

Problem Set 1

1. You can invest in either a mutual fund or a money market fund. In the next year, the mutual fund will increase 20%, increase 10%, not change or decrease 10%. The money market fund will definitely increase by 5%.
  - a. What actions are available to you?
  - b. What states of nature could occur?
  - c. What are the possible outcomes?

There is a 10% chance that the mutual fund will increase by 20%. There is a 60% chance the mutual fund will increase by 10%. There is a 20% chance that there will be no change in the mutual fund and there is a 10% chance that the mutual fund will decrease by 10%. In which fund would you invest given each of the below criteria.

- d. Maximax.
  - e. Maximin.
  - f. Minimax regret.
  - g. First order stochastic dominance.
  - h. The expected value criterion.
2. Repeat question 1 parts (a) – (c) if you have \$100 to allocate between the two funds. For instance, you now have the option of investing \$25 in the mutual fund and \$75 in the money market fund. You could do any other split where you invest a non-negative amount in each fund so that your total investment is \$100.
3. Consider the problem of choosing an insurance plan. Plan A costs \$500 and fully covers all expenses. Plan B costs \$100 and covers 80% of your expenses. You also can self insure, that is, you pay no costs up front but you pay the full cost of any incidents.

There are four possible states: The probability is 0.1 for a major incident which costs \$5000. The probability is 0.3 for a normal incident which costs \$1000. The probability is 0.2 for a minor incident which costs \$100 and the probability is 0.4 for no incident which does not cost anything.

Which insurance plan (or the option to self insure) would you choose under each of the following criteria?

- a. Maximax.
- b. Maximin.
- c. Minimax regret.
- d. First order stochastic dominance.

- e. The expected value criterion.
4. Prove that it is not possible to choose one lottery under the maximax criterion and another under first order stochastic dominance.
  5. Explain the difference between statistical and non-statistical events. Give two examples of each.
  6. Suppose you are considering a mutual fund to hold it for two years. Each year there is a 80% chance that the mutual fund will increase by 10% and there is a 20% chance that the mutual fund will decrease by 5%. Assume the performance in each year is independent of the other.

Let  $X$  be your total return over the two years.

- a. What values can  $X$  take?
  - b. Describe the probability distribution for  $X$ .
  - c. Give the cdf for  $X$ .
  - d. Find  $E[X]$  and  $\text{Var}[X]$ .
7.  $X$  is distributed uniformly from 40 to 60.  $Y$  is distributed uniformly from 50 to 70.
    - a. Find  $E[X]$  and  $E[Y]$ .
    - b. Find  $\text{Var}[X]$  and  $\text{Var}[Y]$ .
    - c. Give the cdf for  $X$  and the cdf for  $Y$ .
    - d. Does  $X$  or  $Y$  exhibit first order stochastic dominance over the other?
  8. Suppose you roll a die two times. In each roll you have an equal probability of getting an integer from 1 to 6.
    - a. Let  $X$  be the number of sixes you roll. Find  $E[X]$  and  $\text{Var}[X]$ .
  9. Suppose 1% of the people in California have a certain type of cancer. The state is considering testing everyone in the state for the cancer. A person with the cancer has a 95% chance of testing positive. A person without the cancer has a 10% chance of testing positive. If a doctor observes a positive test, what is the probability that a patient has the cancer?
  10. State whether the following functions are concave, convex or neither.
    - a.  $e^x$
    - b.  $x^3 + 2x^2 + 4$
    - c.  $\ln(x)$  for  $x > 0$

11. Lottery A pays 0 20% of the time and pays 10 80% of the time. Lottery B pays -10 40% of the time and pays 10 60% of the time. Consider a probability mixture of these two lotteries,  $\alpha A$  and  $(1 - \alpha)B$  where  $0 < \alpha < 1$ .
- Find a simple lottery that's equivalent to the mixture. (Your answer will be in terms of  $\alpha$ .)
  - For what value of  $\alpha$  is the expected payoff of the mixture equal to 6?