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INFORMATION IMPACT AND ALLOCATION RULES IN AUCTIONS WITH AFFILIATED PRIVATE VALUES: A LABORATORY STUDY[1]

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In affiliated private value auctions, each bidder has perfect information regarding his/her own value for the object at auction, but higher values of the item for one bidder make higher values for other bidders more likely. We report on a series of experiments examining three key implications of these auctions: (i) in a first-price auction, public information about rivals' values increases expected revenue, (ii) an English auction increases expected revenue compared to a first-price auction, and (iii) a second-price auction is isomorphic to an English auction. In examining these issues, we compare predictions of some ad hoc bidding models with Nash equilibrium predictions.

In the first-price auction experiments, Nash equilibrium bidding theory organizes the data better than either of two ad hoc bidding models. Public information about others' valuations does increase average revenue, but the increase in revenue is smaller and less reliable than predicted under risk neutral Nash equilibrium bidding. Lower average revenue might be attributed to risk aversion, while the high variability is attributed to a sizable frequency of individual bidding errors relative to the theory.

Bidding theory precisely organizes English auction outcomes after a brief initial learning period. The dominant strategy equilibrium does not organize second-price auctions nearly as well, as market prices persistently exceed predicted prices. The difference between English and second-price outcomes is attributed to effects of different information flows, inherent in the structure of the two institutions, on eliminating bidding errors. Revenue impacts of these two institutions, relative to a first-price auction, are examined in light of observed bidding patterns.

KEYWORDS: Auction institutions, laboratory experiments, private values, affiliation, information processing.

1. INTRODUCTION

A SERIES OF AUCTION EXPERIMENTS are reported in which a single indivisible item is auctioned off among six bidders under different information conditions and using different allocation rules. The induced valuations are private and satisfy the criterion of strict positive affiliation (Milgrom and Weber, 1982). With private values each bidder has perfect information concerning the value of the object at auction for him/herself; with affiliation, a higher value of the item for one bidder makes higher values for other bidders more likely. A simple example of an auction with affiliated private values would be a charity fundraiser of consumer perishables, where an item unusually appealing to you is typically more appealing to other bidders as well.

Milgrom and Weber (1982) provide general characterizations of auctions with affiliated variables. Our experiments afford a test of three important implications

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of these auctions: (i) In a first-price sealed bid auction,[2] public information about rivals' values announced prior to bidding increases expected revenue, in risk-neutral symmetric equilibrium. (ii) An English auction[3] attains higher expected revenue than a first-price auction, again evaluated at the risk-neutral symmetric Nash equilibrium in the first-price auction, and the dominant strategy equilibrium in the English auction. (iii) A second-price auction[4] results in the same dominant strategy outcome as an English institution, with the same expected revenue advantage over a first-price auction. In conducting these tests, we specify alternative ad hoc (rule of thumb) bidding models and their implications for the experimental manipulations employed. In this way we can compare Nash equilibrium bidding models with specific alternative rival formulations, rather than merely test whether the data satisfy point predictions of the Nash formulation.

In the first-price auction experiments, Nash equilibrium bidding theory organizes the data better than either of two ad hoc bidding models, embodying simple or sophisticated discounting behavior. A simple fixed discount rule fails to account for the fact that bids decrease with increases in the interval from which private values are drawn. Large doses of public information raise average revenue, but the resulting revenue increases are lower and considerably less reliable than predicted under the risk-neutral symmetric Nash equilibrium. Observed increases in revenue, and adjustments in individual bids resulting from release of public information, correspond more closely to the predictions of a risk averse Nash equilibrium bidding model than to a sophisticated ad hoc discounting rule. The failure of public information to raise average revenue as much as predicted under risk neutrality might largely be accounted for by risk aversion. However, individual bid patterns and variability in revenue increases are attributable to a sizable frequency of individual bidding errors (relative to Nash theory) in responding to the release of public information.

Nash equilibrium bidding theory precisely organizes English auction outcomes after a brief initial learning period. The dominant strategy equilibrium does not organize second-price auction outcomes nearly as well, as market prices persistently exceed predicted prices. Overbidding in second-price auctions involves bidding errors, relative to theory, of a somewhat different nature than those associated with the release of public information in first-price auctions. The difference between English and second-price auction outcomes appears attributable to differential information flows inherent in the structure of the two institutions. Revenue-raising effects of these two auction institutions, relative to a first-price auction, are examined in light of observed bidding patterns.

The paper is organized as follows. Section 2 characterizes the experimental procedures. Section 3 specifies the ad hoc bidding models, the Nash equilibrium bidding models and the differential predictions of the models given our experi-

[2] Bidders submit sealed bids, with the high bidder obtaining the item and paying the amount bid.

[3] The announced price increases regularly, with the last remaining bidder obtaining the item at the price where the next-to-last bidder dropped out of competition.

[4] Bidders submit sealed bids, with the high bidder obtaining the item at a price equal to the second-highest bid.

mental design. The results of the experiments are reported in Section 4. The concluding section summarizes our results.

2. STRUCTURE OF THE AUCTIONS

2.1. *First Price Auctions*

2.1.a. *Private Information Conditions*: Each experiment had several auction periods with 6 subjects bidding for a single unit of a commodity under a first-price, sealed-bid procedure.[5] Subjects' valuations of the item were determined randomly each period according to procedures described below. In each auction the high bidder earned profit equal to his/her value of the item less the high bid; other subjects earned zero profit.

Subjects were told that their private values (x_i) would be determined according to a two step procedure. First, a random number (x_0) would be drawn from a uniform distribution on $[\underline{x}, \bar{x}] = [\$25.00, \$125.00]$. Once x_0 was determined, private values x_1 through x_6, one for each bidder, were randomly drawn from a uniform distribution centered on x_0 with upper bound $x_0 + \varepsilon$ and lower bound $x_0 - \varepsilon$ (all ε are measured in dollars). Subject i, if he or she wins the auction has a price redeemable in the amount x_i. Since subject i is told the value of x_i at the outset, he/she has perfect information about the value of the object at auction to him/herself. Moreover, the level of ε was posted and announced prior to each auction period. Subjects were not told the value of x_0 under these conditions (which we refer to as the private information conditions). Private values are independently drawn relative to x_0, yet are strictly positively affiliated, as bidders' private values are positively correlated relative to the set of possible valuations.

Bids were restricted to be nonnegative and rounded to the nearest penny. After all bids were collected, they were posted on the blackboard in descending order and the high bid noted. Thus subjects had full information about each others' bids, but not about private values underlying bids. The level of ε varied under these private information conditions (see Table I).

2.1.b. *Public Information Conditions*: Approximately half way through each experiment subjects began bidding in two separate auction markets simultaneously. Bidding in the first auction market continued as before under private information conditions. After these bids were collected, but before they were posted, we introduced a public information signal and asked subjects to bid again, with the same private values. We employed two types of public information. In experiments designated x_+ in Table I (experiments 6 and 7), the public information consisted of randomly drawing an additional private value, which we refer to as x_+, from the same interval as subjects' private values were drawn, and posting it on the blackboard. In experiments designated x_0 in Table I (1-5), public information consisted of posting x_0 on the blackboard, along with lower

[5] Antecedent first-price, private-value auction experiments with independent private values (Cox, Roberson and Smith, 1982) indicated that, with 6 bidders and 1 unit, players were clearly noncooperative.

TABLE I

DESIGN PARAMETERS[a]

Experiment	Auction Periods ε $6	$12	$24	Dual Market Periods with Public Information
First-Price Auction Experiments x_0 Design				
1&2	1-5	6-16, 25-26	17-24	12-26
3&4	1-5	6-16, 24-25	17-23	12-25
5	—	1-14	15-28	10-28 (public information only periods 25-28)
x_+ Design				
6	1-6	7-16	17-24	12-24
7	1-6	7-18, 26-27	19-25	13-27
English Auction Experiments				
8	1-5	6-10	—	
9	1-5	6-12	13-15	
Second-Price Auction Experiments				
10	1-5	6-15, 23-27	16-22, 28-30	
11[b]	1-6	7-16, 25-28	17-24, 29-31	

[a] Experiments 1-3, 6, 8, and 9 had one dry run with no monetary payoffs. Experiments 7 and 10 had two dry runs. Experiments 4 and 5 had no dry runs.

[b] From period 23 on there were only 5 bidders (see Section 4.2.a below).

and upper bounds of the interval from which private values were drawn, $[x_0-\varepsilon, x_0+\varepsilon]$.

Subjects were told that we would only pay profits to the high bidder in one of the two auction markets to be determined by a coin flip *after* bids under *both* information conditions had been collected. Subjects were told that they were under no obligation to submit the same or different bids in the two markets but should bid in a way they thought would "generate greatest profits." All bids from both markets were posted in descending order, side by side.

This dual market bidding procedure, involving the same set of bidders with the same item value and the same set of private information signals, has the advantage of directly controlling for between subject variability and extraneous variability resulting from variations in item value and private information signals.[6] Some critics have argued that under the dual market procedure, the optimal bid in one market will affect bids in the other market. However, under the expected utility hypothesis, which underlies our analysis of the first-price auctions, and

[6] This tends to improve the power of tests of the effects of releasing public information, as it permits a within, rather than between, group analysis of variance. In testing market level predictions, comparable improvements in power can be obtained using a paired experiment design, where the same set of private values are distributed under different informational/institutional arrangements to different groups in different experiments (Cox, Roberson, and Smith (1982) effectively employ this procedure). The dual market procedure clearly affords more power than the paired experiment design with respect to tests of individual subject behavior, such as the Proposition in Section 3.1.d. In this case the paired experiment design would require accounting for variations in individual bid functions, as different subjects are almost bound to receive different private valuations across experiments.

using the coin flip rule to determine which market to pay in, this conclusion is unwarranted irrespective of the form of the utility function: the optimal strategy in the private information market is unaffected by bids in the public information market, and vice versa.[7]

Of course, it is another matter entirely whether the dual market procedure actually affects bids. The tactic of beginning each experiment with a number of auction periods with private information only, before introducing the dual market bidding procedure, and of ending experiment 5 with bidding in the public information market only, permit us to test for these effects. Using individual subject data, a fixed-effects regression model was fit to each experiment. The sign of a single dummy variable, which distinguished auction periods with dual markets, was found to vary between experiments, but did not differ significantly from zero except in experiment 7 (using independent t tests and a 5 per cent significance level).[8] Detailed examination of experiment 7 showed that one bidder markedly reduced his bidding, relative to valuations received, with the introduction of the dual market procedure, effectively withdrawing from the auction. This one bidder was equally passive in the public information market, however, effectively withdrawing from the bidding process there as well. Thus our analysis of the effects of public information remains internally consistent even for this experiment, albeit for an auction market with effectively 5 rather than 6 bidders.

2.2. *Second-Price/English Auctions*

The second-price and English auction procedures were designed to match as closely as possible the first-price auction experiments with private information.

[7] The case for treating each member of a series of auctions, $t = 1, 2, \ldots, n$ as a single shot auction (which we do) may be less compelling. If a subject's utility function over the net profits in an experiment exhibits constant absolute risk aversion (CARA), then utility of profits in individual auctions is multiplicatively separable, and yields a CARA function of the same degree in each auction period. Alternatively, there is some empirical evidence (Kahneman and Tversky, 1979), that people evaluate gambles in terms of deviations from the current status quo so that gambles in each auction period are independent of outcomes in other periods.

[8] The fixed-effects regression model employed was suggested by the bid function (8) in the text (cf., p. 1283). Individual subject bids in each auction period served as the dependent variable, with right-hand-side variables including individual subject private values, the variable Y/N in equation (8), dummy variables for changes in ε, dummy variables for private values above and below the interval in equation (3), and subject-by-variable interaction terms for all of the above. A single dummy variable, taking the value of 1 under dual market conditions, was employed to test for any systematic effects. Coefficient values (with associated t statistics) for experiments 1-6 were: 0.00 (0.0), −.30 (−1.06), .90 (1.67), .26 (.56), −.37 (−1.36), −.51 (−1.08). Under the hypothesis that the mean value for the t statistic in the sample population is z,

$$z = \frac{\sum t_i}{\sqrt{\sum [f_i/(f_i - 2)]}}$$

where f_i is the degrees of freedom associated with t_i, has a sampling distribution which is approximately normal (0, 1) (Winer, 1971). Using this statistic, we are unable to reject the null hypothesis of no systematic change in private information auction bids under the dual market procedure ($z = -.21$). A similar simple fixed effects regression model (suggested by the bid function (9) in the text) was used to test for dual market effects on bidding in the public information market in experiment 5. The dummy variable coefficient here was −.72 with a t statistic of 1.53, which is not statistically significant.

Private values were determined using the same procedures as the first price auctions. Subjects knew ε but not the value of x_0 at the time they bid.

Under the second-price auction rules the high bidder won the item and paid the second-high-bid price. All bids were posted in descending order, the high bid noted, and profits of the high bidder computed and posted. Bids were restricted to be nonnegative and rounded to the nearest penny.

The English auction experiments used an ascending clock procedure whereby the price of the item increased at small fixed increments and bidders had to signal their intention to drop out of the bidding. Once withdrawn, bidders could not reenter the auction for that market period. The number of active bidders in the auction was publicly stated at all times, with the last bidder receiving the item at the price when the next-to-last bidder dropped out of the bidding.[9]

The start point for the clock was set at the largest multiple of $(\bar{x}-\underline{x})/4$ which was below $x_0-\varepsilon$. Price increments decreased with the number of active bidders: once two bidders were left, price increments were fixed at 5¢ for $\varepsilon = \$6$, 10¢ for $\varepsilon = \$12$, and 20¢ for $\varepsilon = \$24$. In cases where the last two bidders dropped out on the same increment, a coin toss was used to determine who earned the item.[10] Bids were not posted here, but the prices where rivals dropped were announced as they occurred. Profits of the high bidder were computed and posted on the blackboard at the end of each auction period.

2.3. *Subjects*

Subjects were drawn primarily from MBA classes in first-price auction experiments 1, 2, 6, and 7 and both second-price auction experiments; and from senior undergraduate economics classes in first-price experiments 3 and 4 and the two English auctions. Experiment 5, conducted last in the first-price auction series, involved experienced subjects who had participated in one earlier first-price auction experiment.

Subjects were paid profits in cash at the end of the experiment plus a $4.00 participation fee.

3. THEORETICAL PREDICTIONS

3.1. *First-Price Auctions*

In a first-price auction, it is clear that bidders should bid below their private resale values. The relevant question is how far below these values to bid (how

[9] Cassady (1967) refers to this procedure as an English clock auction. Milgrom and Weber (1982) refer to it as a "Japanese" English auction, quite distinct from the Japanese auction institution specified in Cassady (1967). Experiment 8 had bidders holding little paddles up for others to see as long as they remained active. Experiment 9 used a row of lights behind the subjects with the experimenters announcing the number of active players as bidders dropped.

[10] The probability that the bids of the two highest resale value holders would be tied, using these increments and assuming the dominant bidding strategy was followed, is .0125.

much to "discount") and what are the operating principles underlying these bids. We consider four different models of bidding behavior here. Two are ad hoc models in which bidders are specified as adopting (more or less) sensible bidding/discount rules. In contrast to Nash equilibrium bidding models, there is no explicit consideration of whether the rule adopted is the best response to rivals' behavior, although the second model discussed is based on sensible considerations concerning rivals' behavior. One might justify bidding schemes of this sort with reference to stringent informational requirements inherent in calculating a best response to rivals' behavior. Certainly, the ad hoc models provide an interesting contrast to the two Nash equilibrium bidding models considered: a risk-neutral symmetric Nash equilibrium bidding model (RNSNE hereafter) and a risk-averse symmetric Nash equilibrium bidding solution (RASNE hereafter). Our analysis focuses on the models' predictions regarding responses to variations in ε and the release of public information.

3.1.a. *Naive Markdown Bidding*: The simplest ad hoc bidding model is one in which subjects merely discount their private values in linear fashion:

$$b(x)=\alpha_0+\alpha_1 x \tag{1}$$

where $\alpha_0 \leqslant 0$ and $0<\alpha_1 \leqslant 1$. Such simple markdown bidders are insensitive to strategic implications of varying ε and to informational content of announcing x_+ or x_0. While not advocated, this model represents a null hypothesis against which to evaluate predictions of more sophisticated bidding theories.

3.1.b. *Sophisticated Markdown Bidding*: Consider a bidder who is aware of rudimentary strategic implications inherent in the auction, such as effects of varying ε on the closeness of potential rivals' values to his. Furthermore, this bidder bids as if he holds the highest private value conditional on his subjective evaluation of the potential distribution of rivals' private values, and discounts his bid accordingly:

$$b(x)=x-\alpha\left[\frac{x-\tilde{E}(x_0-\varepsilon)}{N}\right]=\tilde{E}(x_0-\varepsilon)+\frac{N-\alpha}{N}[x-\tilde{E}(x_0-\varepsilon)] \tag{2}$$

where $\tilde{E}(x_0-\varepsilon)$ is the subjective expected value of $x_0-\varepsilon$, α is a discount factor restricted to $\alpha>0$, and N is the number of bidders. Restricting our analysis to values in the range

$$\underline{x}+\varepsilon \leqslant x \leqslant \bar{x}-\varepsilon, \tag{3}$$

there are two interesting cases to consider, depending on how the bidder estimates the lowest possible value, $x_0-\varepsilon$, from his private information.

First consider the bidder whose subjective expectation matches the objective assessment presuming he has drawn the highest of the N private values (denoted $x = x_1$):[11]

$$E[x_0 - \varepsilon \,|\, x = x_1] = x - \frac{N}{N+1} 2\varepsilon. \tag{4}$$

This results in the bid function

$$b(x) = x - \alpha\left(\frac{2\varepsilon}{N+1}\right). \tag{5}$$

Over most of the range (3), this entails bidding more than the RNSNE (equation 8 below) when $\alpha \leq 1$. Further the predicted response to announcement of public information, x_0 (which is the case we will concentrate on in our analysis) is for individual bidders to increase (decrease) their bids whenever $x_0 - \varepsilon$ is greater (smaller) than (4). Unlike the RNSNE bidding model, however, public information would leave expected revenue unaltered. This follows directly from the ad hoc nature of the discount rule which, while it takes account of number of bidders and ε, does not consider the best response to rivals' behavior.

The second case to consider here is one in which

$$\tilde{E}[x_0 - \varepsilon] = x - \varepsilon. \tag{6}$$

Equation (6) is a naive expectation which ignores the fact that when bidder i wins the auction he/she has the highest (or one of the highest) private resale values. This is a manifestation of a winner's curse effect, a behavioral phenomena commonly cited in the context of common value auctions (Capen, Clapp, and Campbell, 1971; Kagel and Levin, 1986; Milgrom and Weber, 1982), but relevant here as well. Employing (6) in (2),

$$b(x) = x - \alpha\left(\frac{\varepsilon}{N}\right) \tag{7}$$

which involves bidding considerably in excess of RNSNE under private information conditions even with $\alpha = 1$. More importantly, with the announcement of x_0, individual bidders will only increase their bids when $x_0 > x$ and will reduce them whenever $x_0 < x$. Since on average $x_0 < x_1$, public information will reduce expected revenue.

3.1.c. *Nash Equilibrium Bidding With Risk Neutrality*: Theoretical models of auctions have focused upon characterizing risk-neutral symmetric Nash equilibrium.[12] This section presents specification of equilibria in Milgrom and Weber (1982) and Vickery (1961) for the design parameters employed in our experiments. Restricting our analysis to values in the range (3), under private information

[11] This calculation is derived from Bayes' formula with the posterior support

$$\sum [x_0 - \varepsilon | x = x_1] = [x - 2\varepsilon, x].$$

[12] Beginning with Vickrey (1961), continuing with Milgrom and Weber (1982) and most references cited there.

conditions, the risk-neutral equilibrium has every bidder employing the function:[13]

$$(8)\qquad b(x)=x-\frac{2\varepsilon}{N}+\frac{Y}{N}$$

where

$$Y=\frac{2\varepsilon}{(N+1)}\,e^{-(N/2\varepsilon)[x-(\underline{x}+\varepsilon)]};$$

Y contains a negative exponential, and becomes negligible rapidly as x moves beyond $\underline{x}+\varepsilon$. Thus, expected profit to the high bidder under the RNSNE are approximately equal to $2\varepsilon/N$.

Milgrom and Weber (1982) demonstrate that revealing public information raises expected revenue in RNSNE. In their model, public information is an additional variable affiliated with values; this is the treatment in x_+ experiments. The x_0 experiments go further, providing maximal public information about the distribution of values—announcing, in effect, an infinite number of additional private valuations, x_+.

Disaggregating revenue enhancement into predictions of individual behavior is particularly tractable for the x_0 design. With x_0 known, the auction is exactly the independent private values model of Vickrey (1961), with the equilibrium bid function

$$(9)\qquad b(x,x_0)=x_0-\varepsilon+\frac{N-1}{N}[x-(x_0-\varepsilon)]$$

for all x, x_0. Under the RNSNE bidding model, revealing x_0 raises individual bids unless x is extremely near $x_0+\varepsilon$. As in the ad hoc model with rational expectations (4), the high bidder is not surprised on average by the announcement of x_0. Nevertheless, he/she raises their bid out of strategic considerations arising from the effect of x_0 on rivals' bids.

3.1.d. *Nash Equilibrium Bidding with Risk Aversion*: If bidders' evaluations of uncertain profits associated with different bids are not summarized by the mathematical expectation, a model of risk-averse bidding can be explored. A symmetric formulation is to have each bidder maximize $E[u(\Pi)]$, where Π is the profit earned in any single auction, 0 if not high bidder, $u(0)=0$, and u is concave. The introduction of risk aversion will result in bidding in excess of the RNSNE

[13] When x_i is outside $[\underline{x}+\varepsilon,\bar{x}-\varepsilon]$, the conditional distribution generating x_j, $j\neq i$ is no longer symmetric about x_i, which alters equilibrium bidding. For $x_i<\underline{x}+\varepsilon$, substitute the function

$$B(x)=\underline{\underline{x}}+\left(\frac{N}{N+1}\right)(x-\underline{\underline{x}})$$

where $\underline{\underline{x}}=\underline{x}-\varepsilon$. The differential equation characterizing equilibrium is not readily solved for x when $x>\bar{x}-\varepsilon$: We employed the bid function (8) in its place. This introduces a small upward bias in our estimates of RNSNE bids for these observations. An Appendix deriving these RNSNE bid functions is available from the authors.

under private information conditions as bidders trade off lower expected profits for a higher probability of winning the auction (given rivals' bidding strategies).

A particularly tractable special case here is when u exhibits constant relative risk aversion (CRRA):

$$\frac{-\Pi u''(\Pi)}{u'(\Pi)} = r \quad \text{for all } \Pi \geqslant 0.^{14} \tag{10}$$

If the coefficient of relative risk aversion were the same constant for all participants, the equilibrium bid function over range (3) would be

$$b(x, r) = x - \frac{2\varepsilon(1-r)}{N} + \frac{Y_r}{N} \tag{11}$$

where

$$Y_r = \frac{(1-r)^2 2\varepsilon}{(N+1-r)N} e^{[-N[x-(\underline{x}+\varepsilon)]/2\varepsilon(1-r)]}$$

and Y_r becomes negligible rapidly as x moves beyond $\underline{x}+\varepsilon$. Note that (11) degenerates to (8) when $r=0$ and (11) is greater than (8) for all $0<r\leqslant 1$.

With risk aversion, effects of public information on expected revenue depend upon the nature of the utility function and the degree of risk aversion displayed. However, effects on individual bidding depend upon the following striking feature of the x_0 design.

PROPOSITION: *For any concave $u(\Pi)$, over range* (3), *in a Risk-Average Symmetric Nash Equilibrium* (*RASNE*) *all bidders with private values $x < C(x_0)$ raise their bids with public information, where*

$$C(x_0) = x_0 + \frac{N-2}{N}\varepsilon. \tag{12}$$

PROOF: See Appendix.

On average, the highest value is $x_0 + \varepsilon(N-1)/(N+1)$, just above $C(x_0)$. Thus, the Proposition predicts that under a RASNE public information will raise individual bids over virtually the same range of private values that it would under sophisticated markdown bidding with rational expectations (4). Effects of x_0 on bids when values exceed $C(x_0)$, and on expected revenue, depend on the utility function and the degree of risk aversion shown. For example, under CRRA the lower the degree of risk aversion displayed (the closer r is to 0), the wider the interval beyond $C(x_0)$ for which bidders will increase their bids in response to

[14] The bid function is normalized so that $u(0)=0$. Consequently $r=1$ corresponds to infinite risk aversion in this setting. CRRA is not consistent with CARA, one of the potential justifications for treating each member in the series of auctions as a single shot auction. However, the CRRA assumption can be consistent with the single shot assumption as it applies to deviations in income from the current status quo.

x_0. Further, while revenue enhancing effects of announcing x_0 are sharply reduced in the presence of risk aversion, it is only under extremely high degrees of risk aversion ($r > 0.8$) that x_0 will fail to raise revenues (cf. Figure 1). We conjecture that these twin characteristics of the CRRA solution hold for a fairly wide range of concave utility functions, $u(\Pi)$.

The x_+ design experiments are not nearly as rich as the x_0 design in terms of our ability to analyze the impact of public information. Public information must increase revenues, on average, for risk-neutral bidders under our design (Milgrom and Weber, 1982).

3.2. *Second-Price/English Auctions*

Vickrey (1961) established that the bid function

$$b = x \tag{13}$$

is a dominant strategy in both second-price and English auctions, irrespective of attitudes toward risk. Further, Milgrom and Weber (1982) have shown that with strict affiliation between private valuation, as in our experimental design, under the dominant bidding strategy, second-price and English auction institutions will raise expected revenue relative to a first-price auction under the RNSNE. However given an RASNE, or some ad hoc rule resulting in bidding in excess of the RNSNE, and/or deviations from the dominant strategy, it becomes an empirical question as to which institution will raise the most revenue.

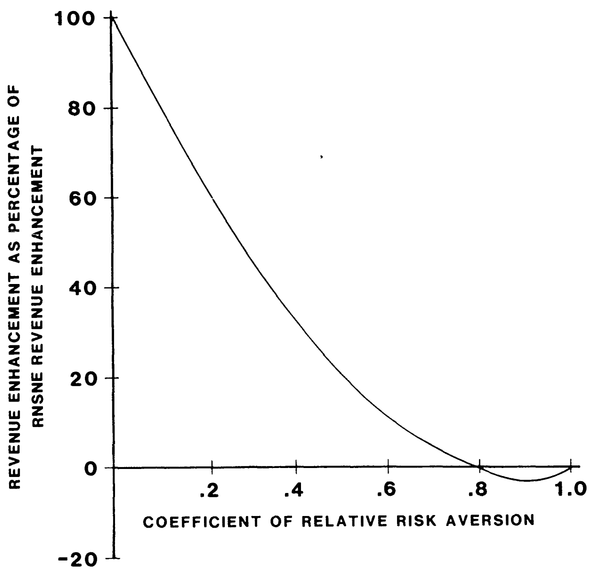

FIGURE 1

In developing ad hoc bidding models for the second-price/English auction institutions, it is clear that there is no injunction, in terms of elementary survival requirements, against bids in excess of private values. With the high bidder paying the second-highest-bid price, bidding in excess of (13), and winning, does not assure losses as it would in a first-price auction. Indeed, consideration of the elementary economic/perceptual forces at work in second-price/English auctions suggests that bids are unlikely to fall below private values: bidding below (13) does nothing to improve profits conditional on winning, and only reduces the chances of making any money. Bidding in excess of (13) has the potential attraction of increasing the probability of winning, with no clear effect on profits given the second-price bid rule. The irrationality of bidding in (modest) excess of (13) only becomes apparent once the question is posed of what is the gain relative to bidding (13)? It may not be natural for bidders to pose this question under second-price and/or English auction procedures. Hence, our intuition suggested that deviations from the dominant bidding strategy would, if anything, result in bids in excess of private values.[15]

This last prediction appears to fly directly in the face of previous experimental research on single unit, independent private value auctions which show mean prices at or below the dominant strategy price under both second-price and English auction institutions (Coppinger, Smith, and Titus, 1980; Cox, Roberson, and Smith, 1982). However, in these earlier private value auction experiments subjects were *not permitted* to bid in excess of their private values. In our second-price/English auction experiments there were no binding ceilings on bids.

4. EXPERIMENTAL RESULTS

4.1. *First Price Auctions*

4.1.a. *Bidding with Private Information Only*: Figures 2–5 graph high market bids over time, in terms of deviations from the RNSNE model's predictions, for the odd-numbered experiments. Table II reports mean deviations from the RNSNE model's predictions for different levels of ε for all experiments. High bids lie scattered about, or slightly below, the RNSNE when $\varepsilon = \$6$, but tend to be well above the RNSNE prediction with $\varepsilon = \$12$ and even further above the RNSNE when $\varepsilon = \$24$. The pooled t statistic for the $\varepsilon = \$6$ condition shows that we can reject the risk-neutral equilibrium hypothesis at the 10 per cent significance level, in favor of risk loving when $\varepsilon = \$6$. If subjects' adjusting to experimental conditions argues for throwing out the first three auction periods, high bids still average 10¢ below RNSNE when $\varepsilon = \$6$, failing to reject the null hypothesis of risk neutral behavior.

[15] In anticipation of this, subjects in these experiments were given \$5.00 starting capital balance to which profits were added and losses subtracted. In cases where the capital balance dropped to zero or less, subjects were no longer permitted to bid. To insure maintaining 6 active bidders in the face of potential brankruptcies, we enrolled 7–8 subjects in these experiments, with a rotation rule used to determine the 6 active bidders in each market period.

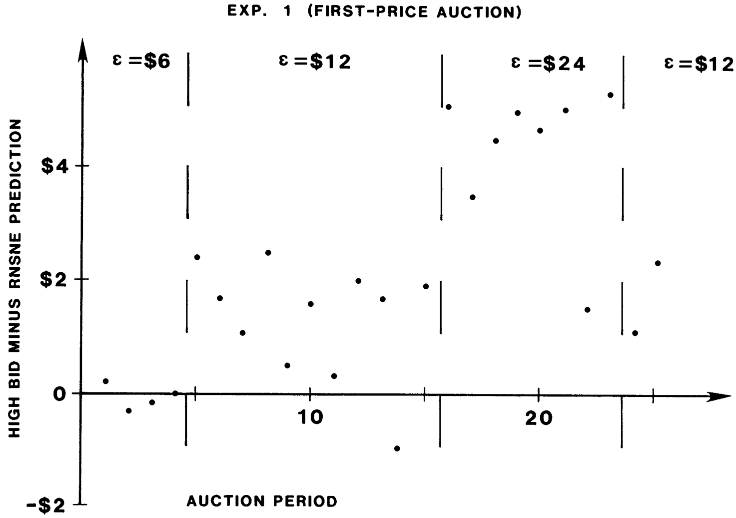

FIGURE 2

Table III shows the effects of ε on profits realized.[16] Under the RNSNE hypothesis, expected profits per auction are approximately \$2.00, \$4.00, and \$8.00

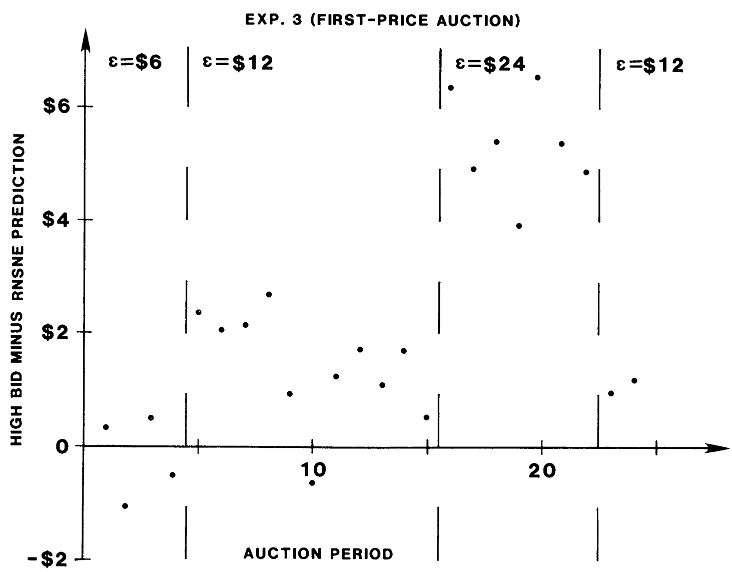

FIGURE 3

[16] Standard deviations in Table III differ slightly from corresponding numbers in Table II, because Table II's construction contrasted the high bid with the RNSNE prediction based upon the high value. Table III subtracts the high bid from the high bidder's value.

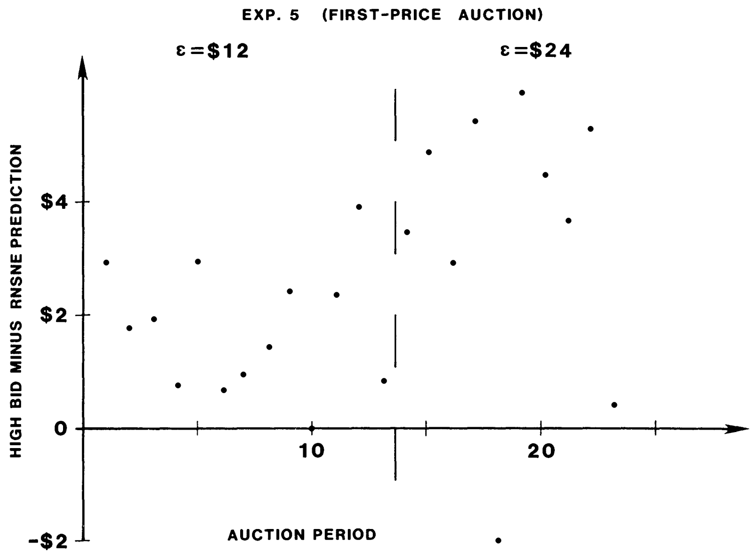

FIGURE 4

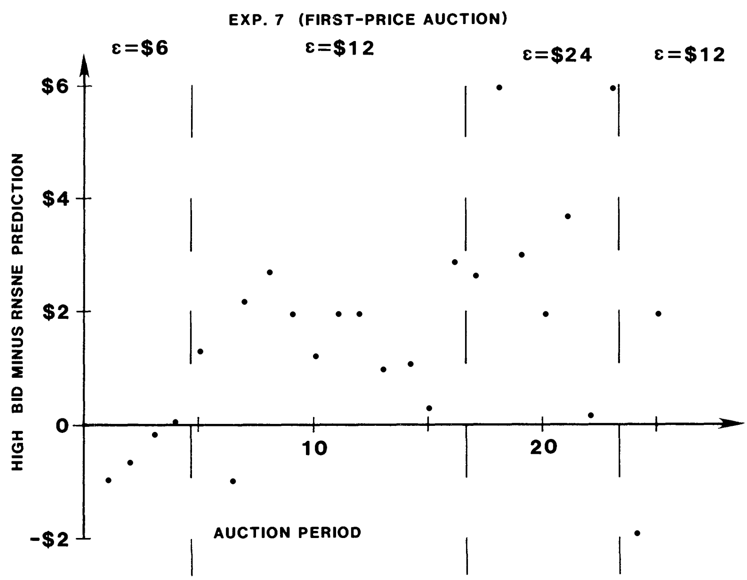

FIGURE 5

TABLE II

DIFFERENCES BETWEEN ACTUAL BID PRICES AND RISK-NEUTRAL SYMMETRIC NASH EQUILIBRIUM BID PRICES: MEAN VALUES WITH STANDARD DEVIATIONS IN PARENTHESES[a]

Experiment	ε \$6	\$12	\$24
x_0 Design			
1	−.12	1.33	4.10
	(.21)	(1.01)	(1.25)
2	−.67	1.49	2.23
	(.71)	(1.43)	(2.84)
3	−.21	1.38	5.48
	(.74)	(.92)	(.93)
4	−.42	1.68	5.01
	(1.28)	(.56)	(1.75)
5	—	1.83	3.49
		(1.15)	(2.51)
x_+ Design			
6	.09	.736	.172
	(.71)	(1.52)	(4.11)
7	−.49	1.21	3.32
	(.47)	(1.36)	(2.12)
Average	−.29	1.40	3.34
[t statistics][b]	[−1.92]	[11.5]	[10.02]

[a] Data for all periods with monetary incentives. Actual bids less predicted bid under RNSNE (cf. equation (8) and footnote 13 for bid functions employed in RNSNE and calculations). All values in dollars.

[b] Test of null hypothesis that mean deviation is zero.

with $\varepsilon = \$6$, \$12, and \$24 respectively. Table III shows actual profits tending to increase as ε increases, but by much less than predicted under the RNSNE. With the notable exception of experiment 6, the increase in bids in going from $\varepsilon = \$6$ to \$12 is sufficient to wipe out virtually all the increase in expected profit. In going from $\varepsilon = \$12$ to \$24, profits increase by \$1.00 or more in all experiments except 3 and 4. Increases here are inconsistent with the naive markdown bidding model which permits no adjustments in the face of changing ε levels.[17]

Both the sophisticated markdown model and the RASNE bidding model are capable of explaining the bidding in excess of the RNSNE with $\varepsilon = \$12$ and \$24. Both models would require modification, however, to account for bidding at the

[17] Formal tests for effects of ε on bidder behavior were restricted to private values within the range (3) for which the term Y/N in (8) was at most \$0.10. The following linear bid function was fit separately to each experiment using a fixed-effects regression model:

$$b_t = \alpha_0 + \alpha_1 x_t + a_2 \varepsilon_t$$

where t refers to the auction period, and each subject's bid in that period generated an observation comingled with other subjects' bids. The coefficient α_2 was negative in all seven regressions, statistically significant at $p < 0.01$ for experiments 1, 2, 5–7.

TABLE III

EFFECTS OF ε ON PROFITS
(Standard Deviations in Parentheses)

	Actual Profits[a]			Actual Profits as a Percentage of Risk Neutral Nash Equilibrium Prediction[b]		
	ε			ε		
Experiment	$6	$12	$24	$6	$12	$24
x_0 Design						
1	2.12 (.21)	2.48 (.96)	3.57 (1.01)	106.0	62.3	45.0
2	2.40 (.75)	2.20 (1.45)	4.80 (2.52)	120.0	56.8	62.7
3	2.18 (.70)	2.27 (.84)	2.40 (.99)	109.0	57.9	30.2
4	1.89 (.64)	2.08 (.64)	2.26 (1.16)	94.5	53.5	29.8
5	—	1.97 (1.06)	4.01 (2.70)	—	50.5	51.7
x_+ Design						
6	1.46 (.56)	3.26 (1.53)	6.73 (3.73)	73.0	81.5	88.6
7	2.30 (.48)	2.30 (1.66)	3.54 (1.89)	115.0	57.8	46.4
Average (standard deviation)	2.03 (.58)	2.33 (1.31)	3.97 (2.28)	101.7	59.3	51.5

[a] Profit calculations based on what high bidder would have earned if paying off in this market in each auction. All values in dollars.
[b] Predicted profits under RNSNE based on bid functions in equation (8) and footnote 13.

RNSNE with $\varepsilon = \$6$. The markdown coefficient, α, would have to vary with ε. To support the Nash equilibrium bidding model the utility function, $u(\Pi)$, would have to exhibit increasing relative risk aversion in gains from an individual auction period.[18]

4.1.b. *Effects of Public Information*: Table IV shows the effects on revenue of public information in the x_0 experiments. The first three columns show the actual average increase in revenue at the different ε levels. The last three columns show the predicted increase in revenue under the RNSNE hypothesis for the x_0 design experiments. There is an average increase in revenue in the x_0 experiments of 22¢ per auction, which is about 30 per cent of the increase predicted under the RNSNE hypothesis. Changes in revenue are quite variable across auctions (more

[18] Alternatively we might assume that subjects have biases in their subjective evaluations of the distribution of rivals' private values which depend systematically on ε. However, at present, we have no independent means to distinguish this explanation from that offered in the text. As such we lump the deviations under the headings of risk aversion. Undoubtedly the true explanation lies somewhere between.

TABLE IV

EFFECTS OF PUBLIC IFORMATION ON REVENUE IN x_0-DESIGN EXPERIMENTS: MEAN VALUES WITH STANDARD DEVIATION IN PARENTHESES

	Realized Change in Revenues[a]			Predicted Increase in Revenues[a,b]		
Experiment	$12	$24	Combined	$12	$24	Combined
x_0 Design						
1	−.21 (1.57)	.18 (1.92)	.00 (1.71)	.67 (.66)	.65 (1.00)	.66 (.83)
2	.01 (1.65)	2.51 (3.36)	1.34 (2.92)	.43 (.46)	1.05 (.90)	.76 (.78)
3	.34 (1.29)	−.26 (1.48)	.04 (1.37)	.44 (.37)	.92 (1.30)	.68 (.95)
4	.63 (.50)	−.57 (1.43)	.03 (1.20)	.39 (.19)	.91 (1.47)	.65 (1.04)
5	−.86 (.84)	−.10 (1.64)	−.35 (1.44)	.30 (.34)	.81 (1.37)	.64 (1.14)
Average [t statistic][c]	.03 [.14]	.37 [1.11]	.22 [1.03]	.45	.86	.68

[a] High bid under public information less high bid under private information conditions. All values in dollars.
[b] From RNSNE formula, for sample of high values actually drawn.
[c] Test of null hypothesis that mean deviation is zero.

so than equilibrium predictions), with most of the increase being accounted for by experiment 2. Experience appears to have little impact upon failure to observe RNSNE predictions: revenue actually decreased 35¢ per period in experiment 5, the only one of the five x_0 experiments where revenue decreased and the only one to employ experienced subjects. While the 22¢ per auction average increase in revenue is not significantly different from zero, it is more in line with predictions of the RASNE bidding model than either of the sophisticated discount models, as these predict no change, or a decrease in revenue, depending upon whether expectations are rational (4) or naive (6).

Table V shows how individual subjects altered their bids upon release of public information in the x_0 design experiments. The first column deals with cases where $x < x_0$. Bids were raised 66.8 per cent of the time here. In most cases where $x < x_0$, if subjects did not raise their bids in response to x_0, they didn't change them at all, so that bids decreased in this case less than 6 per cent of the time.

With the exception of the naive markdown bidding model, all the models considered dictate raising bids when $x < x_0$, even under naive expectations (6) concerning the level of x_0. A partial explanation for the frequency of unchanged bids here undoubtedly lies in subjective transactions costs associated with revising bids: one of our subjects remarked that in cases where $x < x_0$ he didn't bother to change his bid as he didn't see much change of winning the particular auction. With six bidders, his observation regarding the possibility of winning is quite accurate. There is a corresponding tendency for low value holders to "throw

TABLE V

EFFECTS OF PUBLIC INFORMATION ON INDIVIDUAL BIDS[a]

Bids	Private Value Relative to x_0		
	$x<x_0$	$x_0 \leq x \leq C(x_0)$	$x > C(x_0)$
$b(x, x_0) > b(x)$	66.8%	67.0%	38.2%
$b(x, x_0) = b(x)$	27.3%	17.0%	18.2%
$b(x, x_0) < b(x)$	5.9%	16.0%	43.6%
	100%	100%	100%
Total Number of Bids	187	106	55

$$C(x_0) = x_0 + \frac{N-2}{N}\varepsilon$$

[a] Each entry is the percentage of the values drawn, relative to x_0, as in the column heading, for which the subject responded as in the row heading. Only data in range (3) are reported.

away" bids in independent private values auctions (Cox, Roberson, and Smith, 1982; Cox, Smith, and Walker, 1985). Such behavior makes economic sense, once one accounts for subjective costs of calculating a more meaningful bid under the circumstances.

Both the sophisticated discount model with rational expectations and the RASNE bidding model dictate that bidders with private values in the range

$$(14) \qquad x_0 \leq x \leq C(x_0)$$

will increase their bids on release of public information. The middle column in Table V reports behavior in these cases. In 67 per cent of all such cases bids increased with public information, well above the frequency expected by chance factors alone. When bids in range (14) did not increase, about half the time they decreased, while half the time they remained constant. The 16 per cent of bids which were lower represents a marked increase relative to when $x < x_0$. Further, arguments for holding bids constant here on the grounds of subjective transactions costs, in conjunction with a low probability of winning the auction, are on substantially weaker footing than comparable arguments in cases where $x < x_0$. As such we would argue that the 33 per cent of all bids in range (14) which do not increase, represent clear violations of the sophisticated discount model with rational expectations, as well as the RNSNE and RASNE models.

Note that our interpretation of violations of Nash equilibrium bidding theory here depends critically on all bidders having identical risk attitudes, as this was central to proving the proposition in Section 3.1.c. Without assuming symmetry, no method remains to isolate a critical value below which announcing x_0 must raise bids. Since we did not control for bidders' risk preferences, and there undoubtedly were some differences in risk attitudes across bidders, the increased bidding here might be attributable to a breakdown in our symmetry assumption, and is not necessarily inconsistent with Nash behavior. On the other hand, the interval over which bids are required to increase, $x < C(x_0)$, is quite conservative, as our proposition applies to any concave utility function. As such, we interpret the increased frequency with which bids decreased in response to public information in the range (14), as resulting in large part from subjective expectations of x_0 being less than fully rational (4), as well as failures to act on the strategic implications of public information.

The third column of Table V shows what happens to bids in cases where $x > C(x_0)$. The RASNE model calls for increasing or decreasing bids here, depending upon the degree of subjects' risk aversion and the location of private values in the interval $[C(x_0), x_0 + \varepsilon]$. Since six bidders are sufficient for $C(x_0)$ to approximate the expected location of the high value, the sophisticated discount model with rational expectations calls for almost all bidders reducing their bids here. Instead, a sizable portion increase their bids, contrary to the prediction of that model.[19]

Results in Table V, in conjunction with the average increase in revenue reported in Table IV, suggest that the Nash equilibrium bidding model, with risk aversion, does a better job of organizing the data than either the naive or sophisticated discount models with or without rational expectations. The RASNE bidding model, however, falls far short of providing a complete characterization of the data: the bids in range (14) (the middle column of Table V) suggest a sizable proportion of individuals whose subjective expectations deviate from fully rational expectations, and/or who fail to act on strategic implications of public information. To obtain some sense of the relative role of risk aversion vs. bidders' errors on the revenue raising effects of public information, we consider a symmetric constant relative risk aversion bidding model (CRRA).

With public information announced, the bid function under CRRA is

$$b(x, x_0) = x_0 - \varepsilon + \frac{N-1}{N-r}[x - (x_0 - \varepsilon)] \tag{15}$$

which can be used to estimate the coefficient of relative risk aversion from the data.[20] This estimate yields a prediction of the level of revenue enhancement due to public information. The results of this exercise are reported in Table VI. Comparing the predicted impact on revenue in Table VI with the risk-neutral predictions in Table IV shows that the revenue-enhancing effects of public information are sharply curtailed on the basis of the degree of risk aversion

[19] The frequency with which bids increase in the interval $x > C(x_0)$ is clearly greater than one would expect under the sophisticated markdown bidding model, as it predicts no increase in bidding here. Further, if we assume that there is an irreducible "error" rate in bidding of 6 per cent (the frequency with which bids decrease in the interval $x < x_0$), the frequency with which bids increase still exceeds what one would expect under the sophisticated discount model. That is, assuming a binomial distribution where there is a 6 per cent average chance of bids increasing, there is virtually no chance of observing 21 out of 55 bids increasing in the interval $x > C(x_0)$, which is what we observed.

[20] There are clear specification errors involved in assuming CRRA here. It is nevertheless a readily tractable alternative to the RNSNE model for our purposes. Estimates of r were obtained using OLS procedures, individual subject data from each experiment, and the estimating equation

$$b_t^* = \alpha x_t^* + e_t$$

where in terms of equation (15) in the text $b_t^* = b(x, x_0) - x_0 + \varepsilon$, $x_t^* = [x - (x_0 - \varepsilon)]$, and e_t is a random error term. Estimates of r obtained from α are biased, but consistent. Note however that our estimates are based on relatively large sample sizes.

TABLE VI

EFFECTS OF PUBLIC INFORMATION ON REVENUE UNDER THE HYPOTHESIS OF CONSTANT RELATIVE RISK AVERSION

Experiment x_0 Design	Estimated Value of r	Predicted Impact on Revenue[a]	Actual Impact on Revenue[b]
1	.45	.14	.00
2	.46	.22	1.34
3	.51	.11	.04
4	.64	.06	.03
5	.38	.20	−.35
Average[c]	.49	.15	.22

[a] For sample of high private values drawn in the experiment. All values in dollars.
[b] From Table IV. All values in dollars.
[c] Average revenue impacts weighted by number of auctions in each experiment.

observed. In fact there is slight difference, averaged across all auction periods, between the average predicted revenue increase of 15¢ per auction period under the CRRA bidding model and the observed increase of 22¢ per auction.

Table V identified errors in individual bidder behavior in terms of failure to increase bids appropriately in the presence of public information. Undoubtedly some of the increases in bids reported in Table V, and/or the magnitude of the increase in bids, involved errors in assessing and acting on strategic implications of public information as well. The results in Table VI suggest that in this case these errors tend to cancel out: sometimes they reduce revenue enhancing possibilities of public information, but at other times they improve these possibilities.

Public information in the form of x_0 corresponds to an infinite number of announcements of an additional private valuation x_+, and resolves all uncertainty regarding the possible distribution of rivals' valuations. Given that the average response to this maximal dose of public information was relatively small, and quite variable, one would anticipate that responses to public announcement of a single private value in the x_+ design experiments would be even less reliable. Table VII reports these results. Across the two experiments, public information reduces average revenue, although these results are not statistically significant. Moreover, the standard deviation of the change in revenue within these experiments tends to be larger than in the x_0-design experiments.

Table VIII shows the effects of public information on efficiency in the auctions. The efficiency index is defined as

$$E_t = 100[W_t - (x_0 - \varepsilon)] \div [V_t - (x_0 - \varepsilon)]$$

where W_t is the winning bidder's value, V_t the highest of the 6 values drawn;[21] these are the same in a Pareto-efficient ($E_t = 100$) outcome. Otherwise, unrealized gains from exchange remain. Efficiency is not an issue in the theoretical models which analyze symmetric equilibria, thereby assuming Pareto efficiency. Symmetry

[21] Subtracting $x_0 - \varepsilon$ in numerator and denominator makes his formula correspond to the efficiency measure in Cox, Roberson, and Smith (1982).

TABLE VII

EFFECTS OF PUBLIC INFORMATION ON REVENUE IN x_+-DESIGN EXPERIMENTS: MEAN VALUES WITH STANDARD DEVIATIONS IN PARENTHESES

Experiment x_+ Design	Realized Change in Revenue[a] ε 12	24	Combined
6	−1.15 (1.06)	1.50 (6.70)	.48 (5.32)
7	−1.05 (3.55)	−1.43 (1.94)	−1.22 (2.82)
Average [t statistic][b]	−1.08 [−1.34]	.13 [.09]	−.43 [−.55]

[a] High bid under public information less high bid under private information. All values in dollars.
[b] Test of null hypothesis that mean deviation is zero.

of α parameters in (1) and (2) would yield Pareto-efficient predictions from the ad hoc bidding models, so equilibrium forces are not the source of efficient outcomes.

An asymmetric Nash equilibrium could generate inefficient outcomes from asymmetric risk preferences. This still does not make efficiency an issue, as public information serves no role to overcome risk asymmetries.

Table VIII shows, nonetheless, that announcing x_0 raises efficiency. Assymetries in information processing must be the source of these efficiency gains: announcing x_0 must disproportionately assist some bidders to avoid the perceptual errors associated with (6) above. The small dose of public information in x_+ experiments reduces efficiency. This suggests a differential impact conditional on the type of public information announced, and corresponds to our priors: there is more room for error, relative to theory, in assessing and acting on public information in x_+ as compared to x_0 designs.

TABLE VIII

EFFECTS OF PUBLIC INFORMATION ON EFFICIENCY

Experiment	Private Information Only[a] Percentage of Auctions Pareto Efficient	Private Information Only[a] Mean Efficiency Index	Public and Private Information Percentage of Auctions Pareto Efficient	Public and Private Information Mean Efficiency Index
x_0 Design				
1	73.3	98.78	93.3	99.91
2	73.3	98.44	86.7	98.93
3	92.9	99.92	100.0	100.00
4	85.7	99.47	85.7	99.47
5	86.7	99.60	86.7	99.60
x_+ Design				
6	84.6	99.29	84.6	97.36
7	80.0	98.51	66.7	95.29

[a] Data reported for dual bidding procedure auctions only.

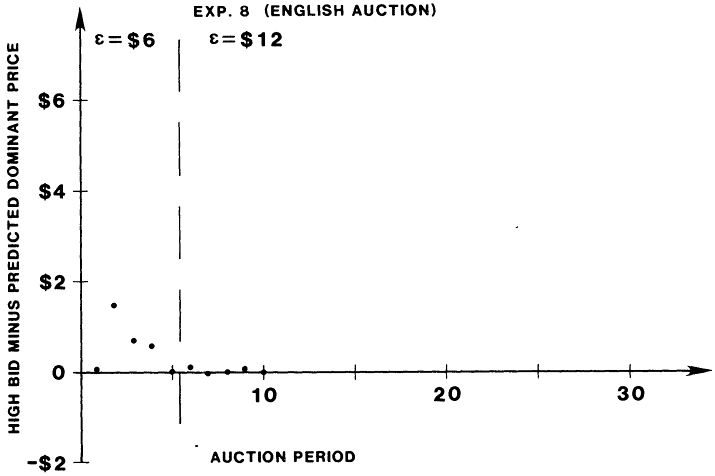

FIGURE 6

4.2. *Second-Price/English Auctions*

4.2.a. *Bid Patterns Over Time*: Figures 6–9 graph market prices over time in terms of deviations from the dominant strategy price. Examination of the figures shows clear differences in bidding patterns between the English auction and second-price sealed bid auctions. Under the English institution (Figures 6–7) prices were quite close to the predicted dominant strategy. In 76 per cent of all auction periods the difference between actual and predicted prices was less than or equal to the clock increment for that period. Experiment 8 shows a brief run up in prices, relative to the dominant strategy price, in early auction periods. But this occurrence was followed by collapse back to predicted prices, which then

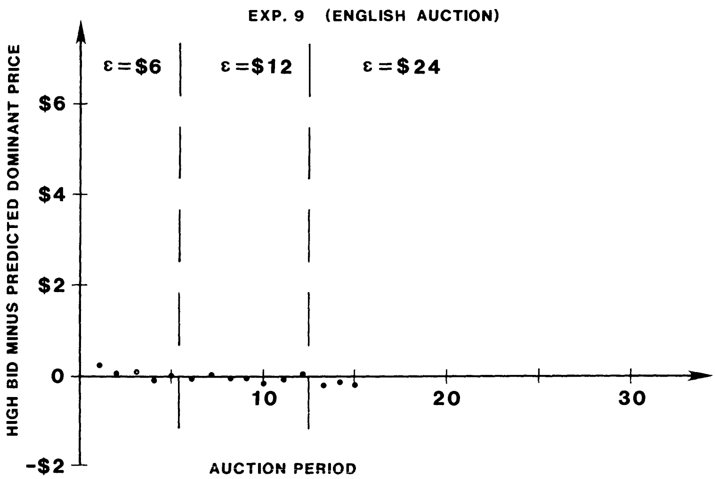

FIGURE 7

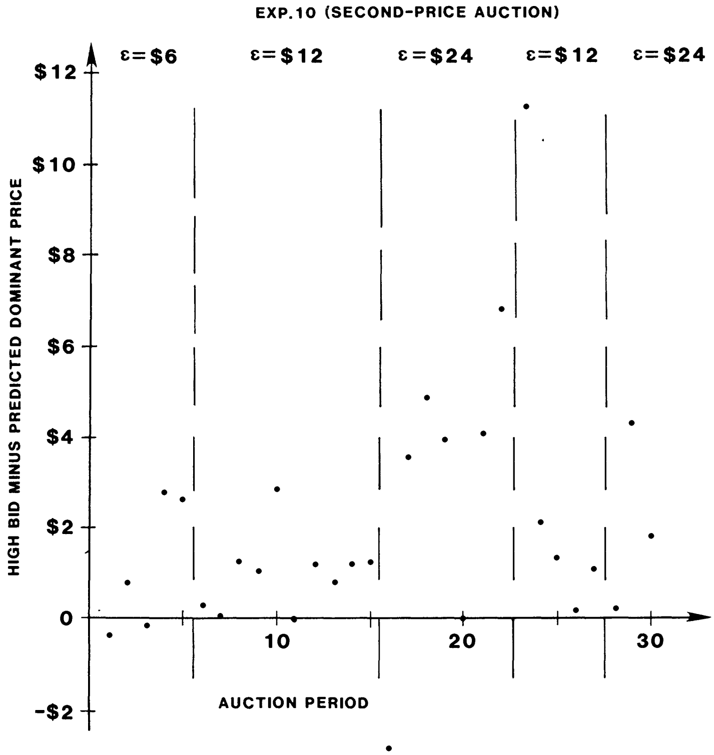

FIGURE 8

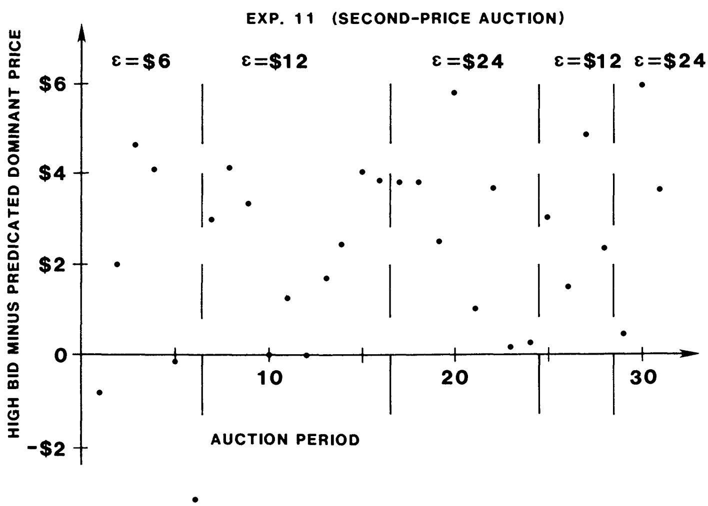

FIGURE 9

persisted. In experiment 9 the market price was $2.00 in excess of the dominant strategy price in the initial dry run, collapsed to within 25¢ of the predicted price in the first auction period with monetary payoffs, and did not deviate again after that. Thus, both experiments suggest an initial learning period with prices in excess of dominant strategy prices, followed by a steady state at the predicted equilibrium price.

Under the second-price auction institution average market prices were well in excess of the predicted dominant strategy price for all values of ε (cf. Table IX). In 80 per cent of all auction periods, market prices exceeded the dominant strategy price by more than the minimal increments employed in the English clock auctions. Moreover, no obvious tendency for prices to converge to the dominant bid price over time was observed.

This persistent excess of market price above the dominant strategy price stands in marked contrast to reports of second-price sealed bid auctions with independent private values (Coppinger, Smith, and Titus, 1980; Cox, Roberson, and Smith, 1982). Results from those experiments show average market price consistently below the dominant strategy price, with varying significance levels dependent on the number of active bidders. The key institutional feature responsible for these different outcomes is, we believe, that those earlier second-price auction experiments did not permit bidding in excess of private valuations. As noted earlier, our second-price experiments had no such restriction, in anticipation that ad hoc forces could induce bidding in excess of private values, at least in the initial auction market periods.[22] Whatever the ultimate explanation of these differences, the fact remains that: (i) permitting bidding in excess of private values seems essential to testing the prediction that market prices will equal (or converge to) the dominant strategy price, and (ii) when such bidding is allowed, average market prices uniformly exceed the dominant strategy price, and there is no tendency for these price differences to be eliminated over time.[23]

[22] It is unlikely that positive affiliation is responsible for these differences. We have conducted one second-price experiment with independent private values which showed average market prices in excess of the predicted dominant strategy price. Further, recently published nondiscriminatory, multiple unit sealed bid auctions with independent private values, where the dominant strategy is to bid one's value, show a substantial (and persistent) percentage of all bids in excess of private values (Cox, Smith, and Walker, 1985).

[23] Note that our first-price auction market procedures did *not permit* bidding in excess of private valuations. In light of our second-price results this restriction may be subject to criticism. It does not appear, however, to invalidate our first-price results or comparisons across auction institutions: (i) Persistent bidding in excess of private values in first-price auctions will soon be extinguished as it *must* result in losses in the event of winning. In contrast, modest bidding in excess of private values in second-price auctions does not generally result in losses when winning the auction. Indeed, we conjecture (see the text) that it is the general absence of such feedback that sustains the behavior. (ii) Battalio, Meyer, and Ormiston (1985) have recently completed a series of first-price private value auctions with no binding ceiling on bids. In 5 experiments involving 45 inexperienced bidders, they observed a total of 4 subjects bidding in excess of their values. All such bids occurred in early auction periods and resulted in immediate bankruptcy for 3 of the 4 subjects. Our rationalization for employing a bid ceiling in first-price auctions was that bidding in excess of private values here was likely to involve clear confuision regarding procedures on a subject's part, and that the prohibition provided a convenient vehicle for clarifying procedures, thereby speeding up the learning process. The results of Battalio, et. al. support this interpretation.

Bidding in excess of x in the second-price auctions would have to be labeled as a clear mistake, since bidding x is a dominant strategy irrespective of risk attitudes. Bidding in excess of x is likely based on the illusion that it improves the probability of winning with no real cost to the bidder as the second-high-bid price is paid.[24] The degree of overbidding observed was sustainable in the sense that average expected profits were positive at all values of ε (average expected profits under the dominant bidding strategy are $2\varepsilon/(N+1)$). To the extent that more precise conformity with dominant strategy bidding results from learning through observation, or from real reinforcement effects, these forces are weak under the sealed bid procedures. First, the idea that bidding modestly in excess of x only increases the chances of winning the auction when you don't want to win is far from obvious under the sealed bid procedure. Second, the real costs of such overbidding are weak as well. For symmetric bid functions of the sort $x+k\varepsilon$ with k equal to the average overbid per auction for $\varepsilon=\$12$ and $\varepsilon=\$24$, reported in Table IX, the probability of losing money conditional on winning the auction averages .36, with the overall probability of losing money averaging .06. These punishment probabilities are weak, given that bidders start with the illusion that bids in excess of x increase the probability of winning without materially affecting the price paid, and the majority of the time the auction outcomes support this supposition. Finally, the obvious question arises as to why we don't observe bidding even further in excess of private values than reported in Table IX. We can only presume that bidders were responding, to some extent,

TABLE IX

DIFFERENCES BETWEEN ACTUAL BID PRICE AND DOMINANT STRATEGY PRICE IN SECOND PRICE AUCTIONS: MEAN VALUES WITH STANDARD DEVIATION IN PARENTHESIS[a]

	ε		
Experiment	\$6	\$12	\$24
10	1.16	1.76	2.70
	(1.50)	(2.72)	(2.81)
11	1.14	2.40	2.86
	(3.04)	(1.43)	(2.10)
Average	1.15	2.07	2.78
[t test][b]	[1.62]	[5.10]	[5.32]

[a] Actual bid less predicted bid under dominant strategy. All values in dollars.
[b] Test of null hypothesis that mean deviation is zero.

[24] In this respect, bidding in excess of x here undoubtedly shares some motivational base with overbidding relative to the RNSNE, in the first-price auctions. In a first-price auction, however, subjects are clearly cognizant of the fact that increased bids reduced profits conditional on winning the auction. The results for the second-price auctions surely support our earlier suggestion (see footnote 18) that some of what we label risk aversion in the first-price auctions may well result from systematic biases in subjective estimates of the distribution of rivals' private valuations. The source of these biases and their effects on bidding in the first-price auctions is far from clear, and the development of consistent models incorporating such effects lies well beyond the scope of the present paper.

to the forces underlying the dominant strategy: with more substantial overbidding, the likelihood of winning when you don't want to win increases substantially.

The structure of the English clock auctions makes it particularly clear to bidders that they don't want to bid above their private values.[25] Once the clock price exceeds a bidder's value, it is clear that competing further to win necessarily involves losing money. Even then, some early overbidding is observed, only to collapse immediately after negative profits (losses) resulted (or would have resulted in the case of experiment 9). The "real time" nature of the English auction is ideal for producing observational learning, learning without experiencing the punishing effects of actually losing money consequent on bidding in excess of x. This enhanced capacity of the English clock institution to produce observational learning distinguishes it most clearly, on a behavioral level, from the second-price institution.[26]

4.2.b. *Effects of Auction Institution on Revenue*: With risk aversion in first-price auctions, and deviations from the dominant bidding strategy in second-price auctions, the enhanced revenue raising possibilities of English/second-price auction institutions relative to a first-price auction need no longer hold (Milgrom and Weber, 1982); it becomes an empirical question which institution is likely to raise the most revenue. To contrast English and first-price auctions we compared average prices in our first-price auction experiments (experiments 1–7) with predicted prices under the English auction's dominant bidding strategy in these same experiments. If we accept the conclusion that, with learning, market prices converge to the dominant strategy price in English auctions, then this comparison is relevant to steady state behavior once learning has been effected in the English auctions.

The second column of Table X reports the results of this comparison. The first column compares, theoretically, revenue between the two institutions (assuming risk neutrality) for the same series of auctions. Consistent with Milgrom and Weber's proposition, under risk neutrality and Nash equilibrium bidding, an English auction would have raised more revenue than a first-price auction for all levels of ε. However, the risk attitudes of our bidders are such that with $\varepsilon = \$6$ an English auction would have raised revenues somewhat more sharply than predicted under risk neutrality. This follows directly from the dominant bidding strategy, in conjunction with observed bidding slightly below the RNSNE in

[25] Under our design, the dominant bidding strategy in both second-price and English auctions rests on first-degree stochastic dominance arguments, rather than the stronger requirements of expected utility theory. Karni and Safra (1986) show that in second-price and English auction institutions, where the object being sold is a lottery that assigns objectively known (non-degenerate) probabilities to a given set of money prizes, it is possible for a non-expected-utility maximizer to deviate rationally from the dominant strategy in the second-price auction, while adhering to it under the English clock institution. The non-expected-utility formulations used to obtain these results imply first degree stochastic dominance. Hence, the Karni and Safra formulation cannot be used to explain our results.

[26] Independent private value auction experiments in Cox, Roberson, and Smith (1982) fail to observe a behavioral isomorphism between the first-price and Dutch auction institutions which theory predicts. Here too the "real time" auction (the Dutch auction) produces lower bid prices than the sealed bid institution. Whether these differences result from the same behavioral mechanisms as those underlying the differences reported here is an open question at this point.

TABLE X

EFFECTS ON REVENUE OF ALTERNATIVE AUCTION INSTITUTIONS: MEAN DIFFERENCES WITH STANDARD DEVIATIONS IN PARENTHESES

ε	Predicted[a] Second-Price/English less First Price	Actual English less First-Price[b]	Actual Second-Price Less First-Price[c]
6	.69	.98	2.13
	(1.10)	(1.26)	(1.66)
12	.08	−1.33	.74
	(3.46)	(3.64)	(3.34)
24	.42	−2.98	−.20
	(5.84)	(6.37)	(5.58)

[a] Calculated from sample of private values drawn in experiments 1-7 assuming a RNSNE in first-price auctions and the dominant bidding strategy in second-price/English auctions. All values in dollars.

[b] Calculated from experiments 1-7 assuming the dominant bidding strategy in English auctions less observed high bid in first-price auctions with private information. All values in dollars.

[c] Mean difference between actual bids and dominant strategy bids in experiments 10 and 11 less mean difference in actual first-price bids and dominant strategy second-price bids in experiments 1-7. All values in dollars.

first-price auctions. With $\varepsilon = \$12$ or \$24 though, the first-price institution would have raised substantially more revenues, due to bidding in excess of the RNSNE in these auctions.

The last column of Table X compares the revenue-raising effects of our second-price sealed bid auctions with the first-price auctions. To control for price differences between auctions, we have computed these revenue-raising effects on the basis of differences between actual and predicted second-price outcomes in both sets of experiments. Risk attitudes of our bidders and deviations from the dominant strategy are such that with $\varepsilon = \$6$ the second-price institution would have raised substantially more revenue than the first-price institution. With $\varepsilon = \$12$ or \$24, the overbidding relative to the RNSNE in first-price auctions tends to be offset by bidding in excess of the dominant strategy in second-price auctions. The net effect is a modest increase in revenue for second-price over first-price with $\varepsilon = \$12$, and a modest decrease with $\varepsilon = \$24$. Neither of these differences is statistically significant.

We do not maintain that there is any exact analogue between our experimental auction markets and field settings, rather that the basic economic forces at work in the laboratory are likely to be observed in the field as well.[27] If this is the case then the general conclusions that can be reached on the basis of Table X are: (i) The revenue-raising possibilities inherent in English/second-price auction institutions, relative to a first-price institution, with positively affiliated private values, are severely compromised by the potential for risk-averse bidding, and

[27] See Kagel and Levin (1986) for some striking evidence on parallels between laboratory and field auction outcomes in cases where the laboratory observations are squarely at odds with Nash equilibrium predictions.

(ii) this compromise is more severe under the English auction institution, as it induces closer conformity to dominant strategy bidding.

5. SUMMARY AND CONCLUSIONS

A series of auction experiments were conducted in which a single indivisible commodity was auctioned off among six bidders with positively affiliated private values under a variety of allocation rules. Under a first-price sealed-bid allocation rule the Nash equilibrium bidding theory outperformed two ad hoc bidding models involving both simple and sophisticated discount rules. Large doses of public information raised revenue, but these increases were lower on average and considerably less reliable than predicted. Lower average revenues may largely be attributable to risk aversion. Variability in revenue raised results from a sizable frequency of persistent individual bidding errors (relative to theory) in response to release of public information.

The dominant strategy equilibrium accurately organized English auction outcomes after a brief learning period. The dominant strategy equilibrium does not organize second-price auction outcomes, as bids consistently exceeded private values. The breakdown of the isomorphism between English and second-price institutions on a behavioral level can be attributed to differential information flows inherent in the structure of the two auctions.

Evaluating results across auction institutions and experimental manipulations indicates that bidders are sensitive to the strategic implications underlying private value auctions, and captured in Nash equilibrium bidding theory. Although we cannot definitively rule out the sophisticated discounting model in first-price auctions, the effects of public information on average revenue, and the directional changes in individual bids, come closer to the predictions of the RASNE. Further, there is much closer correspondence between Nash equilibrium bidding theory, and behavior, in first-price private value auctions then in first-price common value auctions (Kagel and Levin, 1986).

Ad hoc reasoning in second-price and English auctions call for overbidding, or underbidding, relative to the dominant strategy. These predictions are falsified in English auctions. Although we observe persistent overbidding in second-price auctions, clear economic forces are at work limiting the size of the overbid. Unlike deviations from theoretical predictions in first-price auctions, second-price bidding errors cannot be explained by references to asymmetric, risk-averse or disequilibrium behavior. Second-price errors are not robust, however, since a thoretically transparent treatment, the English auction, eliminates them.

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APPENDIX

The risk-averse symmetric Nash equilibrium can be characterized as follows:

$$\text{(A1)}\qquad b'(x_i)=[N/(x_i+\varepsilon-\underline{x})]u[x_i-b(x_i)]/u'[x_i-b(x_i)],\quad \text{if}\quad x_i\in(\underline{x}-\varepsilon,\underline{x}+\varepsilon]$$

and

$$\text{(A2)}\qquad b'(x_i)=(N/2\varepsilon)u[x_i-b(x_i)]/u'[x_i-b(x_i)],\quad \text{if}\quad x_i\in[\underline{x}+\varepsilon,\bar{x}-\varepsilon],$$

with initial condition

$$\text{(A3)}\qquad b_1(\underline{x}-\varepsilon)=\underline{x}-\varepsilon$$

and continuity at $x_i=\underline{x}+\varepsilon$. With public information x_0 announced, the RASNE satisfies

$$\text{(A4)}\qquad \partial b/\partial x_i|_{(x_i,x_0)}=\left(\frac{N-1}{x_i+\varepsilon-x_0}\right)\left(\frac{u[x_i-b(x_i,x_0)]}{u'[x_i-b(x_i,x_0)]}\right)$$

with

$$\text{(A5)}\qquad b(x_0-\varepsilon,x_0)=x_0-\varepsilon.$$

These characterizations follow from Theorem 14 in Milgrom and Weber (1982); derivations are omitted. For $x_i>\underline{x}-\varepsilon$, the bid $\hat{b}=(x_i+\underline{x}-\varepsilon)/2$ yields positive expected utility, so the best response must satisfy $b(x)<x$ on $(\underline{x}+\varepsilon,\bar{x}-\varepsilon]$.

Let $C(x_0)=x_0+\varepsilon(N-2)/N$. Note that $C(x_0)>x_0$ for $N\geq 3$.

PROPOSITION: *If $\underline{x}+\varepsilon\leq x_i\leq\bar{x}-\varepsilon$, for any x_0, any $u(\cdot)$, $x_i<C(x_0)$ implies $b(x_i)<b(x_i,x_0)$, i.e., a signal below the critical value $C(x_0)$ is associated in Nash equilibrium with a higher bid under public information.*

PROOF: As noted above, $b(x_0-\varepsilon)<x_0-\varepsilon=b(x_0-\varepsilon,x_0)$. By (A2) and (A4):

$$\text{(A6)}\qquad \partial b(x_i,x_0)/\partial x_i-b'(x_i)=\left(\frac{N-1}{x_i+\varepsilon-x_0}\right)\left(\frac{u}{u'}\right)\bigg|_{x_i-b(x_i,x_0)}-\left(\frac{N}{2\varepsilon}\right)\left(\frac{u}{u'}\right)\bigg|_{x_i-b(x_i)}.$$

Under the assumption that $b(x_i)=b(x_i,x_0)$, (A6) becomes:

$$\begin{aligned}\partial b(x_i,x_0)/\partial x_i-b'(x_i)&=\left[\frac{N-1}{x_i+\varepsilon-x_0}-\frac{N}{2\varepsilon}\right]\left(\frac{u}{u'}\right)\bigg|_{x_i-b(x_i)}\\&>\left[\frac{N-1}{C(x_0)+\varepsilon-x_0}-\frac{N}{2\varepsilon}\right]\left(\frac{u}{u'}\right)\bigg|_{x_i-b(x_i)}=0\end{aligned}$$

where the inequality results from substituting the larger $C(x_0)$ for x_i in the denominator.

Thus, $b(x_i,x_0)$ starts out above $b(x_i)$ at $x_i=x_0-\varepsilon$, and assuming the two curves intersect before $x_i=C(x_0)$ yields the contradictory inference that $b(x_i,x_0)$ crosses $b(x_i)$ from below. *Q.E.D.*

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