

Regrade Requests

- If you took your exam in pencil you need to hand it back in in a few minutes to keep the option for a regrade.
- Regrade requests must be made through a signed statement before the start of class on Thursday, May 1.
 - I may regrade your entire exam and your score could go up or down.

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Regrades

- Least likely to have the entire exam regraded:
 - You have a correct answer that we didn't consider.
 - You made a mistake at the beginning of a question that affected later parts (assuming you worked the later parts correctly).
- Moderately likely to have the entire exam regraded:
 - Significant error in the grading.
- Very likely to have the entire exam regraded:
 - Minor error in grading or a judgment issue.

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Economics 171 Decisions Under Uncertainty

4. Risk and Risk Aversion

Spring 2008
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Readings

- Lindley
 - Ch. 5
- Additional References:
 - Kreps (1990), *A Course in Microeconomic Theory*, Princeton University Press: Ch 3.2.
 - Machina and Rothschild (forthcoming), "Risk", *The New Palgrave Dictionary of Economics, 2nd edition*.
 - http://econ.ucsd.edu/~mmachina/papers/Machina_Palgrave_2007_Risk.pdf

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Outline

- Certainty equivalents, risk premiums and attitudes towards risk.
- Measuring risk aversion.
- Risk.
- Formulations of increasing risk.

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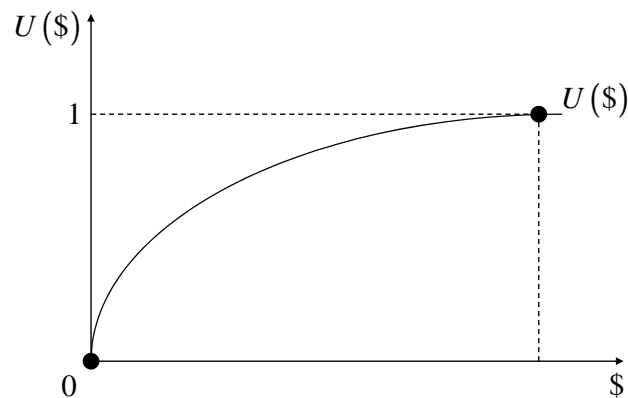
General

- Axioms of expected utility are logical/mathematical.
 - ie., Transitivity, Independence, etc.
 - Can be deduced.
- Attitudes towards risk are more empirical.
 - Different (rational) people have different attitudes towards risk.
 - There's no (logical) reason to think people should be risk averse/risk loving.

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Expected utility of any gamble between 0 and \$200.

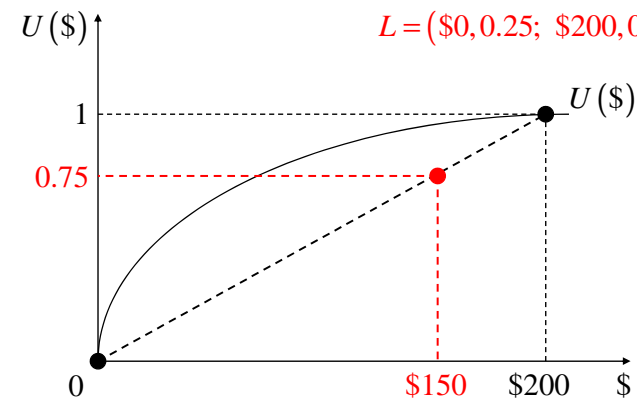
$(0, 1-p; \$200, p)$



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A risk averse individual prefers the expected value of a gamble (with certainty) to the gamble.

$L = (\$0, 0.25; \$200, 0.75)$



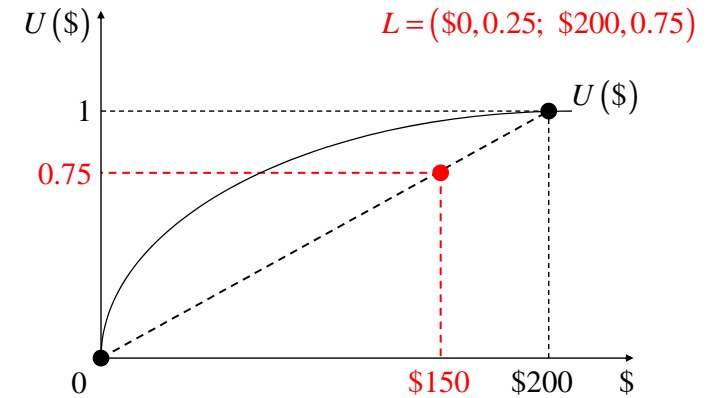
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Certainty Equivalents

- A certainty equivalent for a lottery (L) is any prize (x) such that the individual is indifferent between L and x .
 - When we construct utility functions we use certainty equivalents.
 - A certainty equivalent exists for every lottery if we have mixture continuity.
 - A more risk averse individual will have a lower certainty equivalent for a given lottery (compared to a less risk averse individual).

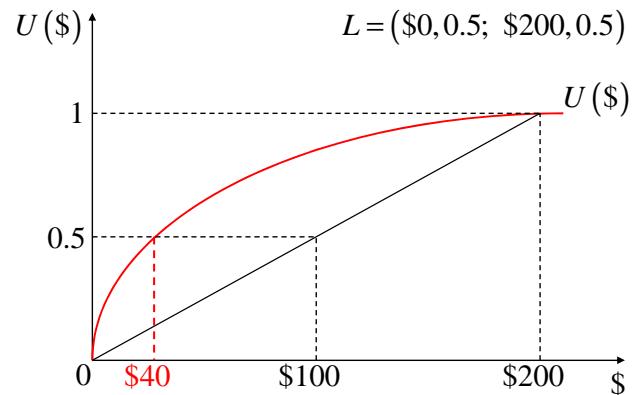
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The certainty equivalent of a gamble is the sure thing where an individual is indifferent between the two.



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Certainty equivalent for less risk averse



More risk averse: $U(0, 0.5; \$200, 0.5) = 0.5 = U(\$40)$

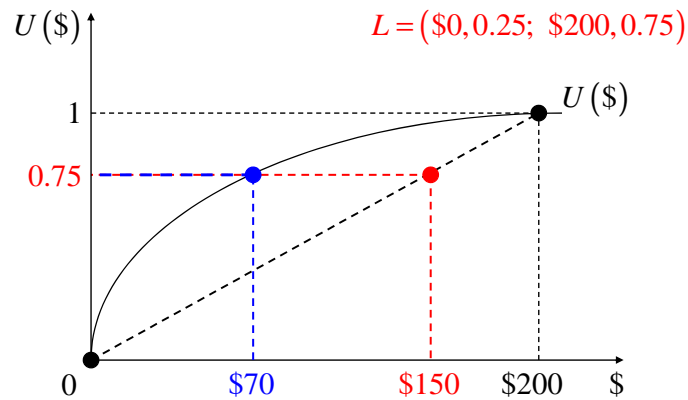
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Risk Premiums

- Difference between the expected value of a lottery and its certainty equivalent.
 - Insurance premium for a policy.
 - Risk premium for a riskier portfolio.

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The risk premium of a gamble tells us how much an individual is willing to pay to avoid the gamble.



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Attitudes Towards Risk

	Shape of Utility Fn	Certainty Equivalent	Risk Premium
Risk Averse	Concave	$C(L) < E(L)$	+
Risk Neutral	Linear	$C(L) = E(L)$	0
Risk Loving	Convex	$C(L) > E(L)$	-

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Measuring Risk Aversion

- We'd like to find a way to measure an individual's risk aversion (given her utility function).
 - Measure the concavity.
- Give us a way to compare risk aversion:
 - Across individuals.
 - For an individual at different levels of wealth.

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Absolute Risk Aversion

Assume an individual has a von Neumann-Morgenstern utility function, u , with the following properties:

Strictly increasing.

Concave.

First and second derivatives are continuous.

For any lottery, p :

Certainty equivalent, $C(p) \leq E[p]$.

Risk premium, $R(p) = E[p] - C(p) \geq 0$.

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Absolute Risk Aversion

We'd like to compare a lottery when an individual has different levels of wealth.

Think about lotteries p and $p \oplus z$.

$p \oplus z$ is the same as p if each prize is increased by z .

$$p = \{0, 0.3; \$100, 0.5; \$200, 0.2\}$$

We're defining the lotteries in terms of ex-post wealth.

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For an individual with utility function u :

If $R(p \oplus z)$ is nonincreasing in z , the consumer is nonincreasingly risk averse.

If $R(p \oplus z)$ is constant in z , the consumer is constantly risk averse.

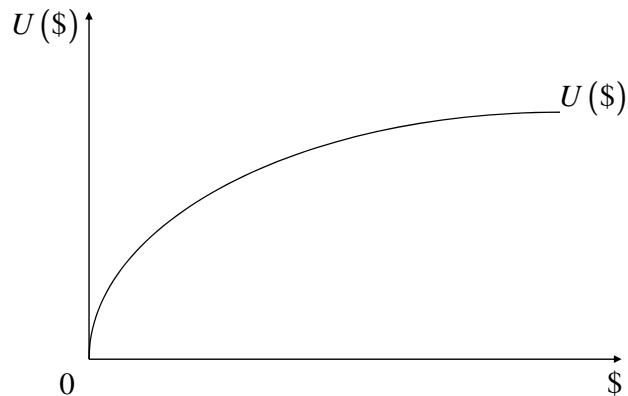
If $R(p \oplus z)$ is nondecreasing in z , the consumer is nondecreasingly risk averse.

The first statement is the same general idea as:

If $R(p \oplus z)$ is decreasing in z , the consumer is decreasingly risk averse.

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$$p = \{0, 0.5; \$70, 0.5\}$$



$R(p \oplus z)$ is nonincreasing in z ,

\Rightarrow the consumer is nonincreasingly risk averse.

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It seems reasonable that people are nonincreasingly risk averse.

Consider a lottery $\{-\$100, 0.5; \$110, 0.5\}$.

We expect a person with an initial wealth of \$10,000 is more likely to accept this gamble than a person with \$500 of wealth.

$$p = \{\$400, 0.5; \$610, 0.5\}$$

$$R(p \oplus \$9,500) \leq R(p)$$

We should show this for any gamble, p and all values of z .

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Absolute Risk Aversion

Take a utility function, u , that satisfies our earlier assumptions.

Define the coefficient of absolute risk aversion, λ :

$$\lambda(x) = -\frac{u''(x)}{u'(x)}$$

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Absolute Risk Aversion

$$u(x) = x^{\frac{1}{4}}$$

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Absolute Risk Aversion

An individual is nonincreasingly risk averse if and only if

λ is a nonincreasing function of x .

An individual is nondecreasingly risk averse if and only if

λ is a nondecreasing function of x .

An individual is constantly risk averse if and only if

λ is a constant function of x .

u can be written in the form of $u = -e^{-\lambda x}$

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Absolute Risk Aversion

$$u(x) = x^{\frac{1}{4}}; \quad \lambda(x) = \frac{3}{4x}$$

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Absolute Risk Aversion

Suppose we want to compare risk aversion across people.

Both have vNM utility functions that satisfy our assumptions.

First one has utility function u with $\lambda(x) = -\frac{u''(x)}{u'(x)}$.

Second one has utility function v with $\eta(x) = -\frac{v''(x)}{v'(x)}$.

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Absolute Risk Aversion

The first individual is at least as risk averse as the second if, for every lottery p and sure thing x , such that the first individual weakly prefers p to x , then the second individual will also prefer p to x .

The first individual is at least as risk averse as the second if and only if $\lambda(x) \geq \eta(x)$ for every x , which is equivalent to the statement that $u(\cdot) \equiv f(v(\cdot))$ for some strictly increasing, concave function f .

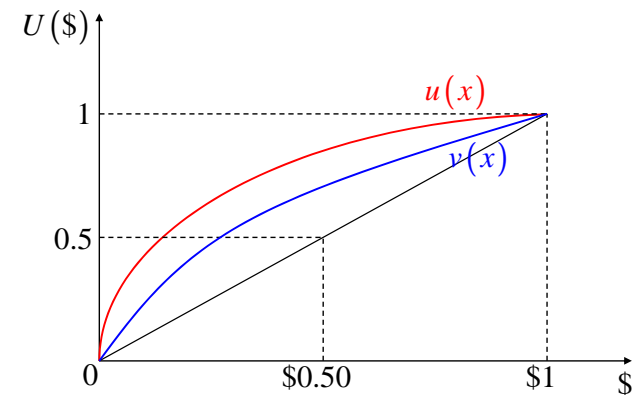
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Absolute Risk Aversion

$$u(x) = x^{\frac{1}{4}}, \quad \lambda(x) = \frac{3}{4}x^{-1}; \quad v(x) = x^{\frac{1}{2}}, \quad \eta(x) = \frac{1}{2}x^{-1}$$

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$$p = \{0, 0.5; \$1, 0.5\}$$



First person (u) prefers p to any sure x up to \$0.06.
 \Rightarrow Second (v) also prefers p to any x up to \$0.06.
 (Second (v) prefers p to any x up to \$0.25.)

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Risk

- Randomness given in the form of objective probabilities.
 - ie., A fair coin described by $\{H, 0.5; T, 0.5\}$.
- One measure of risk is the variance of a random variable.
 - Larger variances correspond to larger risks.

$$\text{Var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

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Mean/Variance Analysis

- Capital asset pricing model assumes preferences are determined solely on gambles' means and variances.
 - Provides a complete preference ordering.
 - Works for a vNM utility maximizer if and only if utility is quadratic.
- $$U(x) = ax - bx^2$$
- It also works in the special case where the gambles are all normally distributed.
 - It's possible to find indifferent points in mean/variance space where one gamble first order stochastically dominates the other.

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Mean/Variance Analysis

Example:

Let p be some lottery.

$$V(p) = 100 + E(p) - 2\text{Var}(p)$$

Compare: $p = (\$5, 0.5; \$15, 0.5)$ and $q = (\$6, 0.5; \$20, 0.5)$.

$$E(p) = 10, \text{Var}(p) = 25; \quad E(q) = 13, \text{Var}(q) = 49$$

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Mean/Variance Analysis



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Mean/Variance Analysis

$$p = \{\$2, 0.2; \$12, 0.8\}, \quad q = \{\$8, 0.8; \$18, 0.2\}$$

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Increasing Risk

- More modern approach.
 - Look at entire distribution.
 - Typically through CDF.
 - Suitable for comparative statics.
 - Problem: Only provides a partial ordering.
 - This is a similar problem to the use of first order stochastic dominance as a decision making criteria.
 - » Maximax always makes a decision.
 - » FOSD doesn't rank many lotteries.

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