Location Choice and Commuting Behavior in Cities with Decentralized Employment*

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This paper explores residential and job location patterns and commuting behavior in a monocentric urban model with decentralized employment. We show that, in some cases, identical households choose different residential rings depending on their workers' job locations. This means that high-income households whose workers work near the central business district (CBD) may choose residential locations near the CBD rather than in the suburbs if their workers have CBD job locations. In this case there will be a nonmonotonic relationship in the city between income level and residential distance from the CBD. Identical workers also are shown to choose different residential locations depending on their job locations. Wage gradients for workers as a function of job location are derived. The model also suggests that the skill composition of a firm's workforce affects its incentive to move to the suburbs. Implications of the model for commuting patterns are explored. © 1988 Academic Press, Inc.

1. INTRODUCTION

In this paper we explore the determination of residential and job locations and the pattern of commuting behavior in an urban model with decentralized employment. Various authors have explored models of cities with decentralized employment, but they have tended to make very specialized assumptions concerning either the pattern of firm location or the nature of household behavior and have often been forced to use numerical simulation techniques to get results. In this paper we try to make very general assumptions concerning firm and household behavior. This approach yields fewer definite answers, but it enables me to consider some new questions.[1]

[1] Models of cities with decentralized employment include Moses [8], Muth [9], Mills [7], White [15], Beckman [1], Capozza [3], Brueckner [2], Ogawa and Fujita [10] and [11], and Straszheim [14].

The most interesting result of the model is that, under some circumstances, households having identical tastes, income, and number of workers occupy different rings around the city center depending on their job locations. This means that cities with decentralized employment may not have the usual residential location pattern in which successively more distant rings around the center are occupied by higher income households. Instead, in this model, households' residential location choice depends on both their workers' job locations and on their income levels. Thus a ring pattern is likely to emerge in which households whose income levels are high but whose job locations are near the center occupy residential rings that are close to the center. They may be surrounded by rings occupied by lower income households whose jobs are further out. Thus the nonmonocentric city has a nonmonotonic pattern of income level changes with greater distance from the center. This richer location pattern is more realistic than the simpler results of the traditional urban model. It provides, for example, a formal model of the "gentrified" location pattern observed in various cities in which high income households locate both near the center and in the far suburbs.

The model also implies that otherwise identical workers have different job locations depending on their residential locations. In particular workers whose households have closer-in residential locations demand a smaller wage premium for extra commuting than do otherwise identical workers whose households live further out. The conditions under which workers who live further from the center hold more suburbanized jobs are developed. The model also yields interesting insights concerning how workers' wage offer curves for jobs at particular locations differ according to their skill or income levels. It suggests that the composition of a firm's workforce, i.e., its relative mix of high-wage versus low-wage workers, is important in determining whether the firm will move out of the central business district (CBD) and, if so, how far.

Finally, the model moves toward a more general theory of commuting behavior which synthesize two previously conflicting views of commuting: that emerging from urban economics, which argues that longer commuting journeys are compensated by lower housing prices, and that emerging from labor economics, which argues that longer commuting journeys are compensated by higher wages.[2] The results suggest that both wages and housing prices adjust to compensate for extra commuting in different circumstances, depending on the direction in which the commuting journey is changed and whether the household moves its job location, its housing location, or both.

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[2] See Mills [7], Rees and Shultz [12], and Madden and White [6].

Section 2 of the paper briefly reviews the theory of a city with centralized employment. Section 3 discusses the incentives facing firms making location choices within a metropolitan area and the commuting pattern that results for workers holding suburban jobs. Section 4 introduces decentralized employment into the residential location choice problem and explores its implications for the spatial variation of wages and land prices. Section 5 explores the model's implications for commuting behavior and considers avenues for further research.

2. RESIDENTIAL LOCATION CHOICE IN CITIES WITH CENTRALIZED EMPLOYMENT

We start by briefly reviewing the theory of a circular city when all employment is at the CBD. Assume that the city is located on a flat featureless plain and that the CBD is a point at its center. All households living in the city have one worker. They have identical utility functions which depend on consumption of land for housing, h, consumption of other goods, x, and leisure time of the worker, l.[3] All of these vary with residential distance from the center, denoted u. Thus the utility function is

$$U = U(h(u), x(u), l(u)). \qquad (1)$$

Workers receive a fixed wage rate of w^* per hour for working at the CBD at jobs whose hours are assumed to be continuously variable. Monetary commuting cost is m dollars per mile traveled in either direction and is constant at all locations. Commuting speed is $1/s$ miles per hour and is also constant at all locations. Thus round trip monetary commuting costs are $2mu$ and round trip commuting time is $2su$. The rent per unit of land for housing at distance u is $p(u)$. Households have a budget constraint and workers are assumed to have a time constraint. Total time available, 24 hours per day, must be split among leisure, work, and commuting. Combining the household's time and money constraints results in the "full income" constraint,

$$p(u)h(u) + x(u) + w^*l(u) = 24w^* - 2(sw^* + m)u, \qquad (2)$$

where $24w^* - 2(sw^* + m)u$ is full income or the amount that workers could earn net of time and money commuting expenses if they worked 24 hours a day.

Households maximize utility over $h(u)$, $x(u)$, and $l(u)$, subject to the full income constraint. The full income constraint must hold at all loca-

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[3] The capital used in housing, i.e., the structure itself, is assumed to be part of the composite good.

tions. Therefore we can substitute (2) into (1) to get

$$U[h(u), 24w^* - 2(sw^* + m)u - p(u)h(u) - w^*l(u), l(u)].$$

Totally differentiating the utility function using the envelope theorem, we can derive the condition on rent levels at different distances from the center such that a household achieves equal utility at all residential locations. We get the following well-known condition which is referred to as the household's rent offer curve for housing:

$$p_u(u) = \frac{-2(sw^* + m)}{h(u)}, \tag{3}$$

where the subscript indicates a partial derivative.

Since work hours are flexible, the value of time at the margin must be w^*. Thus the numerator of (3) is the total cost of commuting an extra mile round trip, including the monetary outlay and the value of time. Since the right-hand side of (3) is negative, the rent offer curve must always decline with distance from the city center.

Differentiating (3) with respect to u and substituting (3) into the resulting expression, we get

$$\frac{\partial p_u(u)/\partial u}{p_u(u)} = -\frac{h_u(u)}{h(u)}. \tag{4}$$

The percentage change in the rate of decrease of the rent offer curve must equal minus the percentage change in housing consumption with increased residential suburbanization. Since consumption of land for housing rises in the suburbs where rents are lower, h_u is positive. The left-hand side of (4) is therefore negative, so $\partial p_u/\partial u$ must be positive. The slope of the rent offer curve for housing thus gets flatter as u increases.

Equation (3) describes a necessary condition on the rent offer curve such that households having the utility function and full income constraint in (1) and (2) achieve equal utility at all locations in a city having centralized employment. But if all households in the city have identical tastes and incomes and if all have one worker whose job is at the center, then the market equilibrium rent gradient must also satisfy (3) and (4). The market equilibrium rent gradient is denoted $p^m(u)$.

Extending the model, suppose households still have identical tastes, but some workers are skilled and some are unskilled. Skilled workers receive a wage rate of w^{b*} per hour and unskilled workers receive a lower wage rate of w^{d*} per hour. In this case we can derive separate conditions on the rent

offer curves for each of the two groups. The condition for skilled workers is

$$p_u^b(u) = \frac{-2(sw^{b*} + m)}{h^b(u)},\qquad\qquad (5)$$

where the superscripts b denote values of the variables for skilled workers' households. The condition for unskilled workers (with superscripts d) is similar.

Both skilled and unskilled workers' rent offer curves must decline at a decreasing rate with distance. Skilled workers' households have higher demand for housing than unskilled workers' households at any u, which tends to make their rent offer curves flatter. However skilled workers' time is more valuable at the margin, which tends to make their rent offer curve steeper. In general the first effect is usually assumed to be more important, making the rent offer curve flatter for skilled than unskilled workers. Given this assumption, the two rent offer curves are depicted in Fig. 1. Skilled workers' households outbid unskilled workers' households for land at suburban locations and unskilled workers' households outbid skilled workers' households at more central locations. Each group of households is now indifferent to locating over a range of distances from the center, rather than everywhere in the city. The market equilibrium rent gradient is the upper envelope of the two groups' rent offer curves. It is shown as the dashed line in Fig. 1.

With two income groups in the city, unskilled workers' households thus occupy a region shaped like a thick ring around the CBD from distance $u = 0$ to u' in all directions and skilled workers' households occupy a
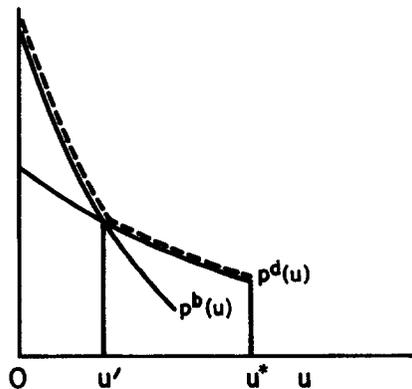

FIG. 1. Rent offer curves of skilled and unskilled workers' households and the market rent gradient.

surrounding thick ring from distances $u = u'$ to the outer edge of the city, u^*. The boundary between the ring occupied by unskilled and skilled workers' households occurs at some location u'. However we have not given the model enough structure to be able to determine the intercepts of the two functions or the distance u' from the center at which they intersect.[4]

3. FIRM LOCATION CHOICE

In this section we examine the problem of firm location choice and the characteristics of labor markets in cities with decentralized employment. In particular we are interested in establishing conditions under which at least some firms in the city have an incentive to move out of the CBD. But we also rule out conditions in which so much firm suburbanization occurs that there is no commuting, because all workers work at home at very small firms. We use v to denote workplace location, where v is measured in miles from the CBD in any direction.

The city is assumed to be located on a flat featureless plain, where land is not specialized for use in housing versus in production. Also there is no zoning or other land use regulation which might set particular regions of the city aside for particular land uses. Firms are assumed to occupy some but not all of the land in all rings around the CBD. Therefore the price that they pay for land in that ring is its value when used for housing, or $p^m(u)$.[5] The assumption that some firms locate at all rings around the center is made for convenience, since it allows workplace location, v, to be treated as a continuous variable. But the general results of the model would not be changed if firms located in some, but not all, suburban rings.

The wage which suburban firms must pay their workers depends on how much workers' commuting journeys are shortened when the firm moves out of the CBD. Firms gain the most from suburbanizing if after the move their workers live both farther from the CBD than the firm itself and in the same direction away from the CBD (i.e., along the same ray from the CBD). We assume that all workers who take jobs at suburban firms satisfy both of these conditions. This could be because workers already live in the relevant region for their particular employer, because they move there when the firm does, or because the firm hires new workers who already live in the relevant region after it moves. The first of these two conditions eliminates the possibility of out-commuting by workers, i.e., it requires that workers live further from the CBD than their jobs. The second condition eliminates

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[4] See Mills [7] for a description of the extra equations needed to close a model similar to this one.

[5] The characteristics of households' rent offer curves for land for housing and the market rent gradient are discussed under Section 2 for the centralized employment case and are discussed under Section 4 below for the decentralized employment case.

circumferential commuting. These assumptions are not as strong as they might appear. For workers to be willing to commute outward or circumferentially to suburban jobs, employers must pay them more than if they commuted in an inward direction to the same jobs. Therefore it is in firms' interest to locate in such a way that all their workers in-commute, i.e., so that all workers live further from the CBD than their employers and along the same ray from the CBD.[6]

Assume that all workers have the same skill level and earn the same wage if they work at jobs in the CBD. The CBD wage is again denoted w^*. Since workers taking suburban jobs have lower commuting costs than workers living at the same residential location who work at CBD jobs, employers at any $v > 0$ are assumed to pay their workers a lower wage than that prevailing at the center. The market wage gradient, denoted $w^m(v)$, therefore must be negatively related to workplace location, v, or $\partial w^m(v)/\partial v < 0$.

In moving to suburban locations, firms are assumed to spread out in all directions around the CBD. This is because once some firms have located, say, exactly north of the CBD, their presence discourages additional firms from also locating exactly north of the CBD, since all firms located along a single ray from the CBD must compete for workers who live further out along the same ray. Other firms instead prefer to locate the same distance from the CBD, but one or more degrees away from north in either direction, where the potential labor supply willing to work for the firm at the wage $w^m(v)$ is likely to be larger. Thus there are agglomeration diseconomies as more firms locate along any particular ray from the CBD, regardless of how far away from the CBD they locate. If more jobs in total are offered by firms located along any particular ray than there are workers living along the same ray, then firms will only be able to attract enough workers if some workers out-commute or commute circumferentially. But this requires that firms pay higher wages than would be necessary if they spread out in different directions, so that all workers could in-commute. These assumptions imply that no suburban subcenters will form which contain enough firms that they exhaust the labor supply composed of workers who commute inward to the firms they work for.

It is interesting to note that firms which suburbanize drastically restrict the labor market area from which they hire workers. While firms located at

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[6]Are these assumptions realistic? In a recent study, Simpson [13] finds that 70% of a sample of households living in London who were not recent movers had jobs closer to the center than their homes and had job locations that were in the same direction away from the center as their residential locations. The criterion used to determine whether households' job and residential locations were in the same direction away from the center was that they both be in a pie-shaped wedge having its center at the city center and an angle of 0.25π radians or one-eight of a circle. See Hamilton [4] and White [18] for further discussion concerning the commuting pattern in U.S. cities.

the CBD can hire workers who live in any direction from the CBD; firms located, say, north of the CBD can only hire workers who live both north of the CBD and further out than the firm. As a result, moving out of the CBD will tend to be most attractive to firms that are relatively small.

For our purposes, it is not necessary to present a formal model of firms' location decision. Such a model would have firms gain from suburbanizing because they pay lower land prices and lower wages in the suburbs, but their goods transport costs or their costs of production could be higher. Firms would maximize profit over their choice of location, where each location is a ring around the CBD. In such a model, it is reasonable to assume that one suburban ring (or a few) would be the profit-maximizing location for any particular firm. This implies that identical firms would concentrate in one or a few suburban rings, rather than distributing themselves over all suburban rings. However if there were firms in many industries located in the city, then firms in each industry would locate in the profit-maximizing ring for that industry, but nonetheless there would be firms of some type located in many or all suburban rings. Note that agglomeration diseconomies also result if many firms attempt to locate in the same ring. In this case firms would bid up the price of land in that ring, giving themselves an incentive to choose locations in adjacent rings which are slightly closer-in or further-out. These adjacent rings have lower demand and therefore lower prices. If firms instead scatter over many rings, then they will pay for land according to the market rent gradient established for land used for housing.[7]

4. RESIDENTIAL AND JOB LOCATION CHOICE IN CITIES WITH DECENTRALIZED EMPLOYMENT

We now generalize the model of household residential and job location choice to allow for decentralized employment. This will enable us to investigate the characteristics of equilibrium market rent and wage gradients in the decentralized city case.

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[7]Other patterns of employment decentralization are also possible, but they may give rise to different spatial wage and price gradients than those considered here. For example, White [15] analyzed a model of a city in which employment was located both at the CBD and at a suburban subcenter at an arbitrarily chosen distance ring. In that model, the concentration of jobs at the subcenter results in a labor shortage, causing wages there to rise in order to attract additional workers who live closer to the CBD than the subcenter and therefore must out-commute. The resulting land price gradient is double-peaked, having a second local maximum at the suburban subcenter. This type of model, in which employment decentralization is profitable even when labor shortages in the suburbs require that wages be raised to attract out-commuters, is only reasonable if agglomeration economies or some important cost savings result from locating at the subcenter. In the ring subcenter model, the cost savings result from the existence of a circumferential highway at the subcenter location.

Households again have a single worker and all workers are assumed to have the same skill level. With decentralized employment, households' demand for goods and leisure depend on both residential and job locations, u and v. Their utility function therefore becomes $U = U(h(u, v), x(u, v), l(u, v))$.

In the decentralized employment case, there will be a market rent gradient, $p^m(u)$, which depends only on residential distance from the center, u. In other words, the rent on land at any u paid by households that live at that u will not depend on the job locations of the households' workers. However we wish to consider the possibility that households whose workers have different job locations may be willing to pay different amounts for land at the same location. Therefore households' rent offer curves, denoted $p(u, v)$, may depend on both residential and workplace location. If households' rent offer curves do vary with job location, then $p^m(u)$ will be the upper envelope of the set of rent offer curves of groups of households having different job locations.

Similarly, there will be a market wage gradient, $w^m(v)$, which depends only on workplace distance from the center. In other words, employers located v miles from the center will pay the same wage to all workers regardless of their residential locations. However we wish to consider the possibility that workers whose residential locations differ may be willing to work at a particular job location for different wage levels. Thus workers' wage offer curves, denoted $w(u, v)$, may depend on both workplace and residential location. If workers' wage offer curves do vary with residential location, then $w^m(v)$ will be the lower envelope of the set of wage offer curves of groups of workers whose households have different residential locations.

Households' full income constraint therefore becomes

$$p(u, v)h(u, v) + x(u, v) + w(u, v)l(u, v)$$
$$= 24w(u, v) - 2(sw(u, v) + m)(u - v), \qquad (6)$$

where $p(u, v)$ and $w(u, v)$ are now the rent offer and wage offer curves for the household and its worker. Since all workers in-commute, one-way commuting distance is $(u - v)$. Full income, $24w(u, v) - 2(sw(u, v) + m)(u - v)$, is denoted $F(u, v)$.

Households maximize utility subject to the full income constraint over their choice of consumption levels of $h(u, v)$, $x(u, v)$, and $l(u, v)$ and over their residential and job locations, u and v. The full income constraint must be satisfied at all residential and workplace locations. Substituting the full income equation into the utility function, we get

$$U\left[h, 24w(u, v) - 2(sw(u, v) + m)(u - v) - p(u, v)h - w(u, v)l, l\right].$$
$$(7)$$

Households must achieve the same utility level at all workplace/residential location pairs that they choose to occupy. Totally differentiating (7) with respect to u and v and using the envelope theorem, we get

$$24w_u - 2sw_u(u - v) - 2(sw + m) - p_uh - w_ul = 0 \tag{8}$$

and

$$24w_v - 2sw_v(u - v) + 2(sw + m) - p_vh - w_vl = 0. \tag{9}$$

These equations respectively characterize households' rent and wage offer curves in the decentralized city case.[8]

Focusing first on (8), we find that it contains terms in w_u. However suppose the household lives at an arbitrary residential location, u'. Also suppose that its worker works at some job location v', where $v' \le u'$. For the worker to work at v', it must be the case that his/her wage offer at v' must be less than or equal to the wage offer at v' of workers living either closer in or further out than u'. Therefore $w_u(u', v')$ must equal zero. (If instead all workers have the same wage offer curves regardless of residential location, then $w_u(u', v')$ still must equal zero.) Therefore solving for the household's rent offer curve, we get

$$p_u(u, v) = \frac{-2(sw(v) + m)}{h(u, v)}. \tag{10}$$

Turn now to (9). It contains terms in p_v. But suppose a worker works at an arbitrary job location v'' and lives at an arbitrary residential location u'', where $u'' \ge v''$. For the worker's household to live at u'', its rent offer at u'' must be greater than or equal to the rent offer at u'' of households whose workers work either closer in or further out than v''. Therefore $p_v(u'', v'')$ must equal zero. Then solving for the worker's wage offer curve, we get

$$w_v(v) = \frac{-2(sw(v) + m)}{24 - 2s(u - v) - l(u, v)}. \tag{11}$$

In the subsection below we discuss rent offer curves and market rent gradients further. The following subsection considers wage offer curves and market wage gradients. A third subsection then reintroduces the possibility of workers having multiple skill levels.

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[8] The rent and wage offer curves can also be derived straightforwardly from households' indirect utility function, making use of Roy's Identity.

4.1. Rent Gradients in Decentralized Cities

How do the characteristics of households' rent offer curves compare in the centralized versus the decentralized employment cases? Examining (3) and (10), we find that rent offer curves in the two cases have the same form, except that the wage rate varies with job location in the latter. Rent offer curves therefore must always have negative slopes, regardless of whether employment is centralized or decentralized.

But do rent offer curves also decline with distance at a decreasing rate in the decentralized employment case? To investigate this question, we differentiate (10) with respect to u, assuming an arbitrary fixed job location, v. (This procedure is followed since the rent offer curve for a household gives its level of willingness to pay for land for housing at different residential locations, given any fixed job location. Not all residential locations will turn out to be feasible for a household having any particular job location, since it may be outbid by other households for land at some locations, but its offer level can be determined nonetheless.) The result is:

$$\frac{\partial p_u}{\partial u} = \frac{-p_u h_u}{h}. \tag{12}$$

Examining (12), since $h_u(u, v)$ must be positive when v remains unchanged, $\partial p_u/\partial u$ must be positive. Therefore rent offer curves decline at a decreasing rate with distance from the center of the city—the same result as in the centralized city case examined above.

We now turn to the question of whether, in a city with decentralized employment, workers' job locations affect households' rent offer curves for land at particular residential locations. In other words, we establish whether or not otherwise identical households whose workers work at different job locations are willing to pay the same amount for land at particular residential locations. If so, then households having different workplace locations will mix together over at least some set of residential locations, as long as all households live further out than their workers' jobs. If not, then households that differ only by workplace location will segregate into different residential rings.

Examining (10) above, we note that households' rent offer curves depend on workplace location, v, as well as on residential location, u. To determine how a household's rent offer curve depends on its worker's job location, we differentiate (10) with respect to v, holding u constant. This allows us to determine how a household's willingness-to-pay schedule for land for housing at all residential locations (including locations at which it will be outbid) varies when its worker's job location changes. The result is:

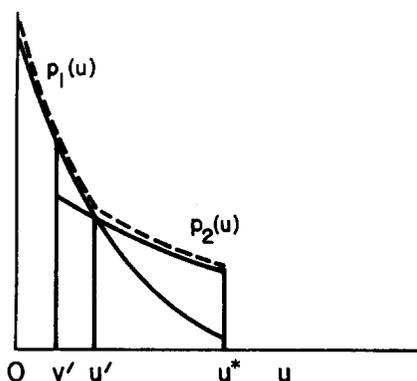$$\frac{\partial p_u}{\partial v} = \frac{1}{h}[-2sw_v - p_u h_v]. \tag{13}$$

FIG. 2.   Rent offer curves of households whose workers work at the CBD and at a suburban job location when only in-commuting occurs.

Equation (13) is difficult to sign with confidence. The first term in square brackets is clearly negative. The second term involves h_v, the variation in housing demand as the household's workplace location becomes more suburbanized. If h_v is positive or zero, then the second term in (13) is also positive or zero and the sign of $\partial p_u/\partial v$ will be positive. If h_v is negative, then the sign of $\partial p_u/\partial v$ can be positive, zero, or negative depending on the size of h_v. In fact, it seems most likely that the sign of h_v is positive or zero, since h_v is the change in housing consumption when workplace location becomes more suburbanized but residential location remains fixed. This means that workers' commuting journey is reduced in both time and money cost and their wage rate is reduced, but not by enough to make full income fall. In this case it seems unlikely that housing consumption would fall and in general the effects on housing consumption of a change in v would seem likely to be small. However, since we are unable to sign $\partial p_u/\partial v$ with confidence, we consider all three possibilities separately below.

Case 1. Assume that $\partial p_u/\partial v$ is positive. This means that households' rent offer curves become flatter as workers' job locations become more suburbanized. This situation is depicted in Fig. 2 for a case in which residential locations vary continuously, but there are assumed to be only two job locations: at the CBD, where $v = 0$, and at some $v = v' > 0$. The steeper rent offer curve, labeled $p_1(u)$, is that of households whose workers work at the CBD, while the flatter rent offer curve, labeled $p_2(u)$, is that of households whose workers work at v'. Note that only households whose

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[9]Actually, suburban workers' households are willing to bid for land closer-in than v'. But from these locations they would have to out-commute to work, causing their rent offer curves to decline as they located further from v' in the direction approaching the CBD.

workers work at the CBD bid for land between $u = 0$ and $u = v'$.[9] Between $u = v'$ and u', both groups of households bid for land, but households whose workers work at the CBD outbid suburban workers' households. Beyond u', both groups again bid for land, but households whose workers work at the suburban job location, v', are willing to bid more. The residential boundary between the two groups of households, u', must be at least as far out as the suburban employment location, or $u' \geq v'$. Its precise location depends on the relative number of workers employed at the CBD versus at the suburban employment ring. The market rent gradient, $p^m(u)$, is the upper envelope of the two groups' rent offer curves. It is shown as the dashed line in Fig. 2.

This result is striking because it indicates that households may segregate into different residential rings even when they have identical utility functions, the same number of workers per household, and identical earning abilities. Previous urban models have only resulted in residential segregation when either household tastes, household incomes, or both are assumed to differ. In this model, workers are indifferent over a range of residential locations and commuting journey lengths, given their job locations. Workers who work at v' will be indifferent among residential locations between u' and u^*, the outer edge of the city. They will therefore be indifferent among commuting journey lengths ranging from a minimum of $(u' - v')$ miles to a maximum of $(u^* - v')$ miles. Workers who work at $v = 0$ will be indifferent among residential locations between $u = 0$ and u' and among commuting journey lengths ranging from zero to u' miles. However all households in the model achieve equal utility and are indifferent across all job location/residential location pairs in the ranges just enumerated.

Case 2. Now suppose $\partial p_u / \partial v = 0$. In this case, households' rent offer curves for land are unaffected by their workers' job locations, as long as only in-commuting occurs. Then all households' rent offer curves are the same, regardless of their workers' job locations. In the context of Fig. 2, the two rent offer curves would be identical. This case leads to the maximum amount of residential mixing possible in a model having decentralized employment. Here, residential locations between the CBD and v' would still be occupied exclusively by households having CBD workers, but residential locations between v' and the outer edge of the city would be occupied by a mix of households whose workers work at the CBD and at the suburban job location.

In this special case, the commuting pattern in the decentralized city has a simple property: workers are indifferent over all commuting journey lengths as long as they in-commute. Increases in the commuting journey length due to a household moving its residential location further out are compensated by lower land prices (and vice versa); while increases in the commuting

journey length due to a worker moving his/her job location further in are compensated by higher wages (and vice versa). As long as all households have identical tastes and incomes, workers are indifferent over a wide range of commuting journey lengths, from 0 miles to u^* miles. In this case, workers or households would only have preferences for particular commuting journey lengths if they had tastes or income levels which differed from the tastes or income levels of households generally, such as having two workers when other households in the city have only one worker.[10]

Case 3. Finally, suppose $\partial p_u / \partial v$ is negative. In this case, rent offer curves by households become steeper as their workers' job locations become more suburbanized. Suppose again that there are only two job locations, at $v = 0$ and v'. Then the two groups' rent offer curves are shown in Fig. 3. Suburban workers' rent offer curve is now the steeper curve labeled $p_1(u)$, while CBD workers' rent offer curve is now the flatter curve labeled $p_2(u)$. Suburban workers' households outbid CBD workers' households for housing at the closer-in region between $u = 0$ and u', while CBD workers' households outbid suburban workers' households for housing in the further-out region beyond u'. The market rent gradient is again shown as a dashed line.

In this case suburban workers living between $u = 0$ and $u = v'$ must out-commute to their jobs. But if out-commuting occurs, then the situation depicted in Fig. 3 cannot be an equilibrium. To see this, compare the situations of two households whose workers both work at v', but who live at $u = v' + 1$ and $u = v' - 1$. Both workers earn the same wage rate and both have the same length commuting journey. Households' indirect utility function is $V = V(p^m(u), w^m(v), 24w^m(v) - 2(sw^m(v) + m)(u - v))$. For the two households, their wage rate and full income levels are the same, but their rent levels are different. Therefore their utility levels, which are $V(p^m(v' + 1), w^m(v'), 24w^m(v') - 2(sw^m(v') + m))$ for the household living at $u = v' + 1$ and $V(p^m(v' - 1), w^m(v'), 24w^m(v') - 2(sw^m(v') + m))$ for the household living at $u = v' - 1$, cannot be the same. In order for the household living at $v' - 1$ to achieve the same utility level as the household living at $v' + 1$, the rent offer curve $p_1(v)$ and the market rent gradient would have to decrease rather than increase for residential locations closer-in than v'. (This is shown as the dotted line in Fig. 3.) Thus Case 3 cannot be an equilibrium if only in-commuting is assumed to occur in the model.[11]

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[10]See White [17] for discussion and estimation of a model of commuting behavior in this case. If there are multiple income/skill levels, then the indifference property discussed above holds within the residential ring chosen by households of that income/skill level.

[11]Note that if out-commuting occurred, there could be a second region just outside the CBD occupied by households whose workers have CBD jobs.
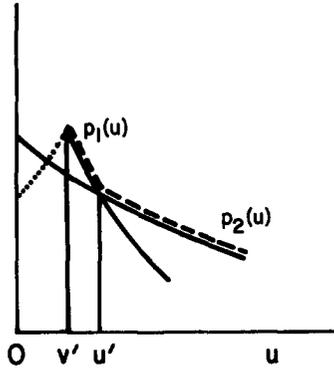
FIG. 3.   Rent offer curves of households whose workers work at the CBD and at a suburban job location when some out-commuting occurs.

To summarize the results of this subsection, we have shown that in an urban model with decentralized job locations, households having identical tastes whose workers have identical skill levels may nonetheless have different rent offer curves for land at the same residential locations. This implies that households whose workers have different job locations may segregate into different residential rings. The most likely pattern (but not the only one possible) appears to be that households locate in concentric residential rings in order of the centrality of their workers' job locations. This means that in equilibrium, households' residential and job locations will be nonnegatively related to each other.

4.2. Wage Gradients in Decentralized Cities

Turn now to the wage gradient. We wish to explore the properties of wage gradients and also to consider whether otherwise identical workers having different residential locations are willing to work for different wage rates at the same workplace location.

We derived an expression for workers' wage offer curves in (11) above. Examining (11), the numerator is the cost of commuting per mile round trip and the denominator is number of hours of work. With decentralized employment, the wage rate falls with increases in employment suburbanization by the reduction in per mile commuting cost per hour of work. The wage gradient must always be negative.

Using the wage and rent offer curves, (8) and (9), we can determine in a relative sense how quickly they each decline with distance from the CBD. Substituting (8) into (9), we get

$$\frac{w_v(u, v)}{w(u, v)} = \left(\frac{ph}{wn}\right)\frac{p_u(u, v)}{p(u, v)},\tag{14}$$

where $n(u, v) = 24 - 2s(u - v) - l(u, v)$ is hours of work. The percentage rate of decrease of the wage offer curve per extra mile of workplace distance from the CBD equals the percentage rate of decrease of the rent offer curve per extra mile of residential distance from the CBD times the ratio of expenditure on residential land to earnings from labor. The ratio of land expenditure to earnings depends on the location variables; however, it must be less than the ratio of housing expenditure to earnings. The latter has been well studied and is usually thought to be around 25%.[12] Thus the wage offer curve should be expected to decline at a much lower proportionate rate with distance from the center than the rent offer curve.

How does the rate of decline of workers' wage offer curves vary as job suburbanization increases, holding residential location constant? To answer this, we differentiate the wage offer curve, (11), with respect to v, assuming an arbitrary fixed residential location, u. This results in

$$\frac{\partial w_v}{\partial v} = \frac{w_v(-2s - n_v)}{n(u, v)}. \tag{15}$$

The expression in parentheses in the numerator of (15) is the reduction in commuting time per extra mile of job suburbanization when residential location is fixed, minus the change in work time per extra mile of job suburbanization. In general we expect that as employment becomes more suburbanized, workers will split the savings in commuting time in some proportion between extra leisure and extra work. If so then n_v is positive, $(-2s - n_v)$ is negative, and the sign of $\partial w_v/\partial v$ must be positive.

Thus workers' wage offer curves must fall at a decreasing rate with greater workplace suburbanization. As long as there is diminishing marginal utility of both leisure and goods, the marginal value of leisure time and work time must increase as the commuting journey gets longer and less time is available for both work and leisure. Leisure time is valued directly and work time is valued because it leads to more income and therefore more nonleisure goods. In order to induce workers to commute farther toward the CBD from fixed residential locations and therefore to give up more leisure and/or work time, the wage gradient must allow for the fact that the value of time at the margin is increasing. The wage gradient thus must increase at an increasing rate as v falls, i.e., as the CBD is approached.

It is interesting to note that a negative linear wage gradient is highly unlikely to occur in the context of the model. Suppose we make the strong simplifying assumption that work hours are fixed at n^*. In this case, the slope of the wage gradient becomes $w_v(v) = -2(sw(v) + m)/n^*$, and $\partial w_v/\partial v = -2sw_v/n^* > 0$. Thus the wage gradient must still decline at a

[12] This relationship was pointed out by Muth [9].

decreasing rate with greater workplace distance from the CBD. As long as leisure time enters the utility function, the resulting wage gradient cannot decrease at a constant rate with workplace distance.[13]

Now consider whether otherwise identical workers whose residential locations differ have different wage offer curves. Examining (11), the wage offer curve clearly depends on residential location, u, as well as on workplace location, v. To determine how workers' wage offers at different job locations vary as residential location changes, we differentiate (11) partially with respect to u, holding v constant. This results in

$$\frac{\partial w_v}{\partial u} = -w_v \frac{n_u}{n}. \qquad (16)$$

Since w_v is negative, the sign of (16) depends on that of n_u, the change in work hours when residential location becomes more suburbanized, holding job location constant. When residential distance from the center increases, commuting journey length increases, leaving less time available for work and leisure together. It seems likely that workers in this situation would share the time loss between reduced hours of work and reduced leisure in some proportion. If so, then n_u must be negative, which implies that $\partial w_v/\partial u$ must be negative. In that case, workers living at more distant residential locations must have steeper wage offer curves. However since this argument is informal, we consider all three possible signs of $\partial w_v/\partial u$ briefly below.

Case 1. Continuing with the assumption that $\partial w_v/\partial u$ is negative, suppose there are only two residential locations, at u^* and u^{**}, where $u^{**} > u^*$, but workplace locations vary continuously. Workers living at u^{**} must have steeper wage offer curves than workers living at u^*. The two groups' wage offer curves are shown in Fig. 4, where the steeper curve labeled $w_1(v)$ is the wage offer curve of workers living at u^{**} and the flatter curve labeled $w_2(v)$ is the wage offer curve of workers living at u^*. Since the market wage gradient is the lower envelope of the wage offer curves, it is shown as the dashed line in Fig. 4. (Note that if only in-commuting occurs, the wage offer curve of workers living at u^* must end at $v = u^*$.) The result in this case is that workers living at the closer residential location get jobs at the closer-in range of workplace locations, from $v = 0$ to v', while workers living at the more distant residential location get jobs at the more distant range of workplace locations, from $v = v'$ to $v = u^{**}$. In this case, all workers in-commute to their jobs.

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[13] Many researchers have assumed or derived linear wage gradients in models of decentralized cities, usually in models in which leisure time does not enter households' utility functions. See White [15] and Ogawa and Fujita [10].
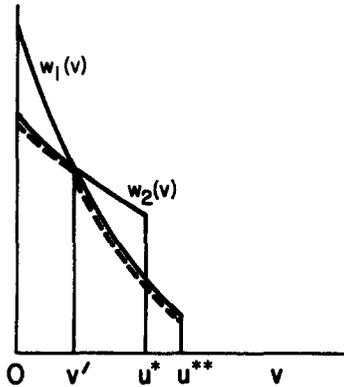
FIG. 4. Wage offer curves of workers whose households live at two different residential locations.

Note that under these assumptions, the market wage gradient, $w^m(v)$, can have any shape as long as its slope is negative. As drawn in Fig. 4, it is approximately negative linear. But depending on the number of wage offer curves and how strongly they vary with residential location, the market wage gradient could be observed to decline at an increasing or decreasing rate with increases in v. This differs from the result for the rent functions, where the market rent gradient declined at a decreasing rate with distance, the same shape as individual households' rent offer curves.

In this case, workers again are indifferent over a range of commuting journey lengths. Workers living at u^* are indifferent over job locations between $v = 0$ and v', and over commuting journey lengths ranging from a minimum of $u^* - v'$ miles to a maximum of u^* miles. Workers living at u^{**} are indifferent over job locations between $v = v'$ and $v = u^{**}$, and over commuting journey lengths ranging from 0 miles to $u^{**} - v'$ miles.

Case 2. Now suppose $\partial w_v / \partial u = 0$. In this case the two wage offer curves in Fig. 4 would be identical and the market wage gradient, $w^m(v)$, would be identical to the wage offer curves. Workers living at both residential locations would be willing to work at any job location for the same wage, as long as they in-commuted to their jobs.

In this case, workers are indifferent over a wider range of commuting journey lengths. For example, if there were again only two residential locations at u^* and u^{**}, then workers living at u^* would be indifferent over job locations between $v = 0$ and $v = u^*$ and over commuting journey lengths ranging from 0 miles to u^* miles. Workers living at u^{**} would be indifferent over job locations between $v = 0$ and $v = u^{**}$ and over commuting journey lengths ranging from 0 miles to u^{**} miles.

Case 3. Finally suppose that $\partial w_v/\partial u < 0$. Then the two wage offer curves shown in Fig. 4 would be reversed. The steeper curve, $w_1(v)$, would be the wage offer curve of workers living at u^* and the flatter curve would be the wage offer curve of workers living at u^{**}. In this case workers living at the more distant residential location would underbid others for the closer-in jobs and workers living closer-in would underbid others for the more distant jobs. In particular, workers living at u^* would both in-commute and out-commute to jobs located between $v = v'$ and $v = u^{**}$.

To show that this outcome cannot occur, examine the situation of two workers whose households both live at u^*. One worker works at $v = u^* - 1$ and thus in-commutes and the other worker works at $v = u^* + 1$ and thus out-commutes. The wage offer curve of workers living at u^* is $w_1(v)$. Note that $w_1(v)$ equals the market wage gradient, $w^m(v)$, from $v = v'$ to the outer edge of the city. The worker who works at $v = u^* - 1$ has a wage rate of $w_1(u^* - 1) = w^m(u^* - 1)$ and the worker who works at $v = u^* + 1$ has a wage rate of $w_1(u^* + 1) = w^m(u^* + 1)$, where $w^m(u^* - 1) > w^m(u^* + 1)$. But the two workers' households cannot achieve equal utility. To see this, note that the indirect utility function for both is $V = V(p^m(u), w^m(v), F(u, v))$, where full income, $F(u, v)$, equals $24w^m(v)$ minus total per mile commuting cost times commuting distance. For the two workers living at u^*, rent is the same, and commuting distance is the same. This means that the only factors entering the indirect utility function which differ are the wage rate and full income. The worker who in-commutes has both a higher wage rate and a higher full income level than the worker who out-commutes. Therefore the worker who in-commutes must be better off than the worker who out-commutes and the situation represented in Case 3 cannot be an equilibrium. In order for out-commuting to occur, the market wage gradient must begin to rise rather than fall for job locations further out than $v = u^*$.

We have shown that in a city with decentralized employment in which all workers in-commute to their jobs, workers living at different residential locations will tend to prefer different job locations. It is interesting to note that the wage offer curves and market wage gradients discussed here in Cases 1 and 2 constitute the maximum possible wage reductions that firms can achieve by moving to suburban locations. With these wage gradients, firms have appropriated all of the gain to workers from shorter commuting journeys when job locations become suburbanized.

4.3. Wage and Rent Gradients When There Are Multiple Skill Levels

Suppose now that we reintroduce the possibility that workers have different skill levels and different wage rates. We wish to investigate how rent and wage offer curves and the location pattern vary with workers' wage/skill level in cities with decentralized employment.
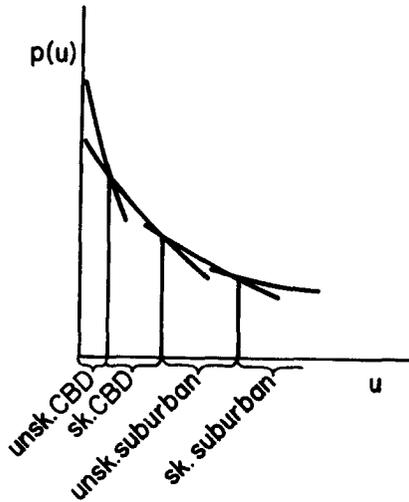
FIG. 5. Possible residential location pattern in a city with two skill classes and two job locations.

Assume that there are two skill groups, skilled workers and unskilled workers. Assume also that there are two employment locations, at $v = 0$ and v'. Then using the results developed under Sections 2 and 4.1, one outcome is that there will be four separate residential rings, one for each skill class and workplace location. Such a situation is shown in Fig. 5. There, unskilled workers with CBD jobs have the steepest rent offer curve. They occupy the innermost residential ring. They are surrounded by a ring of skilled CBD workers, whose rent offer curve declines more steeply. The third ring is occupied by unskilled suburban workers. The suburban employers must be located in a ring somewhere between the CBD and the residential boundary between the second and third rings. The outermost ring is occupied by skilled workers whose jobs are at the same suburban employment ring. The market land price gradient is the upper envelope. The important result here is that income/skill levels do not increase monotonically with residential distance from the CBD, as is usually the result in urban models with centralized employment.

Now turn to workers' wage offer curves. We can investigate how they vary with skill level by differentiating (12) with respect to $w(v)$, holding everything else constant. We get

$$\frac{\partial w_v(v)}{\partial w(v)} = \frac{1}{n}[-2s - n_w w_v]. \qquad (17)$$

The first term in (17) is the effect on the wage offer curve of the increase in the value of time spent commuting, which is negative. As higher wages make time more valuable, the wage offer curve gets steeper. The second term is the effect on the wage offer curve of the change in time spent working. It is positive if n_w is positive, i.e., if time spent working increases when the wage rate rises. (This requires that the positive substitution effect on work time outweigh the negative income effect when the wage rate rises.) If so then the effect of the second term in (17) is also to make the wage offer curve steeper as the wage rate rises.[14] Returning to (17), while the sign of w_v/w is ambiguous, we assume here that it is negative because the commuting cost effect is more important than the labor supply effect. If so, then workers' wage offer curves become steeper as their skill levels rise. In this case, the model predicts that firms have stronger incentives to suburbanize as the average skill level and wage rate of their workers rise. This is because the wage offer curves of highly skilled workers fall more steeply with greater workplace distance from the CBD. In contrast, employers of low-skill workers have much less incentive to suburbanize because low-skilled workers' flatter wage offer curves cause the cost reduction from suburbanization to be smaller. Furthermore, since low-skill workers tend to choose residential locations close to the CBD and there is no out-commuting in the model, employers cannot move very far from the CBD without severely restricting their potential labor supply. This also will tend to keep them close to the CBD.

These results seem realistic in a general way, since we often observe that firms whose workforce contains a high proportion of very skilled workers choose to locate in the suburbs. Research and development branches of large corporations are an example. In contrast, firms that employ a mixture of highly skilled and less skilled workers seem more likely to locate in the CBD. Banks and brokerage firms are examples of the latter.

We can illustrate the variation in the wage offer curves with skill level. Assume that there are two skill groups in the city and that all skilled workers live in more distant residential rings than unskilled workers. The residential boundary between the two groups is at u'. Firms are assumed to be at scattered locations. Also each firm is assumed to hire only one type of worker, not both. Figure 6 shows separate wage offer curves for skilled and unskilled workers. Unskilled workers' wage offer curve is flatter and ends at u', since no firm employing unskilled workers will locate further out than

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[14] It is interesting to note that the two terms in (16) have a similar interpretation to the two terms which result when the rent offer curve changes in response to a change in the wage rate, as discussed under Section 2. The two terms there consist of a value of commuting time effect, which is negative, and a change in housing demand effect, which is positive. The latter is usually assumed to outweigh the former, causing higher wage households to have flatter rent offer curves than lower wage households.
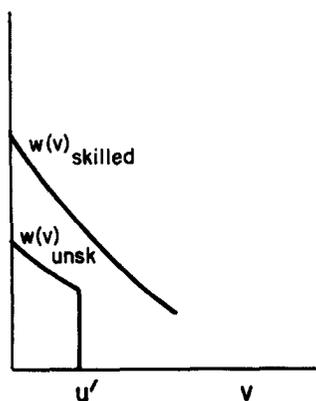
FIG. 6.   Wage offer curves of skilled and unskilled workers.

these workers live. Firms employing skilled workers can locate anywhere in the city. Thus firms employing either type of worker may mix together in the thick ring from $v = 0$ to u', while only firms employing skilled workers will locate in the surrounding thick ring from $v = u'$ to the outer edge of the city. Note that firms are not bidding against each other for sites here, nor are workers of each skill type bidding against each other for jobs. We cannot predict from the wage offer curves that either type of firm outbids the other for sites at any particular location. To answer this question, we would need to determine how much firms of each type could pay for land at any v assuming that the relevant profit conditions were met. The rent offer curve for firms has no necessary relation to their wage gradient.

5. COMMUTING BEHAVIOR AND THE INDIFFERENCE PROPERTY

In a city with multiple skill levels but only CBD employment (shown in Fig. 2), workers of any income level are indifferent over a range of commuting journey lengths. The range of commuting journey lengths covered by the indifference property extends from journeys between the CBD and the innermost location in the thick residential ring occupied by that income group to journeys between the CBD and the outermost location in the same residential ring. Workers are indifferent over this range of commuting journey lengths because a longer commute is compensated by lower housing prices at more distant residential locations.

This research was started with the intent of exploring whether an analogous indifference property exists over commuting journeys of varying lengths in a decentralized city. In fact it seemed that the indifference property applying in decentralized cities would be far more general. With

suburban as well as CBD job locations as possibilities, workers living in any residential ring could potentially be indifferent over commuting journey lengths ranging from working at home (a zero length commute) to working at the CBD to working anywhere in between. Extensions of the commuting journey caused by changes in workplace location would be compensated by changes in the wage rate, while extensions of the commuting journey caused by changes in residential location would be compensated by lower housing prices. Preferences for particular commuting journey lengths would exist only for households with atypical tastes, such as those having two workers in a city where households generally have only one worker.

In fact, this paper has shown that in cities with decentralized employment, a broad commuting indifference property holds only in the special case described under Subsections 4.1 and 4.2, Case 2. In general, we showed that households' rent offer curves for land for housing depend on their workers' job locations. We also showed that workers' wage offer curves for different job locations vary depending on their households' residential locations. These properties lead to a residential pattern in which households segregate themselves into rings depending on their workers' job locations and to a job location pattern in which workers living in particular residential rings have preferences over where they work. Thus the indifference property in an urban model with decentralized employment model is narrower rather than broader than that prevailing in a model with centralized employment. The only indifference property applying over commuting journeys in the decentralized employment case is that workers will be indifferent over all journey lengths between their preferred residential rings and the ring of workplace locations associated with that residential location. If the relevant rings are thin, then the indifference property will apply only over a narrow range of commuting journey lengths. Thus there is no generalized indifference property applying to commuting journey lengths in cities with decentralized employment.[15]

It is difficult to determine from a theoretical model how important this segmentation effect is. In practice, households may be almost indifferent over residential rings regardless of their workplace location, or the incentive to choose different residential rings depending on job location may be strong. A clear next step in this type of research would be a simulation model which could investigate this question. But although the model did not do very well in terms of providing a strong indifference property, it does provide a rich set of possible outcomes concerning patterns of residential location by households of different income levels, enabling researchers to explore more complex urban location patterns. It also suggests that if good

[15]See Simpson [13], White [16], and Madden [5] for other approaches to explaining commuting behavior in decentralized cities.

research on commuting patterns is to proceed, we need better data concerning both residential and job location within metropolitan areas, in addition to the obvious need for data on actual commuting journey length.

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