delta \geqq 0.$$

How do we evaluate this? The assumption that γ_i' is bounded strictly away from 1 implies that there exists a number β such that

$$0 < \beta \leqq \frac{1-\gamma_i'}{\gamma_i'}.$$

It follows that

$$n\beta \leqq \sum_{i=1}^{n} \frac{1-\gamma_i'}{\gamma_i'}.$$

This in turn implies

$$\mathrm{d}G + \Delta \leqq \frac{1}{1+n\beta}\Delta.$$

For large n, the right hand side of this inequality is approximately zero. As the number of contributors approaches infinity an exogenous increase in donations will have no impact on the equilibrium provision of the public good. Even for small numbers of contributors, however, the impact of the exogenous gift can be barely perceptible. This is especially true if γ_i' is small. For example, suppose again that γ_i' is a constant equal to 0.10 for all i. Take $n=10$. One can show that a $\Delta=\$100$ would increase the equilibrium level of giving by only \$1.10. For $n=100$, the same Δ would increase the equilibrium by less than \$0.01. Like section 3.1, this seems to be an absurd implication of the model.

3.3. The invariance proposition

Section 3.1 showed that the total provision of the public good was independent of the distribution of income and that government provision crowds out private provision dollar-for-dollar. When individuals at corners are involved in the tax or transfer, however, neutrality breaks down – a problem which has been examined extensively by Bergstrom, Blume and Varian (1986). This section examines the question which follows naturally from the work of Bergstrom et al.: If a tax or transfer scheme is non-neutral, by *how much* will the equilibrium change?

The answer to this lies in the analysis of section 3.2. Suppose that the government were to tax a non-giver or, equivalently, to tax a contributor an amount greater than his gift. When the government donates the proceeds of this tax to the public good it is identical to the introduction of an exogenous donation Δ in section 3.2. The equilibrium will rise, as Bergstrom, Blume and Varian show, but by only an imperceptibly small amount. This leads to the following startling conclusion: *The only way that the government can have any (significant) impact on the provision of public goods is to completely crowd out private provision. Joint provision is a veil.*[15] In particular, this means that crowding out should be either exactly or approximately dollar-for-dollar. This, obviously, is directly at odds with the observations about joint provision.

Similar implications hold with respect to redistribution. Since a redistribution of income can be reconstructed as a series of tax changes, the above argument can be applied to this phenomenon directly. Any non-neutral redistribution will, therefore, be approximately neutral if the number of contributors to the public good is significant.[16]

[15]This contrasts with Roberts' (1985) view that public and private provision *cannot* coexist because the political clout of the 'indirect demanders', e.g. recipients of charity, leads the government to completely crowd out the 'direct demanders', e.g. the rich altruists.

[16]The exception is when the number of contributors reduces to one. In this case, redistributions to the lone contributor will potentially have significant effects. For example, in an economy of infinite wealth in which all goods are normal goods, transferring all of the resources to one individual will result in an infinite supply of the public good. Barring this, the effect of redistribution will be tiny.

The results derived in section 3 can now be combined with Theorem 1 to form the following:

The invariance proposition of public goods: In a purely altruistic economy, the private supply of a public good is invariant, or approximately invariant, with respect to joint provision with the government, redistributions of income, subsidies to giving, and changes in the population.

4. The limits of altruism

The pure public goods approach to altruism is very limited. This point has been demonstrated in several ways. First, free riding was shown to dominate altruistic economies. In the limit, the proportion of the economy making positive donations to the public good shrinks to zero. The public good will be supplied entirely by the richest individuals. This finding runs counter to the facts about the extent of participation in the provision of collectively consumed goods, and about the high levels of both individual and aggregate giving. Second, the pure altruism model produces several strong neutrality results. Within bounds, the Nash equilibrium is shown to be independent of the distribution of income, direct government provision, and 'distortionary' subsidies to giving. Third, exogenous increases in giving will not have a perceptible effect on the total equilibrium donations – even manna from heaven cannot be expected to increase the budget of the church. Finally, combining the last two results indicates that the total supply of the public good is (approximately) invariant with respect to redistribution, joint provision, and changes in the population. This result runs counter to the empirical finding that government provision only incompletely crowds out private provision.

The clear inference to be drawn from these results is that a new approach must be taken. While the results in this paper do not themselves indicate what that approach should be, there are several obvious alternatives. One possible adjustment is to extend the horizon of the economy. It is well known from the game theory literature that when static games become dynamic, the characteristics of the equilibria can change dramatically. But the question of whether a cooperative equilibrium can be sustained in a large economy is very much like the question of whether an industry can maintain a collusive equilibrium as the number of firms grows to infinity. The fragile nature of the collusive equilibria make one pessimistic about this approach.

A second possibility is to alter the concept of the equilibrium. But both Bergstrom, Blume and Varian (1986) and Sugden (1985) showed that the pathological implications are not remedied by assuming non-Nash conjectures. If anything, the pathologies may be made more severe.

This leaves us to question the basic tenets of the model. We must reexamine the assumptions about preferences and about charitable institutions, and consider non-altruistic motives for giving. In this way, the paper speaks in support of work by Sugden (1984), Margolis (1984), Bernheim, Schleifer and Summers (1985), and Andreoni (1987a). Phenomena such as guilt, repentance, envy, sympathy, emulation, a taste for fairness, or a heuristic for duty may develop naturally from a society of self-interested agents, and institutions may emerge which provide 'selective incentives' for giving [Olson (1965), Posnett and Sandler (1986)]. Economic models which reflect these possibilities may ultimately help us understand both the failures and successes of private giving.

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