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Identifying the free riders

A simple algorithm for determining who will contribute to a public good

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When a heterogeneous group of people provide themselves with a pure public good the resulting Nash equilibrium outcome will divide the group into contributors and free riders. This paper proposes a general algorithm for discovering which individuals in the group fall into which of these two classes. The algorithm is based on identifying, *for each individual*, how much of the public good must be provided *by others* to drive that individual's contribution to zero.

1. Introduction

Economists have manifest a growing concern with how a group provides itself with a public good in the absence of coordination, governance by a tax/expenditure authority, or other forms of coercion. Following Olson's (1965) original investigation, and the application of the theory of voluntary provision of a pure public good to military alliances by Olson and Zeckhauser (1966), economists have found many other applications of this allocation paradigm. Examples range from philanthropy, or intra-household allocation, to the political economy of such bodies as the United Nations, World Bank, and IMF [McGuire and Groth (1985)]. Better understanding of voluntary behavior in groups should become ever more relevant as worldwide environmental and health hazards threaten, and as economic stability itself evolves into a worldwide public good.

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Theoretical interest in these questions was greatly stimulated by Warr's (1983) striking result that income redistribution among contributors to a pure public good has no effect on the Nash equilibrium, as long as such redistribution does not alter the identities of those making positive contributions. In particular, the total provision of the public good, the private good consumption, and the utility of each member of the group (both contributors and free riders) are unchanged by the redistribution. Both for its relation to this neutrality result, and more generally for its relevance to distinguishing free riders form voluntary contributors, the determination of exactly who belongs to which subset in a group merits attention. The question was broached by McGuire (1974) and systematically analyzed by Bergstrom, Blume and Varian (1986) – BBV – as well as by Andreoni (1988). Specifically, BBV (p. 34) suggest the following procedure to identify the set of contributors in Nash equilibrium:

(1) Start by choosing an arbitrary subset of consumers C.

(2) Defining $g_i \geqq 0$ as individual i's contribution to the public good and $G=\sum_{i=1}^{n} g_i$ as the total supply of the public good, calculate the unique value of $G=G^*$ which solves

$$\sum_{i \in C} \phi_i(G)-(c-1)G=\sum_{i \in C} w_i, \tag{1}$$

where w_i is the exogenous wealth of person i, ϕ_i is the inverse of i's demand function f_i in $g_i=f_i(w_i+G_{-i})-G_{-i}$, and G_{-i} is the aggregate public good supplied by the $c-1$ other contributors in the subset C.

(3) Compare G^* so calculated with the Nash solution requirements $G^*=f_i(w_i+G_{-i})$ for i in C and $G^*>f_j(w_j+G_{-j})$ for j not in C.

(4) If all these requirements are satisfied the correct set of contributors has been identified, otherwise a different subset must be assumed and the procedure continued until the G^* calculated does satisfy all of the requirements.

2. Proposed method

While logically correct, this procedure promises to be time-consuming and frustrating. BBV find a simpler method that applies to groups whose members have identical preferences and are distinguished only be their wealth. In a somewhat more general analysis, Andreoni (1988) deals with groups whose members have different utility functions belonging to a finite set of possible functions. In this case it is crucial that the number of utility functions is small relative to the number of individuals in the population. Both BBV and Andreoni use the fact that contributors of a given type can then be identified by a *cut off income level*, w^*. Members of the group who have income w_i below w^* contribute nothing, while those with income above w^* are contributors.

Unfortunately, these methods are not easily generalized to natural cases where, for instance, each member of the population may have a distinct utility function. In this situation, a more important variable for identifying whether a particular individual belongs to the set of contributors will be the level of the public good supplied by others in the group necessary to cause that individual to cease contributing altogether. Let G_i^0 denote this quantity. We can implicitly define G_i^0 as the solution to $G_i^0 = f_i(w_i + G_i^0)$. Hence, when $G_{-i} \geqq G_i^0$ it is optimal for the individual to choose $g_i = 0$; however, when $G_{-i} < G_i^0$ it is optimal to choose $g_i > 0$. This variable G_i^0 will provide the basis for our alternative method.[1]

Note that G_i^0 is specific to each individual. It depends on w_i and on i's preferences.[2] Since G_i^0 denotes the amount of public good provided by others at which individual i is just indifferent between contributing to the public good and not, we will refer to G_i^0 as the individual's 'free rider inducing supply'. The role of G_i^0 can be seen in fig. 1, which illustrates the choice problem of a potential contributor. If the others in i's group provide nothing, i's initial endowment of private and public goods is $(w_i, 0)$, and i will supply his 'isolation purchase' g_i^0. If the others in the group supply G_i^0, then i's initial endowment is (w_i, G_i^0) and i will just be induced to be a free rider, that is, contribute nothing. For intermediate endowments of the public good between $G=0$ and $G=G_i^0$ (along the horizontal through w_i) individual i makes a positive contribution, while for endowments beyond $G=G_i^0$ the individual is at a strict corner solution. The functional relationship between G and g_i is also illustrated at (G_i^*, g_i^*).

To demonstrate our method, begin by solving for the G_i^0 of every person in the economy. Without loss of generality, index all individuals $i = 1, 2, \ldots, n$ such that $j > k$ if and only if $G_j^0 > G_k^0$. Hence G_j^0 increases with the index i. If two people have the same G_i^0 it is acceptable for our purposes to allow them to be indexed by the same number.

Let G^* stand for the Nash equilibrium level of the public good, and let C indicate the equilibrium set of contributors. Then two facts follow trivially from the definition of G_i^0:

Fact 1. $i \in C$ *if and only if* $G^* < G_i^0$.

Fact 2. *If* $i \in C$ *and if* $j \geqq i$, *then* $j \in C$.

[1]We assume that G_i^0 is unique. This is automatically satisfied if the private good is everywhere normal.

[2]Note that, for simplicity, our analysis has followed BBV and others in assuming that the prices of both public and private goods are unity.

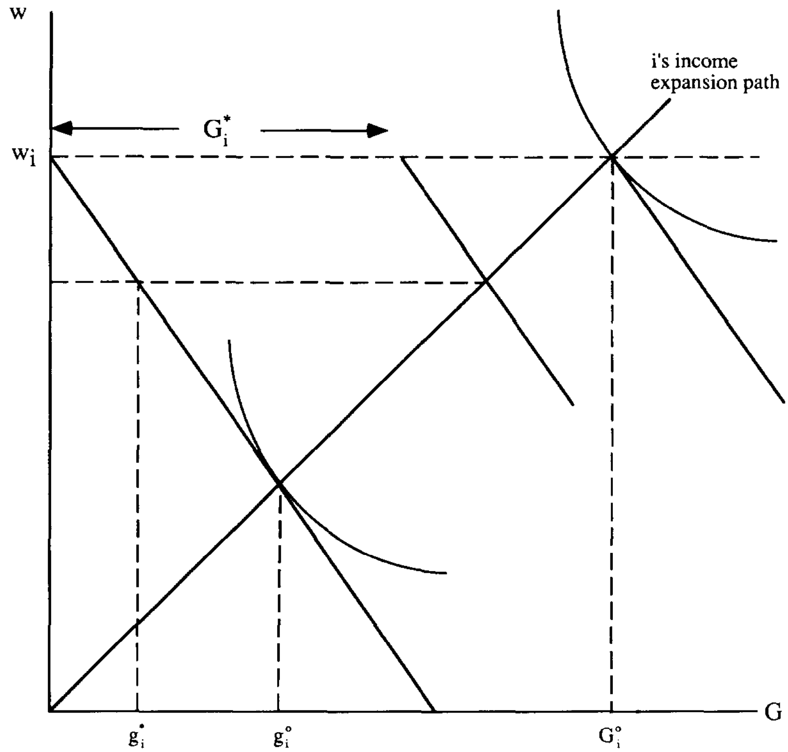

Fig 1. Individual i's isolation purchase, g_i^0, free rider inducing supply, G_i^0, and intermediate contributions. If others contribute nothing, i contributes g_i^0; if others contribute G_i^0, i contributes nothing ; if others contribute an intermediate amount, G_i^*, i contributes g_i^*.

This fact can be used to show the following:

Fact 3. *Let* $G_{\min}^0 = \min\{G_i^0 \text{ s.t. } i \in C\}$. *Then if a person k is added to the economy, none of the original members of C will be replaced as long as* $G_k^0 \leqq G_{\min}^0$, *although person k may be added to C.*

Proof. Suppose $k \notin C$. Then no change in C or G^* will occur. Suppose $k \in C$. By assumption G_k^0 is less than G_i^0 for all i in the original set of contributors. By Fact 2 all of the original members of C are still members of C. □

Fact 3 is useful in the following way. If we begin our search for contributors with those with the highest values of G_i^0, then as we move on to individuals with lower G_i^0's, we know none of the contributors already identified will be replaced by the potential additional contributors.

To establish the next fact we must assume that both the public good and the private good are normal, that is, the function $f_i(\cdot)$ is increasing with $0<f'_i<1$. This assumption is sufficient to guarantee the existence and uniqueness of a Nash equilibrium (see BBV),[3] and follows similar assumptions made by BBV and Andreoni (1988).

Fact. 4. Let S and S′ be two sets of individuals such that $S' \subseteq S$. Let G^* be the Nash equilibrium in S and let G' be the equilibrium in S′. Then $G^* \geqq G'$.

Proof. Suppose not. Then $G'>G^*$. Now consider the members of S' who make positive contributions to the public good, that is with $g'_i>0$. Since $G_i^0>G'>G^*$ for these people, it follows from Fact 1 that they must also be contributors in S; that is $g_i^*>0$. Letting x_i indicate i's consumption of the private good, we can write the individual budget constraints as $w_i=g_i+x_i$. Since the public and private goods are normal, and since by assumption $G'>G^*$, it follows that $x_i^*<x'_i$ for each of these people. However, the assumption that $G'>G^*$ implies $g_i^*<g'_i$ for at least one of the original contributors. This in turn implies $x_i^*>x'_i$ for at least one of the original contributors, which is a contradiction. Hence $G'>G^*$ cannot be an equilibrium. [See Andreoni (1988) for an alternative proof.] □

This fact says, quite naturally, that as more people are added to an economy the level of public good provision cannot fall. Intuitively, since larger numbers of people can collectively provide more alternatives – that is, more social wealth – there is a collective rationality that requires that the original set of individuals cannot be made worse off by adding more prople to the economy. Those in S' should always be able to do at least as well for themselves in S. When all goods are normal, this implies that provision of the public good cannot fall. The next fact, which was already applied in eq. (1), was established by BBV and we refer to them for the proof.

Fact 5. G^* is an equilibrium if and only if

$$\sum_{i\in C} \phi_i(G^*)-(c-1)G^*=\sum_{i\in C} w_i,$$

where c is the number of elements of the set C, and $\phi_i(\cdot)$ is, as before, the inverse of the demand funtion $f_i(\cdot)$.

[3]Given Fact 3, along with the fact that normality guarantees existence and uniqueness for any set of potential contributors, we need not be concerned about membership cycles wherein a new entrant might displace an existing positive contributor. We thank Todd Sandler and the referee on this point. See Cornes and Sandler (1984, esp. pp. 586–587).

The expression given in Fact 5 has the following intuitive interpretation. The left-hand side indicates the aggregate wealth necessary to sustain G^* as an equilibrium in C, given the tastes of those in C. The right-hand side indicates the aggregate wealth actually available in C. In equilibrium these two sides are in balance.

We can now use these five facts to establish the main result. For ease of notation, let Φ_i indicate the wealth that individuals from n down to and including i must possess to sustain G as an equilibrium.[4] That is, $\Phi_i(G)=\sum_{j\geqq i}\phi_j(G)-(n-i)G$. Then if individuals $j\geqq i$ comprise all of the contributors to the public good in equilibrium, Fact 5 implies that $\Phi_i(G)=\sum_{j\geqq i}w_j$. Note also that the assumption of normality, $0<f_i'<1$, implies that $\phi_i'(\cdot)>1$, and hence that $\Phi_i'>0$ for all i.

Proposition. An individual i is in C if and only if $\Phi_i(G_i^0)>\sum_{j\geqq i}w_j$; that is, the income of those in C is insufficient to sustain a G greater than i's free rider inducing supply.

Proof. Suppose $\Phi_i(G_i^0)>\sum_{j\geqq i}w_j$. Then there exists a $G_i^*<G_i^0$ such that $\Phi_i(G_i^*)=\sum_{j\geqq i}w_j$. By Fact 5 this is an equilibrium for the set of people $S_i=\{j \text{ s.t. } j\geqq i\}$. Since i is in the set of contributors for S_i then, by Fact 3, i is in the set of contributors for the entire population.

Next, suppose $i\in C$. Let G^* be the equilibrium level of G. By Fact 1 then $G_i^0>G^*$. By Fact 4, $G^*\geqq G_i^*$. Hence $G_i^0>G_i^*$. Since $\Phi_i'>0$, then $\Phi_i(G_i^0)>\Phi_i(G_i^*)=\sum_{j\geqq i}w_j$, which establishes the result. □

An intuitive interpretation for this proposition follows from Fact 5. If the aggregate wealth of those $j\geqq i$ is sufficient to supply the public good at the level G_i^0 or above, that is $\Phi_i(G_i^0)\leqq\sum_{j\geqq i}w_j$, then i cannot be a member of the contributing set. However, if wealth is insufficient to push i to a corner, that is $\Phi_i(G_i^0)>\sum_{j\geqq i}w_j$, then i will find it optimal to contribute to the public good.

The proposition can now be applied to divide the economy into contributors and non-contributors. The algorithm can be summarized in two steps.

Step 1. Solve for G_i^0 for all i and rank individuals accordingly. Person n is always a member of C.

Step 2. Evaluate $\Phi_i(G_i^0)-\sum_{j\geqq i}w_j$ for $i=n-1$. If this is positive, then $n-1\in C$, and then we can evaluate the expression again for $i=n-2$. Repeat this until we find some $n-k$ such that $\Phi_i(G_{n-k}^0)-\sum_{j\geqq n-k}w_j\leqq 0$. Then $C=\{j \text{ s.t. } j>n-k\}$.

[4]Recall that person n is the individual with the greatest free rider inducing supply, G_n^0.

Note that this method is potentially far simpler than that proposed by BBV. Solving for $\phi_i(\cdot)$ and constructing $\Phi_i(\cdot)$ is necessary for both algorithms. However, our method never requires solving for an equilibrium in order to identify the set of contributors, whereas the BBV method could require one to solve for an equilibrium many times. With our method, one can easily find the set of contributors first, and then calculate the equilibrium G just once. Moreover, this calculation is comparatively easy, since by the definition of G_i^0 the inequality constraints of non-contributors have aleady been checked. In addition, Facts 2, 3 and 4 make it relatively simple to adjust the set of contributors when new people are added to the economy.

3. Applications

While our result may have many applications to theoretical models of public goods provision, two developments on the world scene make the identification of who will contribute to a public good of increasing importance and interest. First, the number of issues possessing large elements of an international public good have increased dramatically in recent history. More and more international institutions are being called upon to deal with regional and world-wide public good and public bad problems, such as rain forest destruction, ozone depletion, refugee affairs, and telecommunications. Second, the political upheavals of recent history have caused the formation of groups to provide public goods to enter a state of flux. How, for instance, will territories be defined over which common sets of laws, currencies, commercial practices, and defense policies prevail? While this paper provides no answers to these questions, it does establish a point of departure. Analysis along the lines of this paper could provide insight into public goods provision, the formation of groups to do so, and the incentives some groups may have to include or exclude others from participating in collective activities.

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