

Christiano, Eichenbaum and
Evans (2005) JPE

and

Smets and Wouters 2003 JEEA
and 2007 AER

CEE (2005)	SW (2003, 2007)
<p>“Can models with moderate degrees of nominal rigidities generate inertial inflation and persistent output movements in response to a monetary policy shock? Our answer to this question is yes.”</p>	<p>Present and estimate a DSGE model with many frictions for the Euro area (2003) and for the US (2007).</p>
<p>Estimate the model parameters by matching impulse responses to monetary shocks.</p>	<p>Estimate the model parameters using Bayesian methods and allowing many types of shocks</p>
<p>Nominal frictions: Calvo price and wage setting; lagged inflation indexing (full)</p>	<p>Nominal frictions: Calvo price and wage setting; partial indexation.</p>
<p>4 new nonstandard features:</p> <ul style="list-style-type: none"> (1) Habit formation in consumption (2) Adjustment costs in investment (3) Variable capital utilization (4) Firms must borrow working capital to finance their wage bill. 	<p>Real features:</p> <ul style="list-style-type: none"> (1) Habit formation in consumption (2) Adjustment costs in investment (3) Variable capital utilization

CEE (2005) Model

The preferences of the j th household are given by

$$E_{t-1}^j \sum_{l=0}^{\infty} \beta^{l-t} [u(c_{t+l} - bc_{t+l-1}) - z(h_{j,t+l}) + v(q_{t+l})]. \quad (11)$$

Here, E_{t-1}^j is the expectation operator, conditional on aggregate and household j 's idiosyncratic information up to, and including, time $t-1$; c_t denotes time t consumption; h_{jt} denotes time t hours worked; $q_t \equiv Q_t/P_t$ denotes real cash balances; and Q_t denotes nominal cash balances. When $b > 0$, (11) allows for habit formation in consumption preferences.

We assume that the functions characterizing utility are given by

$$\begin{aligned} u(\cdot) &= \log(\cdot), \\ z(\cdot) &= \psi_0(\cdot)^2, \\ v(\cdot) &= \psi_q \frac{(\cdot)^{1-\sigma_q}}{1-\sigma_q}. \end{aligned} \quad (19)$$

D. The Wage Decision

As in Erceg et al. (2000), we assume that the household is a monopoly supplier of a differentiated labor service, h_{jt} . It sells this service to a representative, competitive firm that transforms it into an aggregate

Households set their wage rate according to a variant of the mechanism used to model price setting by firms. In each period, a household faces a constant probability, $1 - \xi_w$, of being able to reoptimize its nominal wage. The ability to reoptimize is independent across households and time. If a household cannot reoptimize its wage at time t , it sets W_{jt} according to

$$W_{jt} = \pi_{t-1} W_{jt-1}. \quad (16)$$

SW (2003, 2007) Model

There is a continuum of households indicated by index τ . Households differ in that they supply a differentiated type of labor. So, each household has a monopoly power over the supply of its labor. Each household τ maximizes an intertemporal utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau \quad (1)$$

where β is the discount factor and the instantaneous utility function is separable in consumption and labor (leisure):⁷

$$U_t^\tau = \varepsilon_t^b \left(\frac{1}{1-\sigma_c} (C_t^\tau - H_t)^{1-\sigma_c} - \frac{\varepsilon_t^l}{1+\sigma_l} (\ell_t^\tau)^{1+\sigma_l} \right) \quad (2)$$

Utility depends positively on the consumption of goods, C_t^τ , relative to an external habit variable, H_t , and negatively on labor supply ℓ_t^τ . σ_c is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution; σ_l represents the inverse of the elasticity of work effort with respect to the real wage.

$$H_t = h C_{t-1}$$

2.1.2 Labor Supply Decisions and the Wage Setting Equation. Households act as price-setters in the labor market. Following Kollmann (1997) and Erceg, Henderson, and Levin (2000), we assume that wages can only be optimally adjusted after some random "wage-change signal" is received. The probability that a particular household can change its nominal wage in period t is constant and equal to $1 - \xi_w$. A household τ that receives such a signal in period t , will thus set a new nominal wage, \tilde{w}_t^τ , taking into account the probability that it will not be reoptimized in the near future. In addition, we allow for a partial indexation of the wages that cannot be adjusted to past inflation. More formally, the wages of households that cannot reoptimize adjust according to:

$$W_t^\tau = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}^\tau \quad (8)$$

where γ_w is the degree of wage indexation. When $\gamma_w = 0$, there is no indexation and the wages that can not be reoptimized remain constant. When $\gamma_w = 1$, there is perfect indexation to past inflation.

CEE (2005) Model

The remaining terms in (12), aside from $P_t c_t$, pertain to the stock of installed capital, which we assume is owned by the household. The household's stock of physical capital, \bar{k}_t , evolves according to

$$\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + F(i_t, \bar{k}_t). \quad (13)$$

Here, δ denotes the physical rate of depreciation, and i_t denotes time t purchases of investment goods. The function, F , summarizes the technology that transforms current and past investment into installed capital for use in the following period. We discuss the properties of F below.

Capital services, k_t , are related to the physical stock of capital by

$k_t = u_t \bar{k}_t$. Here, u_t denotes the utilization rate of capital, which we assume is set by the household.⁸ In (12), $R_t^k u_t \bar{k}_t$ represents the household's earnings from supplying capital services. The increasing, convex function $a(u_t) \bar{k}_t$ denotes the cost, in units of consumption goods, of setting the utilization rate to u_t .

SW (2003, 2007) Model

$$K_t = K_{t-1}[1 - \tau] + [1 - S(\varepsilon_t^I I_t / I_{t-1})] I_t,$$

$$r_t^k = \Psi'(z_t) \quad (17)$$

Equation (15) states that the value of installed capital depends on the expected future value taking into account the depreciation rate and the expected future return as captured by the rental rate times the expected rate of capital utilization.

The first-order condition for the utilization rate (17) equates the cost of higher capital utilization with the rental price of capital services. As the rental rate increases it becomes more profitable to use the capital stock more intensively up to the point where the extra gains match the extra output costs. One implication of variable capital utilization is that it reduces the impact of changes in output on the rental rate of capital and therefore smooths the response of marginal cost to fluctuations in output.¹⁴

A. Final-Good Firms

At time t , a final consumption good, Y_t , is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods, indexed by $j \in (0, 1)$, using the technology

$$Y_t = \left(\int_0^1 Y_{jt}^{\lambda_{p,t}} dj \right)^{1/\lambda_{p,t}}, \quad (3)$$

B. Intermediate-Goods Firms

Intermediate good $j \in (0, 1)$ is produced by a monopolist who uses the following technology:

$$Y_{jt} = \begin{cases} k_{jt}^\alpha L_{jt}^{1-\alpha} - \phi & \text{if } k_{jt}^\alpha L_{jt}^{1-\alpha} \geq \phi \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

where $0 < \alpha < 1$. Here, L_{jt} and k_{jt} denote the time t labor and capital services used to produce the j th intermediate good. Also, $\phi > 0$ denotes the fixed cost of production. We rule out entry into and exit out of the production of intermediate good j .

2.2.1 Final-Good Sector. The final good is produced using the intermediate goods in the following technology:

$$Y_t = \left[\int_0^1 (y_t^j)^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}} \quad (18)$$

2.2.2 Intermediate Goods Producers. Each intermediate good j is produced by a firm j using the following technology:

$$y_t^j = \varepsilon_t^j \tilde{K}_{j,t}^\alpha L_{j,t}^{1-\alpha} - \Phi, \quad (21)$$

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random "price-change signal." The probability that a given price can be reoptimized in any particular period is constant and equal to $1 - \xi_p$. Following CEE (2001), prices of firms that do not receive a price signal are indexed to last period's inflation rate. In contrast to CEE (2001), we allow for partial indexation.¹⁵ Profit optimization by producers that are "allowed" to

CEE (2005) Model

Some key parameter estimates:

- Wage contracts last 2.8 quarters on average.
- Price contracts last 2.5 quarters on average.
- Habit parameter is 0.65.
- Very elastic capital utilization.

See paper for many more details.

SW (2003, 2007) Model

Some key parameter estimates:

- Very persistent processes for exogenous driving forces ($\rho = 0.95$)
- Wage contracts last just under 1 year on average.
- Price contracts last 3 quarters on average.
- High cost of changing investment.
- Fixed costs of production are 60%.
- Share of capital is only 0.19.

See paper for many more details.

Some key findings:

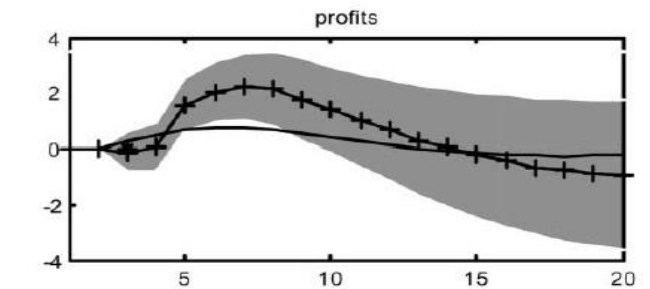
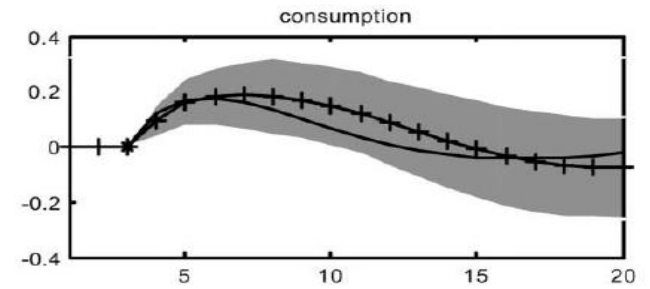
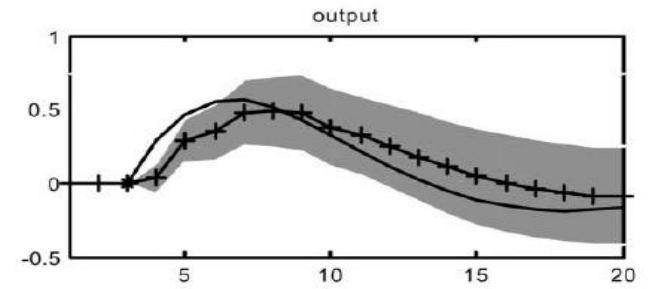
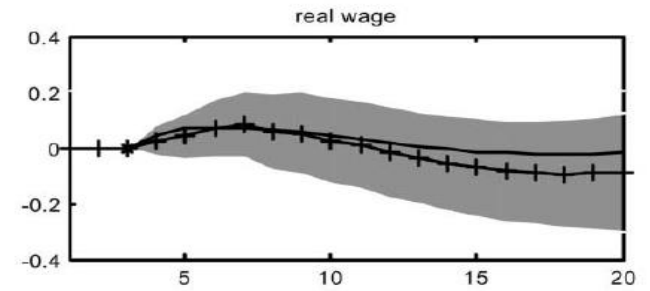
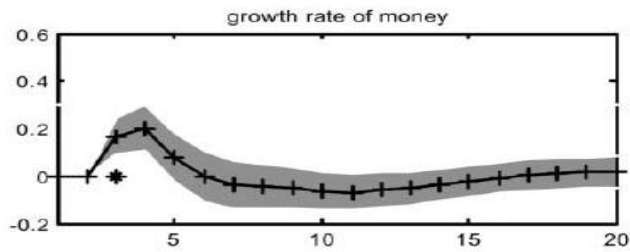
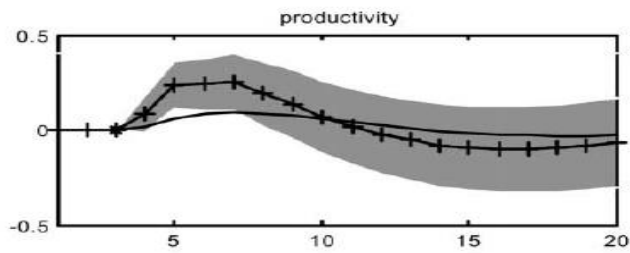
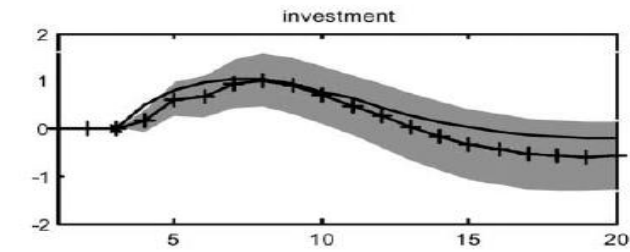
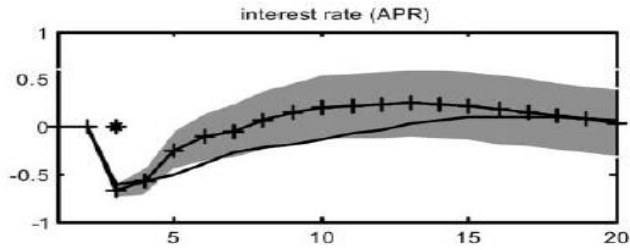
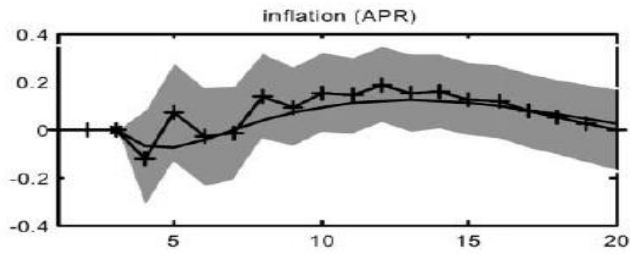
- Wage stickiness more important than price stickiness.
- Capital utilization is very important.

Some key findings:

- Both wage and price stickiness are important, indexing is less important.
- Investment adjustment costs are very important for the marginal likelihood.
- Consumption habits are quite important.
- Capital utilization is not important.
- High fixed cost of production is very important.

CEE (2005)

Solid lines – model
Lines with + data



SW (2007) Driving Forces

1. **Exogenous spending**, e.g. government spending, net exports. (Note that they allow the exogenous spending shock to be correlated with the productivity shock because of possible net export channel.)
2. **Policy rule shock** – shock to Taylor rule
3. **TFP shock** – to intermediate firms' value added production function.
4. **Investment specific technology shock** – relative price of investment goods.
5. **Risk premium shock** – (not in the SW (2003) model) see next page.
6. **Price markup shock** – stochastic parameter on CES intermediate aggregator
7. **Wage markup shock** – stochastic parameter on CES labor aggregator

Risk premium shock in SW (2007)

“Finally, the disturbance term ε_{tb} represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive shock to this wedge increases the required return on assets and reduces current consumption. At the same time, it also increases the cost of capital and reduces the value of capital and investment, as shown below.

This latter effect makes this shock different from a discount factor shock (as in Smets and Wouters 2003), which affects only the intertemporal consumption Euler equation. In contrast to a discount factor shock, the risk premium shock helps to explain the comovement of consumption and investment.”

SW (2007)

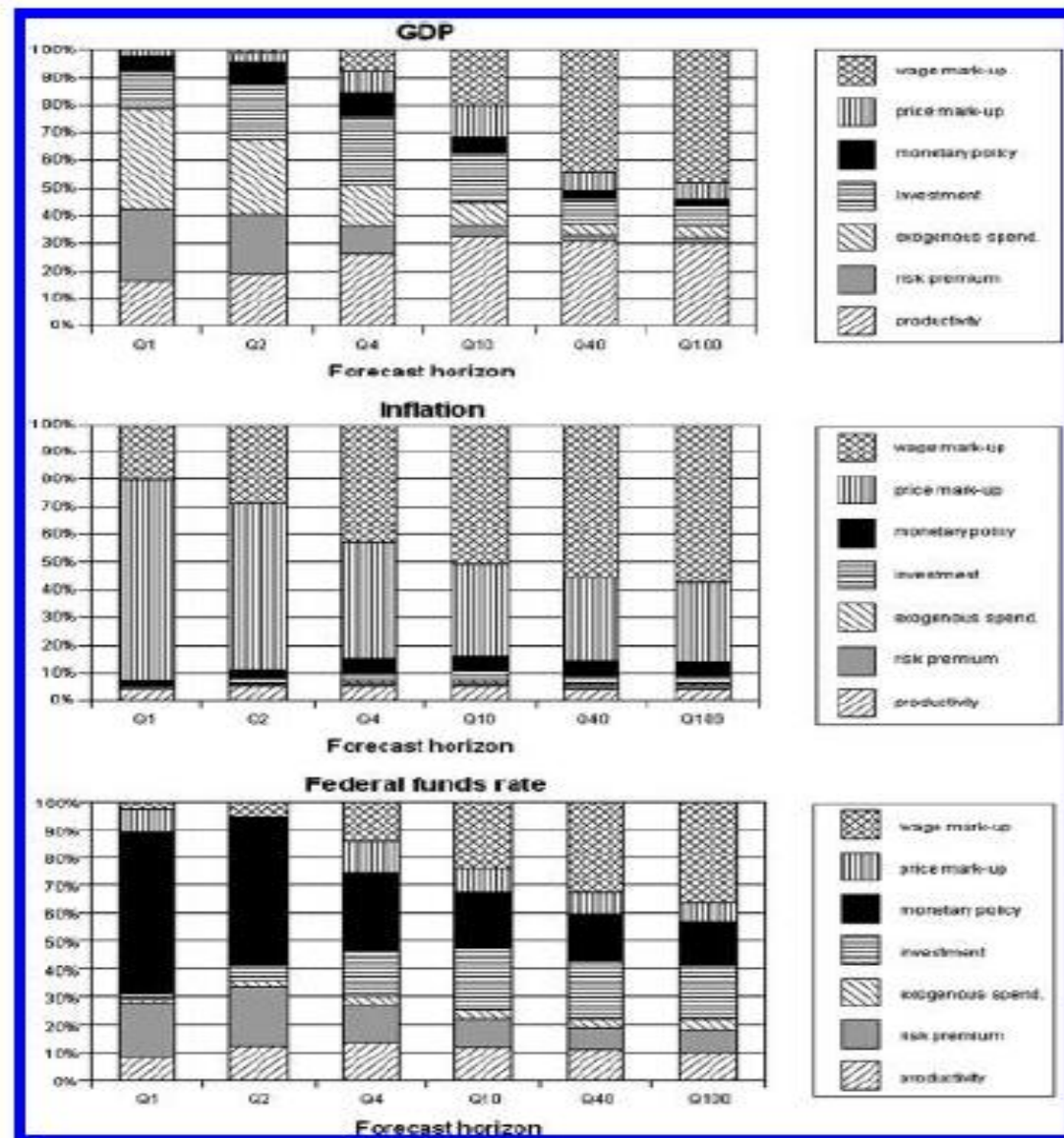
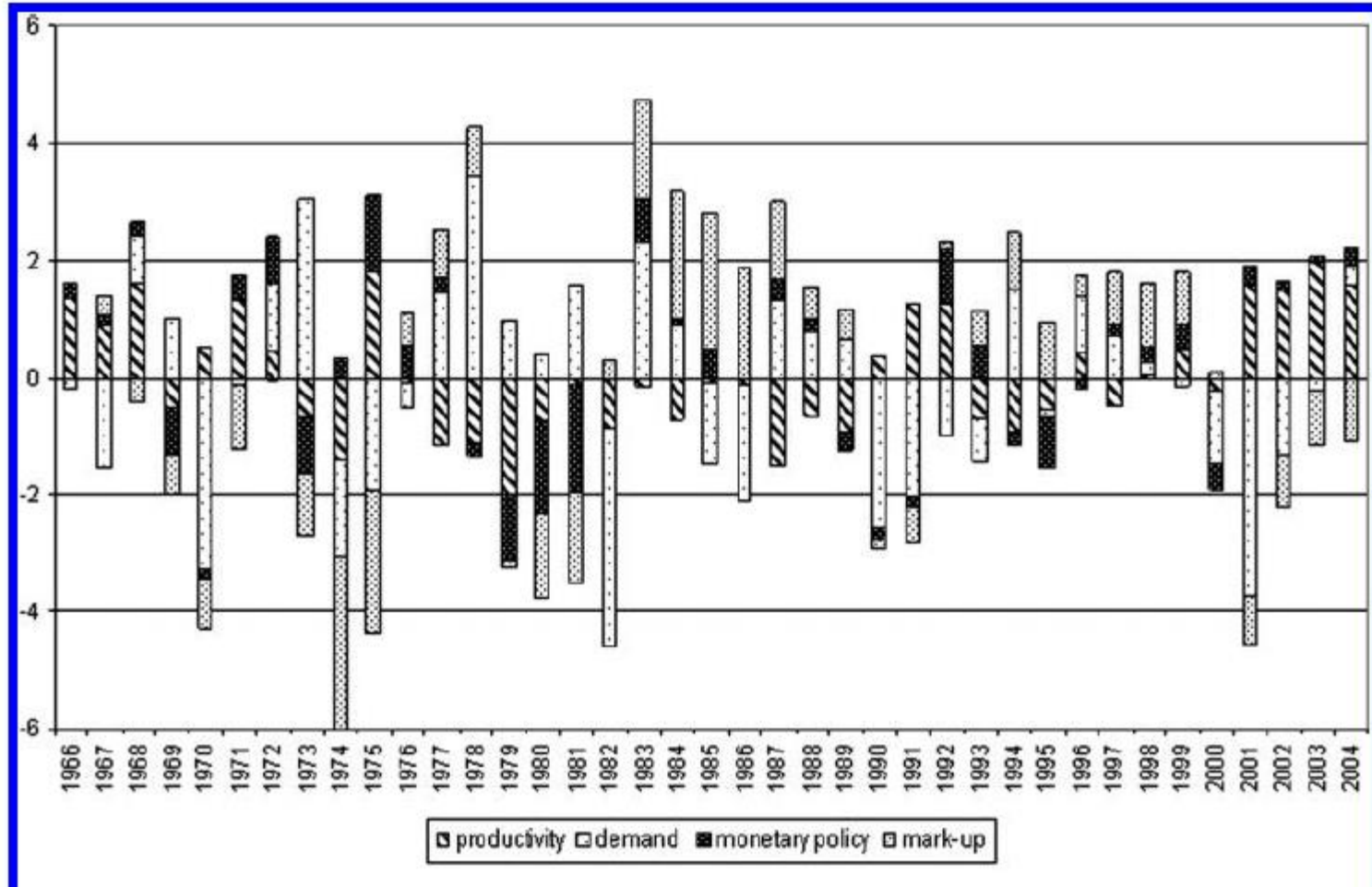


FIGURE 1. FORECAST ERROR VARIANCE DECOMPOSITION
(At the mode of the posterior distribution)

SW (2007) – historical decomposition of GDP



Now let's use Harald Uhlig's Smets-Wouters Toolbox

Note that his Toolbox is for the Smets-Wouters (2003) Euro-area version. It is similar to U.S. 2007 version, except for some shocks:

monopoly power over the supply of its labor. Each household τ maximizes an intertemporal utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau \quad (1)$$

where β is the discount factor and the instantaneous utility function is separable in consumption and labor (leisure):⁷

$$U_t^\tau = \varepsilon_t^b \left(\frac{1}{1 - \sigma_c} (C_t^\tau - H_t)^{1 - \sigma_c} - \frac{\varepsilon_t^L}{1 + \sigma_l} (\ell_t^\tau)^{1 + \sigma_l} \right) \quad (2)$$

Utility depends positively on the consumption of goods, C_t^τ , relative to an external habit variable, H_t , and negatively on labor supply ℓ_t^τ . σ_c is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution; σ_l represents the inverse of the elasticity of work effort with respect to the real wage.

Equation (2) also contains two preference shocks: ε_t^b represents a shock to the discount rate that affects the intertemporal substitution of households (preference shock) and ε_t^L represents a shock to the labor supply. Both shocks are assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ and $\varepsilon_t^L = \rho_L \varepsilon_{t-1}^L + \eta_t^L$.

Parameters	Value	Description
β	0.99	discount factor
τ	0.025	depreciation rate of capital
α	0.3	capital output ratio
ψ	1/0.169	inverse elasticity of cap. util. cost
γ_p	0.469	degree of partial indexation of price
γ_w	0.763	degree of partial indexation of wage
λ_w	0.5	mark up in wage setting
ξ_s^p	0.908	Calvo price stickiness
ξ_s^w	0.737	Calvo wage stickiness
σ_L	2.4	inverse elasticity of labor supply

σ_c	1.353	coeff. of relative risk aversion
h	0.573	habit portion of past consumption
ϕ	1.408	1 + share of fixed cost in prod.
φ	1/6.771	inverse of inv. adj. cost
\bar{r}_k	$1/\beta - 1 + \tau$	steady state return on capital
k_y	8.8	capital output ratio
inv_y	0.22	share of investment to GDP
c_y	0.6	share of consumption to GDP
k_y	inv_y/τ	capital income share, inv. share
g_y	$1 - c_y - inv_y$	government expend. share in GDP
r_π^Δ	0.14	inflation growth coeff.
r_y	0.099	output gap coeff

Parameter	Value	Description
r_y^Δ	0.159	output gap growth coefficient
ρ	0.961	AR for lagged interest rate
r_π	1.684	inflation coefficient
ρ_{ϵ_L}	0.889	AR for labour supply shock
ρ_{ϵ_a}	0.823	AR for productivity shock
ρ_{ϵ_b}	0.855	AR for f preference shock
ρ_G	0.949	AR for government expenditure shock
$\rho_{\bar{\pi}}$	0.924	AR for inflation objective shock
ρ_{ϵ_i}	0.927	AR for investment shock
ρ_{ϵ_r}	0	AR for interest rate shock, IID
ρ_{λ_w}	0	AR for wage markup, IID