Econ 171 Spring 2008

Problem Set 3 – Solutions

1. Maggie's utility function is $u = 2x^{\frac{1}{2}}$. Lisa's utility function is $v = \ln x$ and Bart's utility function is $w = x^3$.

   a. Calculate each person's coefficient of absolute risk aversion. Rank these three people from most to least risk averse.

      Note: For all of these we're assuming $x > 0$.

      Maggie: $u' = x^{-\frac{1}{2}},\ u'' = -\frac{1}{2}x^{-\frac{3}{2}},\ \lambda_M = -\frac{u''}{u'} = -\frac{-\frac{1}{2}x^{-\frac{3}{2}}}{x^{-\frac{1}{2}}} = \frac{1}{2}x^{-1}$

      Lisa: $v' = \frac{1}{x} = x^{-1},\ v'' = -x^{-2},\ \lambda_L = -\frac{v''}{v'} = -\frac{-x^{-2}}{x^{-1}} = x^{-1}$

      Bart: $w' = 3x^2,\ w'' = 6x,\ \lambda_B = -\frac{w''}{w'} = -\frac{-6x}{3x^2} = -2x^{-1}$

      $\lambda_L > \lambda_M > \lambda_B$

      Lisa is the most risk averse; Maggie is in the middle and Bart is the least risk averse.

   b. Show whether Lisa is nonincreasingly, nondecreasingly or constantly risk averse.

      Lisa is nonincreasingly risk averse. $\frac{\partial \lambda_L}{\partial x} = -x^{-2} \le 0$.

   c. Find Maggie's certainty equivalent for lottery $p = \{\$1, 0.4;\ \$36, 0.6\}$. What is her risk premium for lottery p?

      $EU(p) = 0.4\left[2(1)^{\frac{1}{2}}\right] + 0.6\left[2(36)^{\frac{1}{2}}\right] = 8$

      Let $\tilde{x}$ be her certainty equivalent to *p*.

      $u(\tilde{x}) = EU(p)$

      $2(\tilde{x})^{\frac{1}{2}} = 8$

      $\tilde{x} = \$16$

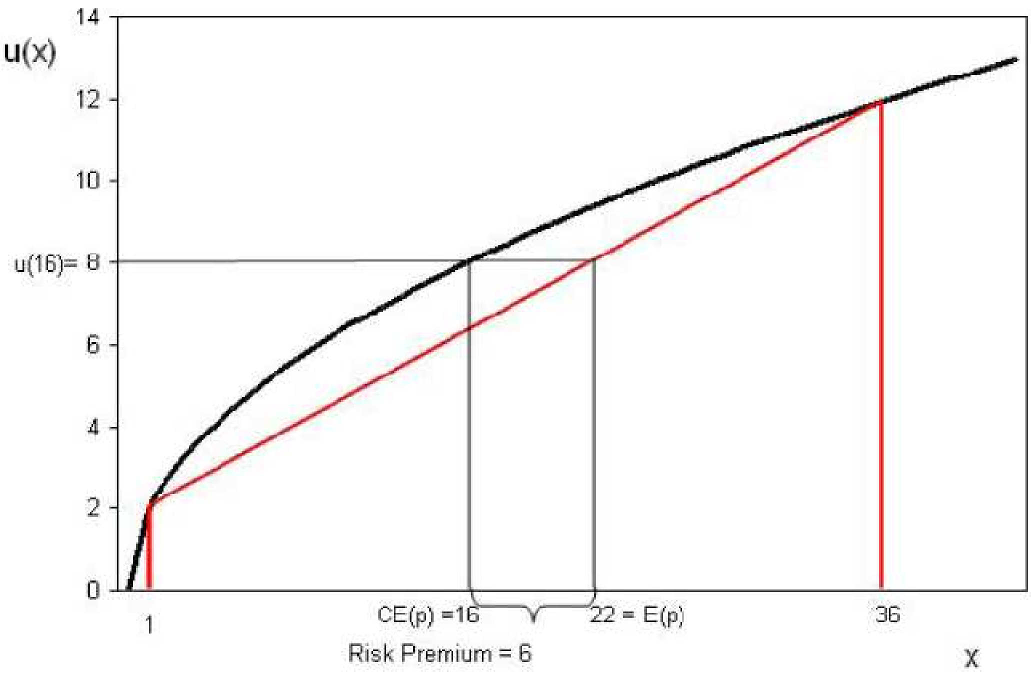
Let R_M be her risk premium for *p*.

$$R_M = E(p) - \tilde{x}$$
$$= [0.4(1) + 0.6(36)] - 16$$
$$= \$6$$

d. Explain how Lisa's and Bart's risk premium's will compare to Maggie's for this lottery. You don't need to calculate the exact values.

$R_L > \$6$ and $R_B < \$6$.
Lisa's risk premium will be greater than Maggie's since Lisa is more risk averse than Maggie.
Bart's risk premium will be less than Maggie's since he is more risk averse than she. (Furthermore Bart's risk premium will be negative since he's risk loving.)

e. Illustrate your answer to part (c) graphically.



f. Find Lisa's risk premium for lottery $q=\{\$1,0.25;\ \$e^3,0.5;\ \$e^5,0.25\}$.

$$EU(q)=0.25\left[\ln(1)\right]+0.5\left[\ln\left(e^3\right)\right]+0.25\left[\ln\left(e^5\right)\right]$$
$$=0.25[0]+0.5[3]+0.25[5]$$
$$=2.75$$

Let $\tilde{x}$ be her certainty equivalent to *q*.

$$v\left(\tilde{x}\right)=EU(p)$$
$$\ln\left(\tilde{x}\right)=2.75$$
$$\tilde{x}\approx\$15.6$$

Let R_L be her risk premium for *q*.

$$R_L=E(q)-\tilde{x}$$
$$=\left[0.25(1)+0.5\left(e^3\right)+0.25\left(e^5\right)\right]-15.6$$
$$=\$31.8$$

2. Lenny and Carl are both von Neumann-Morgenstern utility maximizers. Lenny is at least as risk averse as Carl. Carl is indifferent between a sure payment of \$200 and a lottery $p=\{\$0,0.2;\ \$1000,0.8\}$. What can you say about Lenny's certainty equivalent for lottery $q=\{\$0,0.2;\ \$700,0.2;\ \$1100,0.6\}$.

We need to look at two things: a comparison between Lenny's and Carl's risk premiums; and a comparison between lotteries *p* and *q*.

First observe that lottery *q* has the same mean and more risk than lottery *p*.

E(*p*) = 0.2(0) + 0.8(1000) = 800
E(*q*) = 0.2(0) + 0.2(700) + 0.6(1100) = 800
q = *p* + zero conditional mean noise.

$$q=\begin{cases} p & \text{when } p=\$0 \\ p+\left(-\$300,0.25;\ \$100,0.75\right) & \text{when } p=\$1000 \end{cases}$$

Carl will prefer *p* to *q* so his certainty equivalent for *q* will be lower than his certainty equivalent for *p*. (You could use SOSD to show Carl will prefer *p* to *q*.)

Second observe that Lenny's certainty equivalent for lottery *q* will be less than or equal to Carl's since Lenny is at least as risk averse as Carl.

Lenny's certainty equivalent for lottery *q* will be less than \$200.

It's less than or equal to Carl's for *q* which is less than \$200.

3. Marge is a risk averse, von Neumann-Morgenstern utility maximizer. Determine here preferences between each of the following pairs of lotteries. Explain why.

   a. $a = \{\$0, 0.2;\ \$20, 0.3;\ \$40, 0.5\}, \quad b = \{\$10, 0.2;\ \$20, 0.3;\ \$36, 0.5\}$

      $b \succ a$.

      $E(a) = 0.2(0) + 0.3(20) + 0.5(40) = 26$

      $E(b) = 0.2(10) + 0.3(20) + 0.5(36) = 26$

      a is a mean preserving spread of *b*. We can ignore the common component (\$20, 0.3). The 0.2 mass on \$10 is pushed away from the mean (\$26) to \$0. The 0.5 mass on \$36 is also pushed away from the mean to \$40.

      (You could also show that $b \succ a$ through SOSD.)

   b. $c = \{\$0, 0.2;\ \$20, 0.3;\ \$40, 0.5\}, \quad d = \{\$0, 0.2;\ \$32.5, 0.8\}$

      $d \succ c$.

      $E(c) = 0.2(0) + 0.3(20) + 0.5(40) = 26$

      $E(d) = 0.2(0) + 0.8(32.5) = 26$

      We'll try to write *c* = *d* + zero mean noise.

      $$\varepsilon = \begin{cases} 0 & \text{if } d = \$0 \\ (-\$12.5, p;\ 7.5, 1-p) & \text{if } d = \$32.5 \end{cases}$$

      $E[\varepsilon \mid d] = 0$

      $-12.5p + 7.5(1-p) = 0$

      $p = 0.375$

      c = *d* + ε, where ε = 0 when *d* = \$0.
      = (–\$12.5, 0.375; \$7.5, 0.625) when *d* = \$32.5

      (You should also verify that *c* = the compound lottery *d* + ε.)

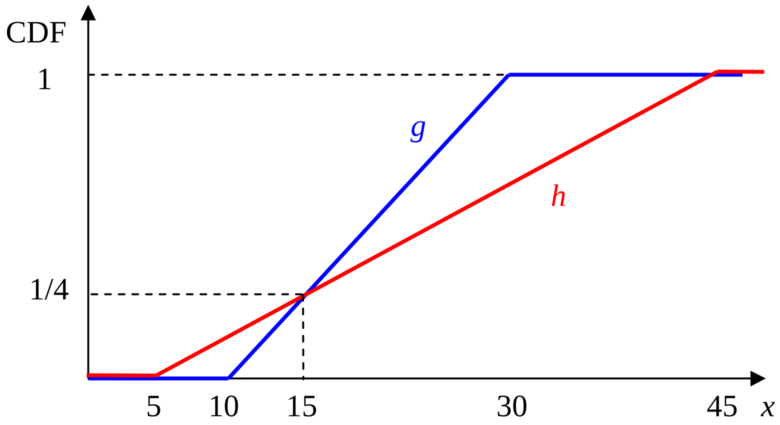
   c. $e = \{\$0, 0.2;\ \$20, 0.3;\ \$40, 0.5\}, \quad f = \{\$10, 0.2;\ \$20, 0.3;\ \$40, 0.5\}$

      $f \succ e$.

      The only difference between these lotteries is that *e* has 0.2 probability of \$0 where *f* has a point two probability of \$10. (You can show that *f* FOSD *e*.)

   d. *g* is distributed uniformly from [\$10, \$30]; *h* is distributed uniformly from [\$5, \$45].

g has a lower mean than *h* ($20 compared to $25) but *g* also seems less risky than *h* (it has less spread). *h* also seems more risky because it's lowest outcome is worse than *g*'s. We probably can't tell her preference without knowing the exact shape of her utility function. We'll check SOSD to be sure.



The equation for g's cdf is $G = \dfrac{x-10}{30-10}$. The equation for h's is $H = \dfrac{x-5}{45-5}$.

(See notes from Probability 1 for more detail.)

The point of intersection will be:

$$\frac{x-10}{20} = \frac{x-5}{40}$$
$$2x-20 = x-5$$
$$x = 15$$
$$G = H = \frac{15-5}{45-5} = \frac{1}{4}$$

$\int_5^x (h-g)\partial s > 0$ when $5 < x < 15$. $\Rightarrow$ $\int_5^x (g-h)\partial s < 0$ when $5 < x < 15$.

$\int_5^{45} (h-g)\partial s =$ The area of the bottom left triangle $-$ the area of the top right triangle.

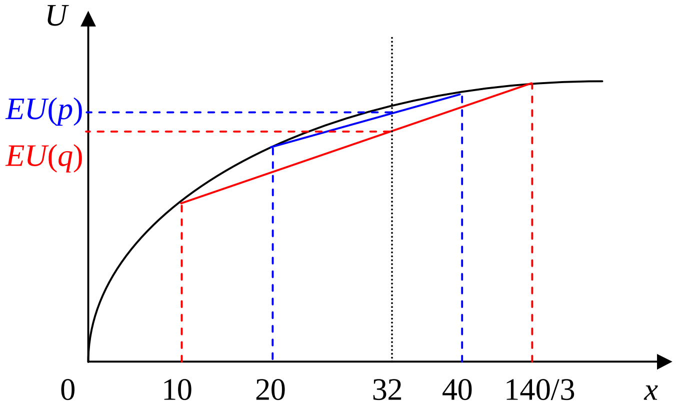
$$= \frac{1}{2}(10-5)\left(\frac{1}{4}-0\right) - \frac{1}{2}(45-30)\left(1-\frac{1}{4}\right)$$
$$= -5$$

$\int_5^x (h-g)\partial s$ won't be $\geq$ 0 for all x. Neither will $\int_5^x (g-h)\partial s$ $\Rightarrow$ We don't have SOSD.

4. $p = \{\$20, 0.4;\ \$40, 0.6\}$. Use a concave utility function to graphically illustrate each of the following:

   a. Show that EU(p) > EU(q) where q is a mean-preserving spread of p. (Note: This will work for any lottery q. Just choose one to illustrate.)

      E(p) = 0.4(20) + 0.6(40) = \$32
      Let's find a mean preserving spread of p. We'll arbitrarily use $q = \{\$10, 0.4;\ \$x, 0.6\}$
      E(q) = 0.4(10) + 0.6(x) = \$32
      $x = 140/3$
      $q = \{\$10, 0.4;\ \$140/3, 0.6\}$



   b. Show that EU(r) > EU(p) where r is a *non* mean-preserving spread of p. (Note: This won't work for all lotteries r. Just find one that does work.)

      E(p) = 0.4(20) + 0.6(40) = \$32
      Let's find a spread that results in a higher mean than p. We'll arbitrarily use $r = \{\$18, 0.2;\ \$50, 0.8\}$
      E(r) = 0.2(18) + 0.8(50) ≈ \$44