

Economics 171: Decisions Under Uncertainty

Midterm 1 – Solutions (Winter 2008)

1. (10 pts) This afternoon it will be either sunny or rainy. Ralph can play on the slide or stay indoors. If he plays on the slide he'll have fun if it's sunny and he'll get hurt if it's rainy. If he stays indoors he'll be bored regardless of the weather.
  - a. What actions are available to Ralph?  
{Play on slide, Stay indoors}
  - b. What states of nature could occur?  
{Sunny, Rainy}
  - c. What are the possible outcomes?  
{Have fun, Get hurt, Be bored}
  
2. (30 pts) Lottery  $P$  pays  $a$  with probability 0.6 and  $b$  with probability 0.4. Lottery  $Q$  pays  $c$  with probability 0.7 and  $d$  with probability 0.3.
  - a. Suppose  $a = \$10$ ,  $b = \$50$ ,  $c = \$20$  and  $d = \$40$ . (Therefore lottery  $P$  is  $(\$10, 0.6; \$50, 0.4)$  and lottery  $Q$  is  $(\$20, 0.7; \$40, 0.3)$ .) Which lottery is chosen according to the maximin criterion?  
 $Q$ . The minimum outcome for  $Q$  ( $\$20$ ) is greater than the minimum outcome for  $P$  ( $\$10$ ).
  - b. Ignore the values given for  $a$ ,  $b$ ,  $c$  and  $d$  in part (a). Assume  $a < b$  and that  $c < d$ . Which lottery is chosen according to the maximin criterion? (You should consider multiple cases depending on the relationship between  $a$ ,  $b$ ,  $c$  and  $d$ .)  
 $P$  is chosen if  $a > c$ .  $Q$  is chosen if  $c > a$ . Either is chosen if  $a = c$ .
  - c. Again, ignore the values given for  $a$ ,  $b$ ,  $c$  and  $d$  in part (a). Assume  $a < b$  and that  $c < d$ . Explain why it's not possible to choose lottery  $P$  under the maximin criterion and lottery  $Q$  under first order stochastic dominance.  
If  $P$  is chosen under the maximin criterion  $a > c$ . Look at the cdfs of both functions at the point  $c$ . The cdf for  $P$  will equal zero and the cdf for  $Q$  will be positive. Since the cdf for  $P$  is below the cdf for  $Q$  at the point  $c$ ,  $Q$  cannot first order stochastically dominate  $P$ .

3. (20 pts)

- a. Lottery  $A$  is  $(\$10, 0.3; \$20, 0.7)$ . Lottery  $B$  is  $(\$0, 0.1; \$20, 0.9)$ . Find a simple lottery equal to a mixture of 60% of lottery  $A$  and 40% of lottery  $B$ .

$$P(\$0 \text{ in the mixture}) = 0.6(0) + 0.4(0.1) = 0.04$$

$$P(\$10 \text{ in the mixture}) = 0.6(0.3) + 0.4(0) = 0.18$$

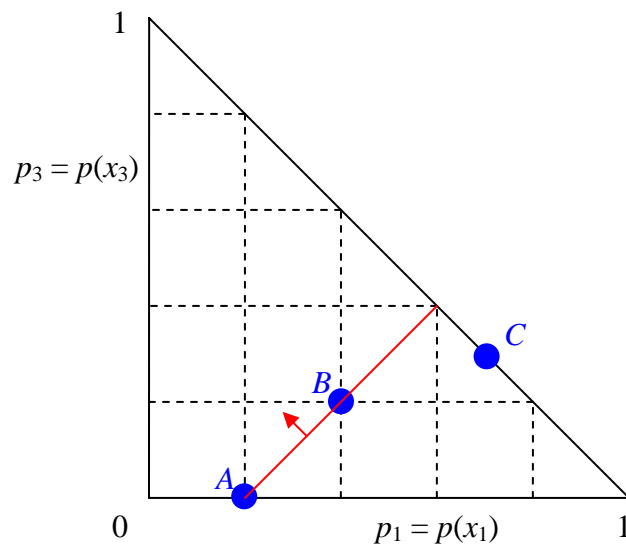
$$P(\$20 \text{ in the mixture}) = 0.6(0.7) + 0.4(0.9) = 0.78$$

- b. Lottery  $C$  is distributed uniformly from 40 to 60. Lottery  $D$  is distributed uniformly from 60 to 80. Find a simple lottery equal to a mixture of 50% of lottery  $C$  and 50% of lottery  $D$ .

Lottery  $C$  has an equal chance at any number between 40 and 60. Lottery  $D$  has an equal chance at any number between 60 and 80. A 50%/50% mixture of the two will have an equal chance at any number between 40 and 80. The mixture is distributed uniformly from 40 to 80.

4. (16 pts) Ellie is a von Neumann-Morgenstern expected utility maximizer. She is indifferent between lotteries  $A = (\$0, 0.2; \$100, 0.8)$  and  $B = (\$0, 0.4; \$100, 0.4; \$500, 0.2)$ . What can you say about her preferences between  $A$  and  $C = (\$0, 0.7; \$500, 0.3)$ . Briefly explain why. (I suggest you use our triangle diagram drawn to scale. You can also use algebraic reasoning.)

$A \succ C$ .



$$EU(A) = 0.2u(0) + 0.8u(100) = 0.4u(0) + 0.4u(100) + 0.2u(500) = EU(B)$$

$$0.2u(500) = -0.2u(0) + 0.4u(100)$$

$$u(500) = -u(0) + 2u(100)$$

$$EU(C) = 0.7u(0) + 0.3u(500)$$

$$= 0.7u(0) + 0.3[-u(0) + 2u(100)]$$

$$= 0.4u(0) + 0.6u(100)$$

$EU(A)$  and  $EU(C)$  are both weighted averages of  $u(0)$  and  $u(100)$ .

$EU(C)$  places more weight on the lower value,  $u(0)$  so  $EU(C) < EU(A)$ .

5. (24 pts) Tim has a choice between lottery  $A = (\$0, 0.8; \$25, 0.2)$  and lottery  $B = (\$9, 0.5; \$16, 0.5)$ . Assume Tim's utility for \$0 is 0, his utility for \$9 is 0.6 and that his utility for \$25 is 1. (Notice that I gave you utility values for three dollar amounts.)

He's indifferent between receiving \$16 with certainty and a lottery equal to (\$0, 0.2; \$25, 0.8).

- a. Give Tim's utility for \$16.

$$u(16) = 0.2u(0) + 0.8u(25)$$

$$= 0.2(0) + 0.8(1)$$

$$= 0.8$$

- b. If he wants to maximize expected utility which lottery should he choose?

He'll choose  $B$ .

$$EU(A) = 0.8(0) + 0.2(1) = 0.2$$

$$EU(B) = 0.5(0.6) + 0.5(0.8) = 0.7$$

- c. Find the value of  $p$  that will make him indifferent between receiving \$9 with certainty and a lottery equal to (\$0,  $1 - p$ ; \$16,  $p$ ).

$$u(9) = (1 - p)u(0) + (p)u(16)$$

$$0.6 = (1 - p)0 + (p)0.8$$

$$p = 0.75$$