Economics 171
Decisions Under Uncertainty

3. Expected Utility Preferences

Spring 2008
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Readings

- Lindley
  - Ch. 4
- Additional References:
  - Kreps (1990), *A Course in Microeconomic Theory*, Princeton University Press: Ch 3 – 3.2.
  - Machina (1987), “Choice under Uncertainty: Problems Solved and Unsolved”, Economic Perspectives: p. 121 – 132
    - http://econ.ucsd.edu/~mmachina/papers/Machina_Problems_Paper.pdf

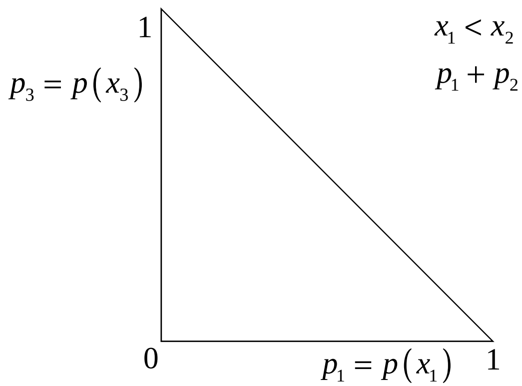
Outline

- Indifference curves for vNM utility functions.
- The expected utility representation theorem.
- Assigning utility values to outcomes.

The Triangle Diagram

$x_1 < x_2 < x_3$

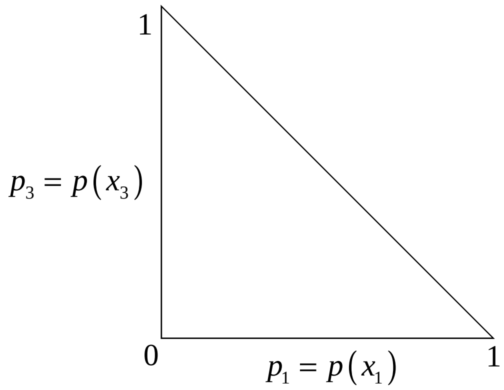
$p_1 + p_2 + p_3 = 1$



Expected Utility:

$$E[U]=\sum_{i=1}^{3}U(x_i)\,p_i$$

The Triangle Diagram

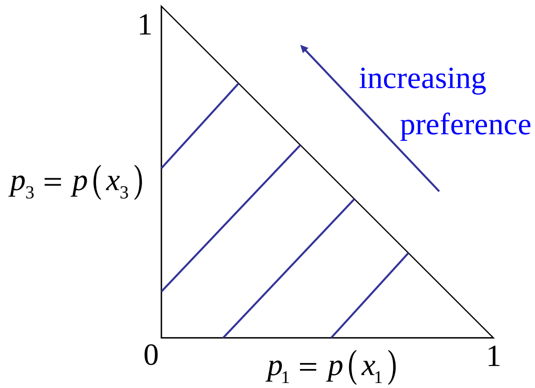


Expected Value:

$$E[X]=\sum_{i=1}^{3}x_i\,p_i$$

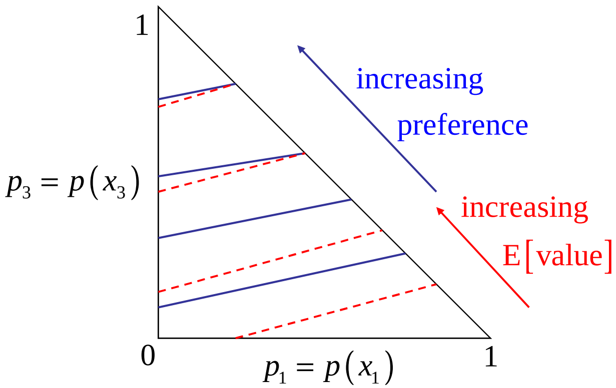
Isovalue Curves: All lotteries (with these three outcomes) that have the same expected value.

The Triangle Diagram



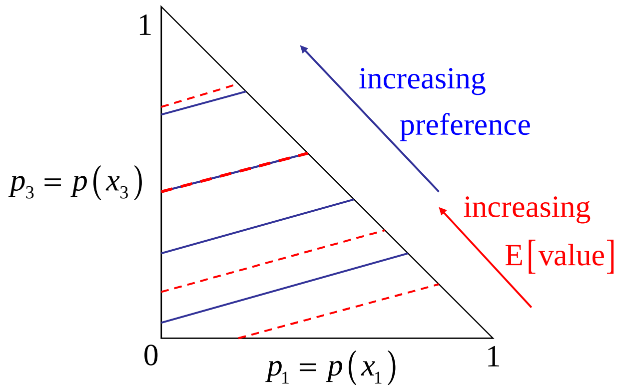
Indifference curves are steeper than isovalue curves.

The Triangle Diagram



Indifference curves are flatter than isovalue curves.

The Triangle Diagram

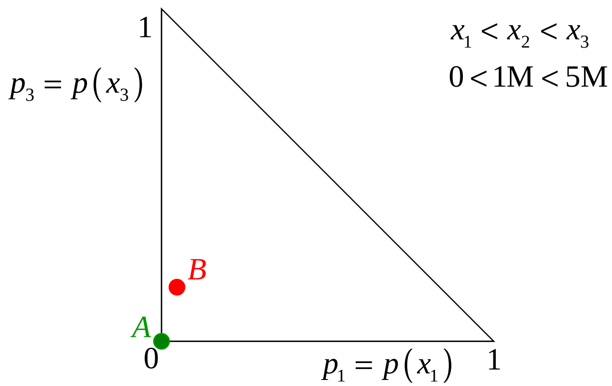


Indifference curves are parallel to isovalue curves.

Choice 1

	0	$1,000,000	$5,000,000
a:	0	100%	0
b:	1%	89%	10%

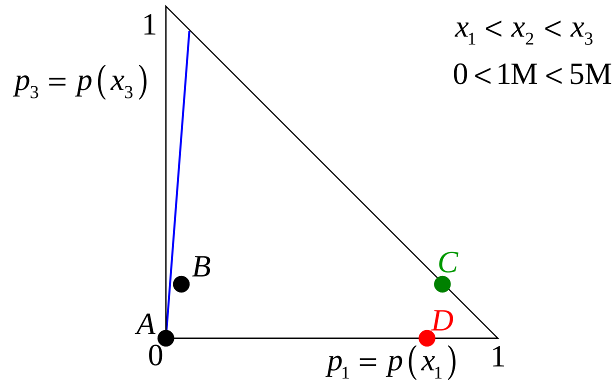
The Triangle Diagram



Choice 2

	0	$1,000,000	$5,000,000
c:	90%	0	10%
d:	89%	11%	0

The Triangle Diagram



The Expected Utility Representation Theorem

An individual's preferences satisfy our four axioms.

Completeness
Transitivity
Mixture Continuity
Independence

This individual's behavior is consistent with expected utility maximization.

Assigning Utility Values to Outcomes

- Suppose we have an individual facing a decision under uncertainty.
  - A choice between two lotteries.
    - Possible outcomes: $x_n \succ x_{n-1} \succ \ldots \succ x_2 \succ x_1$
  - We'd like to find the expected utility maximizing lottery.
  - First we need to find the individual's utility values for the different outcomes.

1. Define utility values for any two outcomes.

We typically choose the best and worst:

$U(x_n) = 1, \quad U(x_1) = 0$

2. Consider each of the other outcomes, x_i, in turn:

Determine a mixture of x_n and x_1 such that the the individual is indifferent between the mixture and x_i.

Find $0 < \lambda_i < 1$ such that

$$\lambda_i x_n + (1-\lambda_i) x_1 \sim x_i$$

Assign $U(x_i) = \lambda_i U(x_n) + (1-\lambda_i) U(x_1)$.

$U(x_i) = \lambda_i$ when $U(x_n) = 1$ and $U(x_1) = 0$

Insurance Problem

	No fire p=0.4	Small fire p=0.3	Medium fire p=0.2	Large fire p=0.1	
Self insure	$0	-10	-60	-150	
Full insure	-40	-40	-40	-40	
Partial 1	-20	-30	-70	-70	
Partial 2	-30	-40	-70	-70	

Insurance Problem

- Based on EV you would self insure.
  - Maximizing expected monetary values does not take account of attitudes toward risk.
- We want to maximize expected utility.
  - We need to assign utility values.

$X = \{0, -10, -20, -30, -40, -60, -70, -150\}$

$U(0) = 1, \quad U(-150) = 0$

$-20 \sim (\$0, 0.97; \ -\$150, 0.03)$

(This mixture is decided by introspection.)

$-60 \sim (\$0, 0.7; \ -\$150, 0.3)$

Repeat this procedure for the other outcomes.

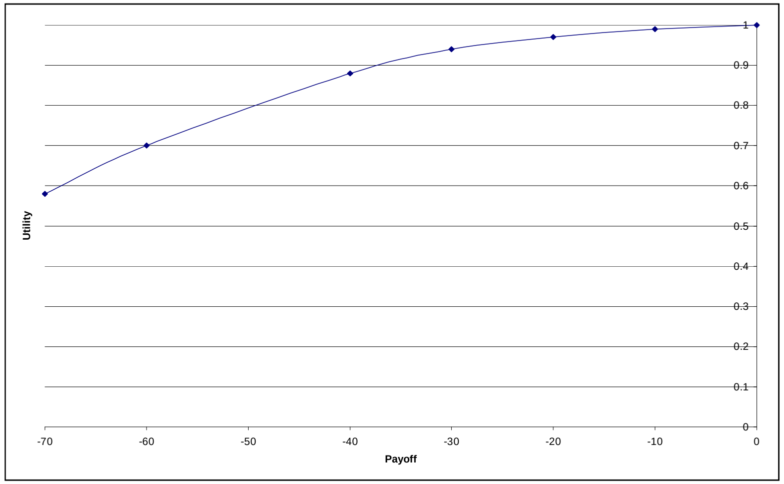
$$X = \{0, -10, -20, -30, -40, -60, -70, -150\}$$

$$U(0) = 1, \quad U(-20) = 0.97, \quad U(-60) = 0.7, \quad U(-150) = 0$$

Once we have more utilities we can use them as well:

$$-10 \sim (\$0, 0.67; \; -\$20, 0.33)$$

The utility function



Insurance Problem – with Utilities

	No fire p=0.4	Small fire p=0.3	Medium fire p=0.2	Large fire p=0.1	
Self insure					
Full insure	0.88	0.88	0.88	0.88	
Partial 1		0.94	0.58	0.58	
Partial 2	0.94	0.88	0.58	0.58	

Insurance Problem

- Someone who satisfies our four axioms (and who has the indifferences we just used) will maximize expected utility.
  - Choose full insurance.
  - Sometimes we'll call this type of person a *von Neumann-Morgenstern utility maximizer*.