Economics 171
Decisions Under Uncertainty

5. Empirical Analysis of Risk Preferences

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Optional Readings

– Kahneman & Tversky (1979), “Prospect Theory: An Analysis of Decision Under Risk,” *Econometrica*, **47**: 2.

Outline

- Expected utility review.
- An alternative to expected utility theory.
- Resolving the Allais paradox.
- A new problem.

Expected Utility

Review

Basic Idea

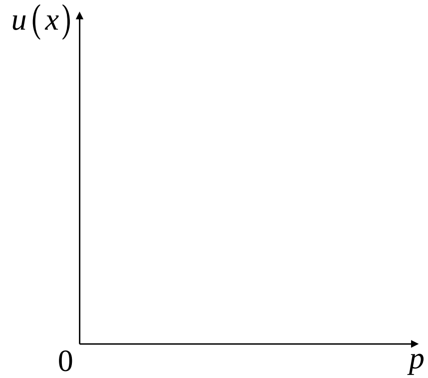
Consider someone's preferences over the following class of lotteries:

$$(\$x, p;\ \$0, 1-p)$$

Under our typical assumptions, this person will prefer:

More x.

Higher values of p.



An expected utility maximizer maximizes $E[U] = pu(x)$.

More generally we could have $U = f(p, x)$

where $\frac{\partial f}{\partial p} > 0$ and $\frac{\partial f}{\partial x} > 0$; i.e. $V = p + u(x)$ or $W = p^2 \ln x$

We'd like to study preferences on more general lotteries.

Graphically even $(x_1, p_1;\ x_2, p_2)$ becomes impractical.

The trick we'll use is fixing x_1 and x_2.

This leaves us with two variables (p_1, p_2).

The laws of probability tell us $p_2 = 1 - p_1$.

So we really just have one variable.

We can add one more (fixed) outcome to our lottery.

We end up with a function of two variables.

$$(p_1, 1 - p_1 - p_3, p_3)$$

More General Idea

Consider someone's preferences over the following class of lotteries:

$$(x_1, p_1;\ x_2, p_2;\ x_3, p_3)$$

where x_1, x_2, x_3 are constants with $x_1 < x_2 < x_3$

Expected utility maximizer maxes $E[U] = p_1 u(x_1) + p_2 u(x_2) + p_3 u(x_3)$.

Someone else might maximize $V = f(p_1, p_2, p_3, x_1, x_2, x_3)$

ie., we could have $V = p_1 p_3 \left(\frac{x_3 - x_2}{x_1} \right)$

Indifference Curves

Expected utility maximizer's indifference curves:

$$E[U] = p_1 u(x_1) + p_2 u(x_2) + p_3 u(x_3) \equiv u_0$$

$$p_1 u(x_1) + (1 - p_1 - p_3) u(x_2) + p_3 u(x_3) \equiv u_0$$

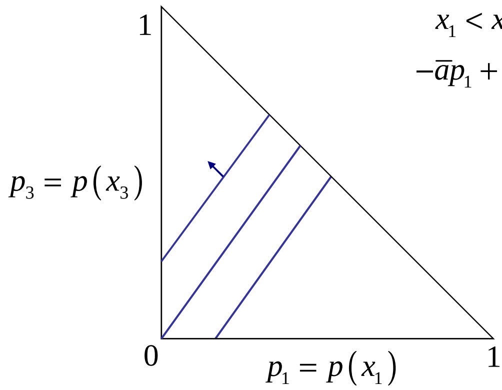
$$\left[u(x_1) - u(x_2)\right] p_1 + \left[u(x_3) - u(x_2)\right] p_3 \equiv u_0 - u(x_2)$$

$$-\bar{a} p_1 + \bar{b} p_3 \equiv \bar{c}$$

An expected utility maximizer's indifference curves are linear (and the slope is constant).

We can test this result to see if people are expected utility maximizers.

The Triangle Diagram



$$x_1 < x_2 < x_3$$

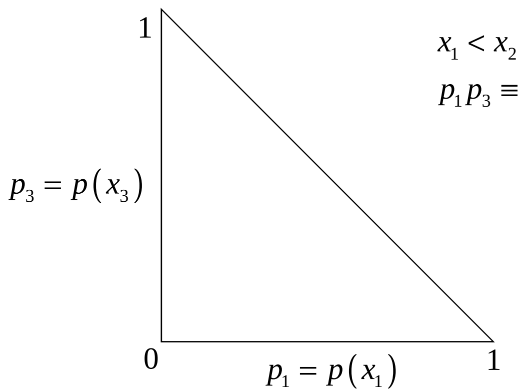
$$-\bar{a} p_1 + \bar{b} p_3 \equiv \bar{c}$$

Indifference Curves

Our other person's indifference curves:

$$V = p_1 p_3 \left(\frac{x_3 - x_2}{x_1} \right) \equiv v_0$$

The Triangle Diagram



$$x_1 < x_2 < x_3$$

$$p_1 p_3 \equiv \bar{c}$$

Violations of Expected Utility Theory

From Kahneman & Tversky (1979)

Allais Style Violation

Problem 1: Choose between

$A = (\$2500, 0.33;\ \$2400, 0.66;\ \$0, 0.01)$

and $B = (\$2400, 1)$.

82% of participants chose B.

Problem 2: Choose between

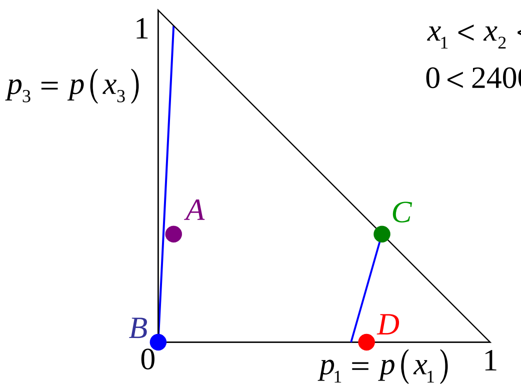
$C = (\$2500, 0.33;\ \$0, 0.67)$

and $D = (\$2400, 0.34;\ \$0, 0.66)$.

83% of participants chose C.

61% of participants chose both B and C.

The Triangle Diagram



$x_1 < x_2 < x_3$

$0 < 2400 < 2500$

Problem 1: Tells us

Problem 2: Tells us

The inequalities cannot both hold.

$\Rightarrow$ Reject $E[U]$ for 61% of the participants.

$$U(2500)0.33 + U(2400)0.66 + U(0)0.01 < U(2400)1$$
$$U(2500)0.33 + U(0)0.01 < U(2400)(1-0.66)$$

$$U(2500)0.33 + U(0)0.67 > U(2400)0.34 + U(0)0.66$$
$$U(2500)0.33 + U(0)0.01 > U(2400)(0.34-0)$$

K&T's explanation:

The difference between Problem 1 and Problem 2 is that 0.66 probability mass is moved from 2400 to 0.

The change in probability from 1 to 0.66 is stronger than the change from 0.34 to 0.

The Weighting Function

One of K&T's main ideas is to use a weighting function.

$\pi(p)$ represents the weight given to each outcome.

$L = (x_1, p_1;\ x_2, p_2;\ x_3, p_3)$

Expected Utility

$$EU(L) = p_1 u(x_1) + p_2 u(x_2) + p_3 u(x_3)$$

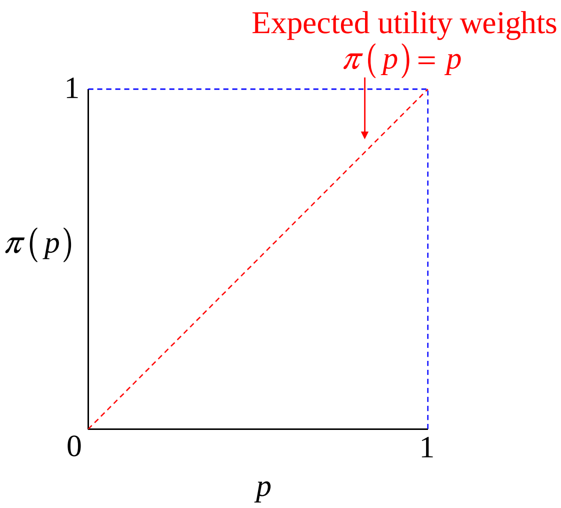
Prospect Theory

$$V(L) = \pi(p_1)v(x_1) + \pi(p_2)v(x_2) + \pi(p_3)v(x_3)$$

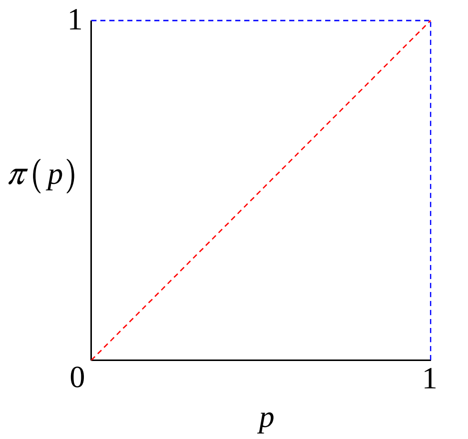
Expected Utility is equivalent to $\pi(p) = p$.

Today we'll work with $\pi(p) = p^2$.

The Weighting Function



The Weighting Function



$$V(\$0, p_1;\ \$2400, p_2;\ \$2500, p_3)$$
$$= v(0)\, p_1^2 + v(2400)\, p_2^2 + v(2500)\, p_3^2$$

Problem 1: Tells us

$$V(A) < V(B)$$
$$v(2500)(0.33)^2 + v(2400)(0.66)^2 + v(0)(0.01)^2 < v(2400)(1)^2$$
$$v(2500)0.1089 + v(0)0.0001 < v(2400)0.5644$$

Problem 2: Tells us

$$V(C) > V(D)$$
$$v(2500)(0.33)^2 + v(0)(0.67)^2 > v(2400)(0.34)^2 + v(0)(0.66)^2$$
$$v(2500)0.1089 + v(0)0.0133 > v(2400)0.1156$$

Problem 1: Tells us

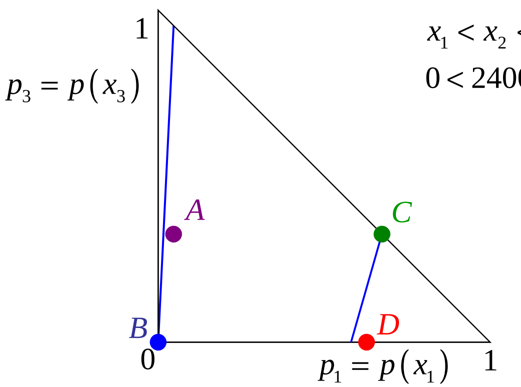
$$v(2500)0.1089 + v(0)0.0001 < v(2400)0.5644$$

Problem 2: Tells us

$$v(2500)0.1089 + v(0)0.0133 > v(2400)0.1156$$

We can normalize $v(0) = 0$ and $v(2500) = 1$.

The Triangle Diagram



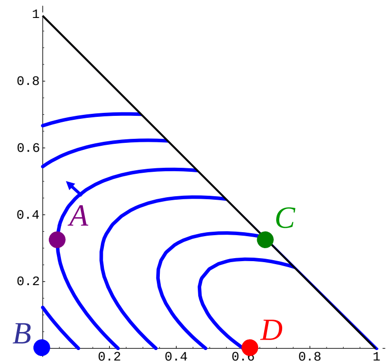
$x_1 < x_2 < x_3$

$0 < 2400 < 2500$

The indifference curves cannot have the same slope.

$\Rightarrow$ 61% of these participants don't maximize $E[U]$.

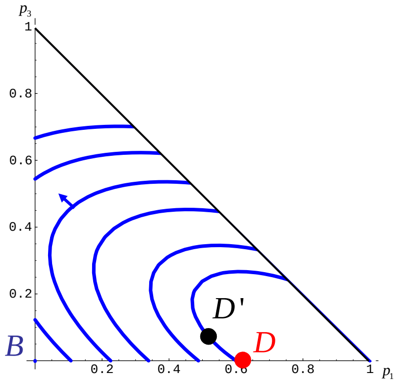
$$V(\$0, p_1;\ \$2400, p_2;\ \$2500, p_3) = v(0)\, p_1^2 + v(2400)\, p_2^2 + v(2500)\, p_3^2$$



Indifference curves:

$$v(0)\, p_1^2 + v(2400)(1 - p_1 - p_3)^2 + v(2500)\, p_3^2 \equiv C$$

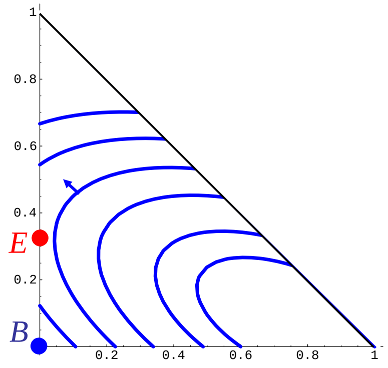
$$V(\$0, p_1;\ \$2400, p_2;\ \$2500, p_3) = v(0)\,p_1^2 + v(2400)\,p_2^2 + v(2500)\,p_3^2$$



As we move left from D:

Then we move up to D':

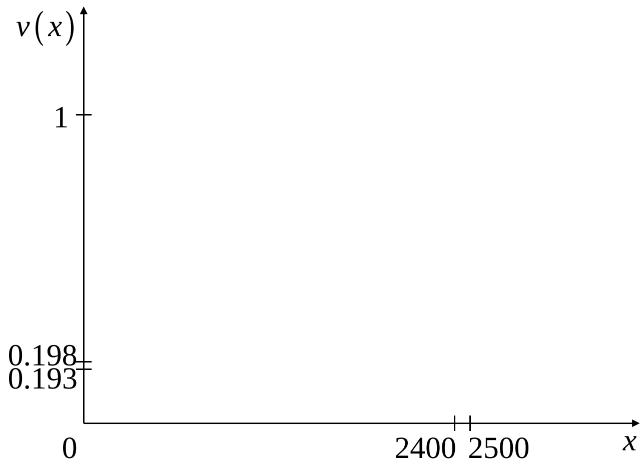
$$V(\$0, p_1;\ \$2400, p_2;\ \$2500, p_3) = v(0)\,p_1^2 + v(2400)\,p_2^2 + v(2500)\,p_3^2$$



$$V(\$0, p_1;\ \$2400, p_2;\ \$2500, p_3)$$
$$= v(0)\,p_1^2 + v(2400)\,p_2^2 + v(2500)\,p_3^2$$

We should have $E \succ B$

We can still fix the Alais paradox and not violate dominance here:



Consider the following two lotteries.

$A = (\$101, 0.5;\ \$102, 0.5);\ B = \$100$ with certainty.

$$v(101)0.5^2 + v(102)0.5^2 > v(100)1^2$$

We can assume $v(0) = 0$ and $v(102) = 1$.

Point of This Lecture

- We want to find an alternative to vNM expected utility maximization that explains people's preferences.
  - Resolve paradoxes.
  - Explain ordinary behavior.
    - Respect dominance.
    - Correct attitudes towards risk.
    - Etc.
  - We'll look at other functional forms for $\pi(p)$ and other paradoxes.

For the Midterm

- You're responsible for all the material in this lecture.
  - You should be able to work problems with the Allais paradox.
    - Graphically and algebraically.
    - Don't worry about the new paradoxes yet.
  - You may see different functional forms for $\pi(p)$.
    - Illustrate properties/problems.
    - Some other ones are on the problem set.