COOPERATION IN STRATEGIC GAMES REVISITED

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Abstract. For two-person strategic games with transferable utility, all major variable-threat bargaining and arbitration solutions coincide. This confluence of solutions by luminaries such as Nash, Raiffa, and Selten, is more than mere coincidence.

The present paper unifies and extends these solution into a complete theory. For two-person strategic games with transferable utility, it presents: (1) a decomposition of a game into cooperative and competitive components, (2) an intuitive and computable closed-form formula for the solution, (3) an axiomatic justification of the solution, and (4) a generalization of the above to games with private signals, along with an arbitration scheme that noncooperatively implements the solution with this type of incomplete information. The objective is to restart research on cooperative solutions to strategic games and their applications.

JEL Codes: C71 - Cooperative Games and C78 Bargaining Theory

1. Introduction

1.1. Background.

Cooperation in strategic environments is a crucial component in much of social interaction. Yet game theory devotes little effort developing *general* solutions that deal with it: existing general strategic (noncooperative) solutions of games ignore the possibility of cooperation and existing general cooperative solutions ignore strategic considerations. Solutions to strategic games in which the players may cooperate, referred to in this paper as *semi-cooperative games* (whose solutions are called *semi-cooperative solutions)*, receive little attention.

Motivated by the importance of cooperation in social interaction, researchers in the implementation and experimentation literatures study such cooperation on an ad hoc basis in specific families of games, such as cooperation in voluntary-contributions games, cooperation in coordination games, and cooperation in trust games. However, little attention is given to semi-cooperative solutions for *general* strategic games.

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This was not always the case. Early in the development of game theory, several important papers presented preliminary solutions for semi-cooperative games. In particular, the "variable threat" papers of Nash (1953), Raiffa (1953), and the later one of Kalai and Rosenthal (1978), henceforth refered to as NRKR, offer solutions to the general class of finite complete-information two-person strategic games in which the players may cooperate through the use of an arbitrator. Still, the subject has seen few applications over the years due to difficulties discussed below.

More specifically, the NRKR solutions were not constructed directly as solutions for strategic games; they came about as modifications of solutions to the cooperative Nash (1950b) bargaining problem,[1] by replacing the exogenously-given *fixed threat points* with equilibria of noncooperative games that endogenously determine them (hence the name *variable-threat* bargaining). Nevertheless, viewed as semi-cooperative solutions, the NRKR solutions are subject to the following difficulties:

D1. Multiple solutions: The three papers offer three different solutions.

D2. No direct definitions: The solutions do not admit closed-form descriptions; rather, they are defined to be the equilibria of specific noncooperative arbitration games.

D3. No direct axiomatic justification: NRKR use axiomatic solutions of the cooperative Nash bargaining problem to construct their arbitration games. But they have no axioms that relate the solution to the strategic game being arbitrated.[2]

D4. Restriction to complete information: Only the paper of Kalai and Rosenthal addresses incomplete information, and their discussion of this topic is restricted to highly specialized environments.

In addition to NRKR, Selten's (1960) Ph.D. dissertation defines and axiomatizes a semi-cooperative value for the class of two-person transferable-utility (see definition below) complete-information games in extensive-form.[3] However, Selten's work did not receive much follow-up, since dealing with the class of extensive-form games requires definitions and axioms that are complex and abstruse.

A generalization of the cooperative fixed-threat bargaining solution of Nash (1950b) to the case of incomplete information was studied by Harsanyi and Selten (1972), and in a follow-up paper, Myerson (1984). But even though Myerson's general cooperative setting may be interpreted to include strategic games as a special case, it follows the cooperative bargaining literature in relying on exogenously selected fixed-threat points that determine the arbitrated solution.[4] And since different selected threat points result in different outcomes, the Myerson (1984) solution is not well-defined as a semi-cooperative solution for strategic games.

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[1]These are the Nash (1950b) solution, the egalitarian solution in Kalai (1977), and the Kalai-Smorodinsky (1975) solutions. See Thomson and Lensberg (1989) for additional discussion and references.

[2]In the three NRKR papers, the authors first use axioms that identify a *cooperative* solution (the Nash, the egalitarian or the Kalai-Smorodinsky, respectively) to serve as a function that assigns a feasible pair of payoffs to any pair of noncooperative threats. Then, they define threir semi-cooperative solution by a two-step process: step 1, have the players play the noncooperative game to be arbitrated, and step 2, use the outcome of step 1 as the threats to which they apply the axiomatically-selected cooperative solution.

In other words, their axioms describe properties of an intermediary step in the arbitration process, rather than properties of the entire process.

[3]See Selten (1964) for a more accessible partial description of his solution.

[4]See also Myerson (1984a), where what a coalition can guarantee is independent of the actions chosen by the complementary coalition, a condition often referred to as "orthogonal coalitions." From the perspective of strategic game theory, the orthogonal- coalitions condition is a serious weakness of cooperative game theory.

Indeed, readers familiar with the history of the bargaining literature may recall that it was the above deficiency that motivated Nash (1953) to replace the fixed-threat solution by the variable-threat solution when dealing with strategic games. From this perspective, the Myerson solution may be viewed as a generalization of the *fixed-threat* (cooperative) solution to games of incomplete information, while the current paper takes a first step in generalizing the *variable-threat* (noncooperative) solution to incomplete information through private signals. Moreover, like the NRKR, the Myerson solution is not described by a closed-form expression.

1.2. **The objective, approach, and contribution.** Building on the literature above, this paper takes an initial step to restart the study of cooperation in strategic games. For a restricted but important class of games, we offer a more complete theory of such cooperation. The theory is centered around a semi-cooperative solution, referred to as the *cooperative-competitive value*, or coco value for short, that significantly overcomes some of the difficulties cited above. The presentation also leads to well-defined questions for further research. To make this paper self-contained, we assume no prior knowledge of the early literature. Further connections to the papers cited above and to additional important literature are deferred to later in this introduction and to remarks throughout the body of the paper.

The paper is restricted to finite two-person incomplete-information strategic games with *side payments*, which are alternatively referred to as transferable-utility (TU).[5] The restriction to TU games allows us to make the following advances:

A1. Unification and generalization: On TU games with complete information, the different NRKR values and the Selten value all coincide. The coco value studied here is a formal generalization of their common TU value from complete to incomplete information.

A2. Closed-form definition: For TU games, we are able to define the coco value by an easy-to-interpret and easy-to-compute closed-form expression that generalizes naturally to games of incomplete information.

A3. Decomposing cooperation and competition: The closed-form expression and the generalization to incomplete information are made possible by a general decomposition of a game into cooperative and competitive components.

A4. Axiomatization: The solution (and decomposition) are justified by a simple and intuitive set of axioms based on principles of efficiency, fairness, and stability.

A5. Noncooperative Implementation: We consider scenarios where the players have private signals regarding the state of nature, and we assume that the realized state of nature is observed after game play. In this case, the first-best outcome determined by the coco value can be implemented in the interim sense.

A6. Other generalizations: The coco value **generalizes the minmax** value of von Neumann (1928) from zero sum to non-zero-sum games; and its axiomatization, when restricted to the class of zero-sum games, is a formal axiomatization of the minmax value. It also **generalizes the maxmax value** from team games (games with identical payoffs) to general games.

[5]Stated differently, the players have quasi-linear utility functions in monetary payoffs. A simple way to view this is that players assign monetary values, say, in dollars, to outcomes, and without loss of generality utilities have been normalized so that the utility for \$x is x.

Much of the discussion of the coco value in this paper is directed at an arbitrator, whose role is to bring about an efficient and fair outcome to a strategic game. But since the formula and computation of the value are straightforward, players may bypass the arbitrator and follow the formula and procedures on their own. For example, in the 2×1 sacrifice game below, an arbitrator who follows the coco theory would prescribe that the row player play *sacrifice* in exchange for a \$2 transfer from the column player, netting each player a \$1 payoff. But once they understand the rationale of this solution, the two players may shake hands, or sign a contract to execute this solution on their own. Indeed, statements in this paper that refer to *arbitration* may be reinterpreted as statements about *self-arbitration*.

Sacrifice game:

decline	\$0,0
sacrifice	-1,3

This paper does not attempt to provide a descriptive interpretation of the coco value. Nonetheless, the coco value may shed light on the tension between competitive and cooperative factors present in many real-world negotiations that possess both cooperative and strategic elements.

Readers who are used to working entirely in noncooperative game theory may wonder why we do not simply model the problem as a strategic contract-signing game that allows for commitments and monetary transfers. However, we know from more recent folk theorems that games with contracts and voluntary commitments possess large sets of Nash equilibria.[6] Thus, it is important for game theory to go further and select one equilibrium as a recommended focul point. Indeed, as discussed in this paper, the coco value may be viewed as the "natural" equilibrium payoff of a noncooperative arbitration game that allows for commitments. The axioms of efficiency, fairness and stability that lead to the coco value, are formal conditions that one may wish to impose on a "natural" equilibrium.

The noncooperative arbitration game discussed in the paper addresses standard implementation questions: Would strategic players come to an agreement leading to the coco value (or agree to use an arbitrator who implements the coco value)? Would they provide the information needed for its implementation?

These questions are addressed under additional requirements of interest. First, unlike earlier literature on arbitration, we discuss voluntary as opposed to obligatory arbitration. Moreover, we address the questions above in the interim sense: Would the players choose to use the arbitrator when they already possess their private signals?

Readers familiar with the implementation literature are aware of the impossibility results of Myerson and Satterthwaite (1983), showing that a general implementation of a first-best outcome in the interim sense may be impossible. Thus implementing the coco value, a first-best outcome that satisfies additional properties, must be subject to significant restrictions.

More specifically, we restrict ourselves to private signals relating to states that can be collectively assessed after the play of the game, and do not deal with individuals' internal states that may remain private forever. For example, differential forecasts about weather and

[6]See, for example, Fershtman et al. (1991), Tennenholtz (2004), Jackson and Wilkie (2005), A. Kalai et al. (2010) and references therein; and see the recent manuscript of Forges (2011) for extensions to games with incomplete information.

market conditions may be in the domain of our analysis, while an individual's satisfaction from owning an object of art falls outside of its scope.[7]

A natural next step for future research would be to extend the theory to second-best solutions of semi-cooperative games in which first-best outcomes cannot be made incentive-compatible. But this may be quite challenging, as one would like to simultaneously extend all aspects of the theory: general unrestricted private information with second-best implementation; a weaker efficiency axiom, with full axiomatic justification of the implemented second-best outcome; and a closed-form expression that defines the selected second-best outcome.

1.3. **Decomposing cooperation and competition.** The definition, axiomatization, and implementation of the coco value all rely on a natural decomposition of two-person strategic TU games to cooperative and competitive components. The natural interpretation of this decomposition is critical for the generalization to incomplete information and to the other aspects of the coco theory.

As an easy illustration of the decomposition, consider a complete information game, where the payoffs of the two players are described by payoff matrices A and B. The game has a natural cooperative/competitive decomposition, or *coco decomposition*[8], described by:

$$\begin{aligned}(A,B) &= \left(\frac{A+B}{2},\frac{A+B}{2}\right)+\left(A-\frac{A+B}{2},B-\frac{A+B}{2}\right)\\ &= \left(\frac{A+B}{2},\frac{A+B}{2}\right)+\left(\frac{A-B}{2},\frac{B-A}{2}\right).\end{aligned}$$

Notice that the first term of the right-hand side is a cooperative team component in which the interests of the players are fully aligned. The second term is a fully competitive zero-sum game in which the player's interests are in direct conflict. Game theory has appealing solutions to these two types of games: the maxmax for cooperative team games and the minmax for zero-sum games. Restricted to the case of complete information, the coco value is defined by:

$$\text{coco-value}(A,B) = \text{maxmax-value}\left(\frac{A+B}{2},\frac{A+B}{2}\right)+\text{minmax-value}\left(\frac{A-B}{2},\frac{B-A}{2}\right).$$

This definition has a natural interpretation in terms of cooperation and one-upmanship. The players play the maxmax strategies of the cooperative component to yield equal (maximal) payoffs, which are efficient in the game (A,B). But then the equal payoffs are adjusted by a zero-sum compensating transfer from the strategically weaker player to the stronger one. To determine the relative strategic strength, one considers a hypothetical *relative advantage* game in which each player's goal is to exhibit how much better he could have done, or equivalently, by how much he could have "outplay" his opponent.[9] The body of the paper builds upon such a decomposition in defining the coco value for general Bayesian games.

[7]See Mezzetti (2004) for an earlier use of such assumptions in other implementation problems.

[8]Even though some people seem to have earlier awarness of this decomposition for complete information games, we are not aware of any published version of it.

[9]This is reminiscent of the "difference game" used by Nash (1953) in his variable threat paper.

The decomposition also plays an important role in facilitating a strategic *implementation* of the coco value.

It is easy to see from the above definition that when starting with a cooperative strategic game (X, X), the coco value is the cooperative maxmax value, and when starting with a zero-sum game $(Y, -Y)$, the coco value is the minmax value.

It is also clear from this definition why the restriction to two-person games is important. While the decomposition above holds for any number of players, the minmax solution is defined only for two-person games. Thus, a value for n-person games would most likely constitute a generalization of the minmax value to n-person zero-sum games.

1.4. **Additional related literature.** Additional important directions that address semi-cooperative solutions can be found in Aumann (1961), Forges et al. (2002), de Clippel and Minelli (2004), Ichiishi and Yamazaki (2006), Biran and Forges (2011), Hart and Mas-Colell (2010), and related references.

Carpente et al. (2006) present an earlier axiomatization of a semi-cooperative value defined for the class of TU complete-information games. It turns out that their value describes a "bridge" solution that has been repeatedly used on an ad hoc basis by many authors, including some of those mentioned in the previous paragraph. As discussed in the body of this paper, the Carpente et al./bridge solution is different from the coco value (and from the NRKR values), even on the restricted class of complete-information TU games: it does not account for important strategic externalities and credibility of threats.

Finally, observations from experimental game theory suggest that players tend towards "playing fair," sometimes even against their selfish material interest.[10] Being fair *and* compatible with individuals' material incentives, the coco value is an attractive focal point for the selection of equilibrium in arbitration games.

1.5. **Organization.** Section 2 contains the formal model and the definition of the coco value. An axiomatic characterization is given in Section 3, while Section 4 is devoted to implementation. Section 5 contains an analysis of a joint venture game between a producer and a distributor that shows how the coco value is related to the impossibility results of Myerson and Satterthwaite (1983). Section 6 offers further elaboration on the possibility of implementation of the coco value. Section 7 elaborates on the axioms discussed in Section 3 and on the computability and robustness of the coco value. Section 8 includes a brief concluding summary and suggests directions for future research.

2. Definition of the coco value

2.1. **Complete information.** Before turning to the general definition, we first illustrate the coco value for a complete-information game, based on the decomposition and the formula discussed in the introduction.

Example 1. ***Vendors with complete information.*** *Simultaneously, each of two hot-dog sellers, P1 and P2, has to choose one of two locations, the airport, A, or the beach, B. The daily demand at A is for* 40 *dogs, while the demand at B is for* 100 *dogs. If they choose different locations, they each sell the quantity demanded at their respective location; and if*

[10] This literature is too large to survey here. See Roth (1979), Rabin (1993), Binmore (1994), Fehr and Schmidt (1999), Camerer (2003), Chaudhuri (2008), and follow-up references.

they choose the same location, they split the local demand equally. P1 nets a profit of \$1/dog sold, while P2 nets a profits of \$2/dog sold.

Below is the representation of this game, followed by its coco decomposition.

	A	B			A	B			A	B
A	*20,40*	*40,200*	=	A	*30,30*	***120,120***	+	A	*-10,10*	*-80,80*
B	*100,80*	*50,100*		B	*90,90*	*75,75*		B	*10,-10*	***-25,25***

The cooperative component, or team game, has a maxmax value of $(120, 120)$, while the competitive component, or the zero-sum game, has a minmax value of $(-25, 25)$. Hence, the coco value is $(95, 145) = (120, 120) + (-25, 25)$.

The coco value achieves the maximum total of 240, which the players obtain by playing (A,B). The play (A,B) is followed by a \$55 side payment from P2 to P1: $(40, 200) + (55, -55) = (95, 145)$. The $(55, -55)$ transfer represents the sum of the two transfers $(80, -80) + (-25, 25)$; the first is needed to equate the payoffs at (A, B), and the second is the correction factor needed to compensate P2 for agreeing to take equal payoffs.

Indeed, since this is a game of complete information, the values proposed by NRKR and by Selten all agree on this solution, even if they do not describe it through the simple closed-form expression above.

We now use the same idea for the a more general case of incomplete information.

2.2. **The general case, with incomplete information.** Unless otherwise specified, there is a set of two players, $N = \{1, 2\}$.[11] A Bayesian game G is defined by: $G = (A = \times_{i \in N} A_i, Y = \times_{i \in N} Y_i, U = \times_{i \in N} U_i, \mu)$ where for each player i, A_i denotes the set of *actions*, Y_i denotes the set of *signals*, and $U_i \subseteq \mathbf{R}^A$ denotes the set of *payoff functions* (utilities), $u_i : A \to \mathbf{R}$. All these sets are assumed to be finite and μ is the *prior* probability distribution over $Y \times U$. To increase readability, we sometimes write $(y, u) \sim \mu$ to indicate that the pair (y, u) is drawn from the distribution μ.

As in much of the literature on Bayesian games, we assume that the game and the prior distribution are commonly known to the players. Game play is as follows. First, the state of the world, $(y, u) \in Y \times U$, is drawn from μ. Each player i then observes her own signal y_i, on the basis of which she chooses (simultaneously with her opponent) an action $a_i \in A_i$. The payoff to player i is $u_i(a)$, where $a = (a_1, a_2)$ is the selected *action profile*.

As is standard, a *mixed action* for player i is a probability distribution $\alpha_i \in \Delta(A_i)$ over her set of actions. We also extend the domain of payoff functions, u_i, to mixed actions by the use of expected values. A (pure) *strategy* for player i, $s_i : Y_i \to A_i$, is a function that specifies the action that player i would choose if her signal were $y_i \in Y_i$; and a (behavioral) *mixed strategy* for player i, $\sigma_i : Y_i \to \Delta(A_i)$, similarly specifies a mixed action to play based upon her knowledge of her own signal. We may also refer to y_i as the *private information* of player i.

A game is *zero-sum* if, with probability 1, $u_1 = -u_2$, and it is a *team game* if, with probability 1, $u_1 = u_2$. Finally, we use the standard convention that a_{-i} and u_{-i} represent, respectively, the actions and payoffs of player i's opponent. A *coordinated strategy* is a function $c : Y \to A$ from signal profiles to action profiles.

[11]Our generic player is a female, and to ease the reading in the case of two players, player 1 is a female and player 2 is a male.

As mentioned, we assume that the players have additive transferable utility (TU) for money, i.e., each player can make arbitrary monetary side payments (or their equivalent) to the other at a one-to-one rate.

We next describe the coco decomposition for Bayesian games.

Definition 1. *For any* $u : A \to \mathbf{R}^N$*, define* $u^{eq}, u^{ad} : A \to \mathbf{R}^N$ *as follows:*

(1) *The* equal, or average, payoff *is* $u_1^{eq}(a) = u_2^{eq}(a) \equiv \frac{u_1(a)+u_2(a)}{2}$.
(2) *The* payoff advantage *of player* i *is* $u_i^{ad}(a) \equiv u_i(a) - u_i^{eq}(a) = \frac{u_i(a)-u_{-i}(a)}{2}$.

Although $u^{\mathrm{eq}}(a) \in \mathbf{R}^2$, we sometimes use $u^{\mathrm{eq}}(a)$ to denote the single equal payoff that it allocates to the players, and thus write $u^{\mathrm{eq}}(a) = u_1^{\mathrm{eq}}(a) = u_2^{\mathrm{eq}}(a)$.

The cooperative-competitive decomposition presented above for complete information extends naturally to incomplete information:

$$u = u^{\mathrm{eq}} + u^{\mathrm{ad}}.$$

But unlike the complete-information case, now the players may also improve the sum (or average) of their expected payoffs by sharing information. To this end, we define the following notions.

Definition 2. *For a game* $G = (A, Y, U, \mu)$*, the* team optimum *of* G *is defined by:*

$$\textit{team-opt}(G) = \max_{c:Y\to A} \mathbf{E}\left[u_1(c(y)) + u_2(c(y))\right].$$

A coordinated (pure) strategy $c : Y \to A$ *is called* optimal *if* $\mathbf{E}\left[u_1(c(y)) + u_2(c(y))\right] = \textit{team-opt}(G)$.

In words, an optimal coordinated strategy is a rule c that the players may use to select, for every pair of realized signals y, a pair of actions $c(y)$ that maximizes the sum of their expected payoffs in G. In most situations this selection is unique, but if there is more than one optimal pair of actions, the coordinated strategy selects one. The team optimum is the maximal sum of expected payoffs that may be generated by such a rule. Notice that this definition assumes that they truthfully share all their information and then coordinate their actions.

Definition 3. *The* relative advantage *of player* i *is defined to be her minmax value in the zero-sum game* $G^{ad} = (A, Y, V, \mu^{ad})$*, where* V *consists of all payoff functions* $v = (v_1, v_2)$ *with each* $v = u^{ad}$ *for some payoff pair of function* u *of* G*, and* $\mu^{ad}(y, v) = \mu(\{(y, u) : v = u^{ad}\})$.

The game G^{ad} is a zero-sum modification of G, which preserves the differences between the two players' payoffs. Each player is simply trying to maximize the difference between her payoff and that of her opponent. Since the advantage game is a zero-sum Bayesian game, it has a unique minmax expected value, which we denote by $\mathrm{minmax}_i(G^{\mathrm{ad}})$. We refer to this value as player i's *competitive advantage, relative advantage,* or just *advantage.*

Definition 4. *The* coco value *of a game* G *to player* i*, denoted by* $\kappa_i(G)$*, is defined by,*

$$\kappa_i(G) = \frac{1}{2}\textit{team-opt}(G) + \textit{minmax}_i(G^{ad}).$$

In parallel to the complete-information case above, one may define a cooperative component of G, G^{eq}, in which the players share both the information they have when they come to the game and the payoffs resulting from any play.[12] The team-opt(G) equals the highest possible (common) expected payoff that may result from any pure strategy of G^{eq}, which may be thought of as the team-value(G^{eq}). Thus, in parallel to the complete-information case, one may think of the coco value of a private-information game as the sum $\kappa(G) = \frac{1}{2}$ team-value(G^{eq})+ minmax-value(G^{ad}).

Note that the coco value is feasible and Pareto optimal for every y (in conditional expectations over Y), i.e., the sum of the payoffs is the maximum (expected) sum that the players can achieve with coordination and sharing of information.

Example 2. ***Vendors with incomplete information.*** *This is the same game as in the (complete-information) vendor example above, except that the demand at B depends on the weather: if sunny, which happens with probability* 1/2*, the demand is for* 200 *dogs; and if cloudy, which happens with probability* 1/2*, there are no customers at the beach. Furthermore, suppose that player 1 is perfectly informed, a priori, of the weather and player 2 has no information.*

The resulting Bayesian game G is described by the payoff tables below:

Sunny: prob 1/2

	A	B
A	20,40	40,400
B	200,80	100,200

Cloudy: prob 1/2

	A	B
A	20,40	40,0
B	0,80	0,0

The team-opt(G) $= \frac{1}{2}(40 + 400) + \frac{1}{2}(0 + 80) = 260$ is the greatest sum of expected payoffs they can generate under honest sharing of all the available information. The advantage game, G^{ad}, is the following:

Sunny: prob 1/2

	A	B
A	-10,10	-180,180
B	60,-60	-50,50

Cloudy: prob 1/2

	A	B
A	-10,10	20,-20
B	-40,40	0,0

This is a zero-sum game with complete information for P1 and no information for P2. To compute its minmax value, notice that by domination, P1 will play B in the left-hand side game, and play A in the right-hand side game. P2's best response is to play B, yielding the minmax(G^{ad}) $= \frac{1}{2}(-50, 50) + \frac{1}{2}(20, -20) = (-15, 15)$. So the coco value $\kappa(G) = \frac{1}{2}(260, 260) + (-15, 15) = (115, 145)$.

Notice that the team optimum is computed under the assumption of full cooperation. The information is fully and truthfully shared before the players choose the optimal action in each state of information. The advantage game is played noncooperatively, which reflects the relative advantage of P2 when both his payoff advantage and his information (dis)advantage are taken into account.

[12] $G^{\text{eq}} = (A, \hat{Y}_1 \times \hat{Y}_2, V, \hat{\mu})$ defined by $\hat{Y}_1 = \hat{Y}_2 = Y_1 \times Y_2$, $V = \{v : v = u^{\text{eq}}$ *for some* $u \in U\}$, and $\hat{\mu}((y, y), v) = \mu(\{(y, u) : u^{\text{eq}} = v\})$.

3. Axiomatic characterization of the coco value

This section discusses simple properties that one may expect from an arbitration scheme, based directly on the payoffs of the underlying strategic game being arbitrated.[13] In other words, the properties allow one to judge whether the outcome of the arbitration has desirable properties such as efficiency, fairness, and stability. Note that, due to the semi-cooperative nature of the solution, standard properties of Nash equilibrium are not preserved. In fact, if they were, we would be left with the Nash equilibrium of the arbitrated game, which is grossly inefficient in many cases. However, the Nash-equilibrium properties are preserved in the play of the strategic implementation game, studied in the next section.

All the properties of the coco value are adopted from the minmax value, and from the NRKR values on the restricted class of complete information games. Further elaborations on these properties are discussed throughout the rest of this paper. However, the six selected properties below are sufficient to identify the coco value as a unique semi-cooperative solution for the class of two-person TU Bayesian games.[14]

A *value* is a function from the set of all finite two-person Bayesian games G to $\mathbf{R}^2$, i.e., $v(G) \in \mathbf{R}^2$ where $v_i(G)$ is the value to player i. We focus on values that satisfy the following properties, or axioms:

(1) **Pareto efficiency**. Players achieve the maximum (first-best) total expected payoff possible with shared information: $v_1(G) + v_2(G) = \max_{c:Y \to A} \mathbf{E}\left[u_1(c(y)) + u_2(c(y))\right]$, i.e., team-opt$(G)$.
(2) **Shift invariance**. The shifting of payoffs in all cells by any pair of constants leads to a corresponding shift in the value. Formally, fix any $c = (c_1, c_2) \in \mathbf{R}^2$. For any u, let $u'(a) = u(a) + c$. Then $v(G') = v(G) + c$ where $G' = (A, Y, U', \mu')$, with $U' = \{u' : u \in U\}$ and $\mu'(y, u') = \mu(y, u)$.
(3) **Monotonicity in actions**. Removing an action of a player cannot increase her value. Formally, for player 1, let $A_1' \subseteq A_1$ and u' be the restriction of any u to $A_1' \times A_2$. Then $v_1(G') \leq v_1(G)$ where $G' = (A_1' \times A_2, Y, U', \mu')$, in which U' consists of the restricted payoff functions from G, and μ' is the induced distribution over (y, u') (i.e., $\mu'(y, u') = \mu(\{(y, u) : u|_{A_1} = u'\})$). The symmetric condition holds for player 2.
(4) **Payoff dominance**. If, under any coordinated pure strategy, a player's expected payoff is strictly larger than her opponent's, then her value should be at least as large as the opponent's. In particular, for player 1, if $\min_{c:Y \to A} \mathbf{E}[u_1(c(y)) - u_2(c(y))] > 0$, then $v_1(G) \geq v_2(G)$. The symmetric condition holds for player 2.
(5) **Invariance to redundant strategies**. For player 1, let $a_1 \in A_1$ and $A_1' = A_1 \setminus \{a_1\}$. We say that a_1 is *redundant* (in expectation) if there exists $\sigma_1 : Y_1 \to \Delta(A_1')$ with the property that for every $y \in Y$, and for every $a_2 \in A_2$ a_1 and σ_1 yield both players the same expected payoffs, i.e., $\mathbf{E}_\mu\big[u(a_1, a_2) \mid y\big] = \mathbf{E}_\mu\big[u(\sigma_1(y_1), a_2) \mid y\big]$. Then removing such a redundant action a_1 does not change the value of the game for either player. The symmetric condition holds for player 2.

[13] As mentioned earlier, Selten (1960, 1964) gives a more involved axiomatization of the analogous value for extensive-form games with complete information. Extended to games of incomplete information, the Selten value does not coincide with the coco value. We are grateful to Moshe Tennenholtz and Dov Monderer for pointing us to Selten's work.

[14] The proof of the theorem that follows may be carried out entirely within the class of two-person TU complete-information games, and within the class of two-person zero-sum Bayesian games. Thus, it provides an axiomatization of the NRKR value and of the von Neuman minmax value.

(6) **Monotonicity in information**. Giving player i strictly less information cannot increase her value. In particular, for player 1, $v_1(G') \leq v_1(G)$, whenever G' is obtained from G by reducing player 1's information through any function $f : Y_1 \to Y_1$ as follows: $G' = (A, Y, U, \mu')$ with $\mu'((y_1', y_2), u) = \mu(\{((y_1, y_2), u) : f(y_1) = y_1'\})$. The symmetric condition holds for player 2.

Theorem 1. *The coco value is the only value that satisfies axioms 1-6 above.*

Before turning to the proof of the theorem, we first offer some intuition through a sketch of the proof for complete information games. Specifically, for any value v that satisfies Axioms 1-5 above, $v(G) = \kappa(G)$ for any complete information game G. Shift invariance implies that it suffices to consider the special case of games G with $\kappa(G) = (0, 0)$. Moreover, to complete the proof of this special case, it suffices to show that $v_1(G) \geq 0$ (because a similar argument would show that $v_2(G) \geq 0$, and Pareto efficiency would imply that $v(G) = (0, 0)$) . Now, when $\kappa(G) = (0, 0)$, the minmax value of G^{ad} must also be zero. Let σ_1^* be any minmax strategy of player 1 in G^{ad}. By the coco decomposition, in the game G this strategy guarantees player 1 an expected payoff at least as large as that of player 2. Now consider the game H in which player 1 is forced to play σ_1^* (i.e., she has only one pure action that corresponds to σ_1^* in G). By axioms 3 and 5, $v_1(G) \geq v_1(H)$. By payoff dominance $v_1(H) \geq v_2(H)$ (see the proof of Theorem 1 for how to address the weak vs. strong inequality),. If the team optimum of H were the same as G, (i.e., if $v_1(H) + v_2(H) = 0$), then these three facts would imply that $v_1(G) \geq 0$, and we would be done.

However, forcing player 1 to play σ_1 may decrease the team optimum. To overcome this difficulty in this complete-information case, before we force player 1 to play σ_1^*, we augment the game G as follows: we add to player 2 a new simple action that yields the constant payoffs $(0, 0)$, no matter what player 1 plays. This does not change the coco value, and axioms 1 and 3 imply that this new strategy cannot increase player 1's value (in particular, player 2 is no worse off while the team optimum remains zero). Furthermore, when player 1 is now forced to play σ_1, player 2's new action guarantees that the team optimum remains zero, and hence the argument in the previous paragraph goes through.

In the case of incomplete information, the approach of the proof above fails for two reasons. First, the addition of a constant $(0, 0)$ action for player 2 could very well change the team optimum and the value of the advantage game, because this action may be taken based upon the player's information. Second, in order to apply the payoff-dominance axiom, we remove all of player 1's information, which might decrease the team optimum. We now show how to address these subtleties.

Lemma 1. *Let G be a finite two-person Bayesian game such that $\kappa(G) = (0, 0)$. Then axioms 1-6 above imply that $v_1(G) \geq 0$.*

Proof. We will construct a sequence of games and argue that $v_1(G) \geq v_1(G') \geq v_1(G'') \geq v_1(H) \geq 0$.

To construct G', we add a new action b_2 ($\notin A_2$) to player 2's set of actions, so that the sets of actions of G' are $A_1' = A_1$ and $A_2' = A_2 \cup \{b_2\}$. Next we define the possible payoff functions U' of G'. We fix any action $a_2^* \in A_2$ for player 2, we fix some team-optimal coordinated strategy $c : Y \to A$ in G (see Definition 2), and we define the gain from cooperation at (a_1, a_2^*) and y to be $g = u_1(c(y)) + u_2(c(y)) - u_1(a_1, a_2^*) - u_2(a_1, a_2^*)$. Every payoff function of G is extended in up to $|Y|$ payoff functions in G' so that when player 2 selects the new action b_2, their payoffs are those of G at (a_1, a_2^*) plus the gain g divided equally between the two players.

Formally, for any $y \in Y$ and $u : A \to \mathbf{R}^2$, define $f^{tu} : A' \to \mathbf{R}^2$ by

$$f^{tu}(a) = \begin{cases} u(a) & \text{if } a_2 \neq b_2, \\ u(a_1, a_2^*) + \left(\frac{g}{2}, \frac{g}{2}\right) & \text{if } a_2 = b_2, \end{cases}$$
$$\text{with } g = u_1(c(y)) + u_2(c(y)) - u_1(a_1, a_2^*) - u_2(a_1, a_2^*).$$

Now the prior probability distribution of G' is the one induced by μ, i.e., $\mu'(y, u') = \mu(\{(y, u) : f^{tu} = u'\})$.

It is easy to see that the team optimum of G' is still zero, because the total achieved by any coordinated strategy in G' can also be achieved in G, so $v_1(G') + v_2(G') = 0$. By monotonicity in actions, $v_2(G') \geq v_2(G)$. Hence, $v_1(G') \leq v_1(G)$.

Next, because $\kappa(G) = (0, 0)$, the value of the advantage game G^{ad} must be $(0, 0)$. Hence, there must exist a mixed strategy for player 1, σ_1^*, which guarantees player 1 at least as much as player 2, in expectation, i.e., $\mathbf{E}_\mu[u_1(\sigma_1^*(y), \sigma_2(y)) - u_2(\sigma_1^*(y), \sigma_2(y))] \geq 0$ for any σ_2. In particular, fix any such σ_1^* which is a minmax optimal strategy for player 1 in G^{ad}. Note that σ_1^* also guarantees player 1 at least as much as player 2 in G', because b_2 is equivalent to a_2^* in terms of the difference in the players' payoffs.

Now, using σ_1^* as defined above, we define a new action $b_1 \notin A_1$; we then consider the game G'', obtained from G by restricting the actions of player 1 to be $A_1'' = \{b_1\}$, with payoffs $u''(b_1, a_2) = u'(\sigma_1^*, a_2)$. Hence, in G'' player 1 must play like σ_1^* in G' (in expectation). By monotonicity in actions, this means that $v_1(G'') \leq v_1(G')$. (To see this formally, one must first consider the game with actions $(A_1 \cup \{b_1\}) \times A_2'$, which has the same value as G' because b_1 is redundant by axiom 5; then remove all remaining actions for player 1.) Finally, team-opt$(G'') = 0$, since when player 2 plays b_2, they achieve the same expected total as when they play c in G.

Now, by design, b_1 guarantees player 1 at least as much as player 2, in expectation. However, to apply payoff dominance, we must argue that, *even if the players coordinate*, player 1 gets *strictly more* than player 2, in expectation. Even though player 1 has only one action in G'', she may have information that can help player 2 achieve an advantage.

To address this coordination problem, we remove all information from player 1. In particular, fix any $y_1^* \in Y_1$ and define the game H by changing only the set of signals of G'' so as to obtain $Y_1^H = \{y_1^*\}$ with $\mu^H((y_1^*, y_2), u'') = \mu''(\{(y_1, y_2), u'') : y_1 \in Y_1\})$.

By axiom 6, $v_1(H) \leq v_1(G'')$. Also, the team optimum of H remains zero, because player 2 still has the option of playing the fixed action b_2. Finally, notice that player 1 is guaranteed an expected amount at least as large as that of player 2, due to our choice of b_1. Coordination is impossible since player 1 has only one action and one possible signal.

We are almost ready to apply the payoff dominance axiom. The remaining issue is that we have a weak inequality rather than a strict one. To complete the proof, imagine translating the payoffs of player 1 up by any constant $\epsilon > 0$. By Axiom 2, this would only shift his value up by ϵ. However, once his payoff has been shifted, Axiom 4 does apply, in which case player 1's value is at least as large as that of player 2. Hence, $v_1(H) + \epsilon \geq v_2(H)$. Since this holds for every $\epsilon > 0$, it follows that $v_1(H) \geq v_2(H)$. The combination of this with $v_1(H) + v_2(H) = 0$ implies that $v_1(H) \geq 0$, and we have already argued that $v_1(G) \geq v_1(G') \geq v_1(G'') \geq v_1(H)$. □

We now prove Theorem 1.

Proof of Theorem 1. First, we argue that the coco value satisfies axioms 1-6. Pareto efficiency is trivially guaranteed by the fact that the advantage game is zero-sum and the team-game value maximizes the expected sum of payoffs. Second, a payoff shift of (w_1, w_2) corresponds to a shift of $\left(\frac{w_1-w_2}{2}, \frac{w_2-w_1}{2}\right)$ in the advantage game and to a shift of $w_1 + w_2$ in the team optimum. Since the value of zero-sum Bayesian games satisfies shift invariance, this corresponds to a shift of (w_1, w_2) in the coco value. Monotonicity in actions and information clearly holds for zero-sum games and the team-opt value, and hence also for the coco value. Similarly, removing a redundant action for i in G corresponds to removing the redundant action in the zero-sum and team games, which does not change their value. Removing a redundant state also does not change the value of a team game or a zero-sum game.

The proof of the converse, namely, that the only value v that satisfies the axioms is $v(G) = \kappa(G)$, follows easily from Lemma 1. Specifically, translate the payoffs of G by $-\kappa(G)$ to get game G' where $\kappa(G') = (0, 0)$. Lemma 1 states that $v_1(G') \geq 0$. Since the axioms are symmetric, the same reasoning implies that $v_2(G') \geq 0$. Pareto efficiency then implies that $v(G') = (0, 0)$. Finally, axiom 2 implies that $v(G) = v(G') + \kappa(G) = \kappa(G)$. □

4. Noncooperative implementations of the coco value

The discussion in this section is with regard to *voluntary*, not obligatory, arbitration.[15] In other words, would unobligated individual players use an arbitrator who prescribes the coco value? Would they be willing to do so before they know their private signal, (*ex-ante implementation)*, and would they be willing to do so after they know it (*interim implementation)*?[16]

As readers familiar with the interim implementation literature know, achieving first-best efficiency in general Bayesian games may be impossible, even if we do not insist on simultaneously achieving the other properties (such as fairness) of the coco value. Thus, a completely general interim implementation of the coco value may not be achievable.

However, there are two special types of games in which private information is easy to deal with (though in opposite ways): strictly cooperative and strictly competitive games. In Bayesian team games (where the players' payoffs are identical), the obvious incentive is to fully disclose all private information, enabling the choice of the mutually best pair of actions. Conversely, in Bayesian zero-sum games (where one player's gain is the other's loss), the obvious incentive is not to disclose any private information, keeping the opponent from gaining any advantage. It follows that through the decomposition of a Bayesian game into the sum of a team game and a zero-sum game, the coco value can also deal with private information, provided that the incentives in the play the two component games are made to be independent of each other.

This decomposition can be exploited in a variety of situations in different ways. In what follows, we discuss general sufficient conditions under which implementations of the coco value are possible, even in the interim sense. But the conditions are only sufficient, and

[15] This is unlike the arbitration games considered by NRKR for the complete-information case.

[16] The *voluntary* decision of a player is whether or not to use the arbitrator. But once a player chooses to use the arbitrator, then he is committed to follow the arbitrator's instructions, even if his opponent did not choose to use the arbitrator. This means that an arbitrator may be used as a commitment device by a player.

the following sections describe other games in which implementation of the coco value is possible.

We use the term *protocol* to describe a two-person procedure that involves communication and simple commitments, without the use of joint randomization devices. To capture the idea of voluntary arbitration, the protocols below provide each player with the option of not participating. We say that a protocol *implements* the coco value if it admits a Bayesian Nash equilibrium with expected payoffs that match the coco value.

The sufficient conditions for implementability are specified by the following definition.

Definition 5. *A game satisfies the* weak revealed-payoff assumption,[17] *if after the play of the game, the realized payoffs obtained at the (purely or randomly) selected action profile, $u(a)$, are observed by the arbitrator and the players.*

A game satisfies the (unrestricted) revealed-payoff assumption, *if after the play of the game, the realized payoff functions $u : A \to \mathbf{R}^2$ become known to the arbitrator and the players.*

For example, "play as you wish and then share the realized profits equally" is a protocol that can be implemented under the weak revealed-payoff assumption. But the (unrestricted) revealed-payoff assumption is needed for a protocol that allocates the final payoff based on payoffs of hypothetical actions that could have been used.

It turns out that the weak revealed-payoff assumption is sufficient for ex-ante implementation of the coco value, and the revealed-payoff assumption is sufficient for interim implementation of the coco value.

Remark 1. *Note that despite the fact that the revealed-payoff assumption is quite demanding, it holds in all games in which the payoff functions depend on states that are observed after the play of the game. These include games in which the payoffs depend on observable weather conditions, as in the vendor games with incomplete information discussed earlier. Other important examples include games in which the payoff functions depend on realized market prices that can be eventually verified, such as the prices of stocks and other commodities.*

The above observation could be formalized by incorporating such states into the model of the game, and assuming that these states are observable after the play of the game. But this would involve additional notations, and since the revealed-payoff assumption is more general and includes all these cases, we proceed formally with it alone.

The particular protocols below are reminiscent of the formation of some real-life partnerships. When two partners agree to share equally the total (net) realized profits of a joint venture, they create individually monotonic payoff functions: the payoff of each increases as a function of total profit realized by the partnership.[18] This monotonicity property gives each partner the incentive to share information truthfully and to take actions that are optimal for the success of the project.

But if the situation is not symmetric – for example, if there are differences in information, resources, and opportunity costs – the partners may agree up-front to make a compensating

[17]Even though we state this as an assumption on the game, it is really an assumption on the environment in which the game is played. We could have two games, G and G', that have an identical description in the formal model presented earlier. Yet G is played in an environment E, where the ex-post payoffs are revealed, and G' is played in an environment E', where the ex-post payoffs are not revealed.

[18]The use of such monotonicity conditions is common in cooperative game theory; see, for example, E. Kalai (1977), Thomson and Myerson (1980) and Thomson and Lensberg (1989).

payoff transfer to correct for such asymmetries. If the size of the transfer is independent of their performance in the joint venture (for example, if they commit to the size of the transfer before the actual play), then the partnership should still be able to achieve first-best efficiency in an incentive-compatible manner.

Throughout the remainder of this section, $G = (A, Y, U, \mu)$ is assumed to be an arbitrary, fixed, two-player finite Bayesian game as discussed above.

4.1. **Ex-ante implementation.** In this subsection, we assume that the decision of whether or not to use the arbitrator is made before the players observe their private signals, and that the game is played in an environment with weakly revealed payoffs. A simple protocol can implement the coco value: the players form a partnership in which they split the total payoffs (positive or negative) equally. This can always be achieved by a side payment, and the sharing of total payoff incentivizes them to coordinate by revealing information and playing actions that maximize the total payoff. However, to compensate for the imposed equal division of payoffs when the game is not symmetric, a second side payment is made from the weaker player to the stronger one. When the two side payments are combined, the coco value is achieved at equilibrium. A direct consequence of this implementability result is that the coco value is ex-ante individually rational.

Ex-ante partnership protocols for an arbitrary, finite, two-player Bayesian game $G = (A, Y, U, \mu)$.

Fix any *optimal coordinated strategy* $c : Y \to A$.

(1) Players simultaneously choose whether to commit to participate or not.
- If either one does not agree to participate, then they play G unmodified, they collect their respective G-payoffs, and the protocol ends.
- Otherwise, they have made a binding agreement to continue, as follows.

(2) A triple (y_1, y_2, u) is drawn by the prior distribution μ, and each player i is informed of her *realized signal* y_i.

(3) Players $i = 1, 2$ simultaneously declare their supposed signals $\tilde{y}_i \in Y_i$.

(4) The players are committed to play the pair of actions $a = c(\tilde{y})$, after which the pair of payoffs $u(a)$ is revealed.

(5) A side payment is made so that the net payoff to player i is $u^{\mathrm{eq}}(a) + \mathrm{val}_i(G^{\mathrm{ad}})$. In other words, she is paid one-half of the total payoffs obtained through the actual play in stage 4, plus her minmax value (positive or negative) of the advantage component-game of G, computed (ex-ante) without knowledge of the signals.

Notice that different optimal coordinated strategies c and c' give rise to different protocols, but the difference is not essential. The induced protocols may differ only in how they break ties, but they result in the same payoffs.

Theorem 2. *The coco value $\kappa(G)$ of any finite two-player Bayesian game $G = (A, Y, U, \mu)$ is the expected payoff vector of a Nash equilibrium in any ex-ante partnership protocol of the game.*

Proof. Consider the following equilibrium strategy for each player i:

- Choose to participate.

- If mutual participation fails, play the (mixed) minmax strategy of G^{ad}, i.e., play G *as if you were playing* G^{ad}.
- If mutual participation holds, truthfully reveal your realized signal, i.e., $\tilde{y}_i = y_i$.

Observe first that no player can benefit by declaring a false signal $\tilde{y}_i \neq y_i$ (given that the other player is being honest), because $\tilde{y} = y$ simultaneously maximizes each player's expected payoff (i.e., it maximizes $\frac{u_1(a)+u_2(a)}{2}$ and has no effect on $\text{val}_i(G^{\text{ad}})$).

Next, observe that player i cannot increase her payoff by not participating. Suppose she does not participate, and instead plays a mixed strategy σ'_i, while her opponent plays his minmax strategy of the game G^{ad}, σ_{-i}. We can nonetheless compute her expected payoffs via the coco decomposition. In particular, her expected payoff is the sum of the expected payoffs in G^{ad} and G^{eq}. Her expected payoff in G^{ad} is at most her (minmax) value of G^{ad}, and her expected payoff of σ in G^{eq} is at most the (team) value of G^{eq}; hence, her total is at most the coco value for i. □

In addition to its direct implementation message, the theorem above serves as an easy way to establish the following:

Corollary 1. *The coco value is individually rational (ex-ante).*

Proof. Notice that a player may decline to participate, and choose to use her G minmax strategy in the ex-ante protocol above, guaranteeing herself her minmax value of G as the payoff in the protocol. Thus, the minmax values of the protocol are at least as high as the minmax values of G. Moreover, as equilibrium payoffs of the protocol game, the coco payoffs must be at least as large as the minmax values of the protocol. Thus, the coco payoffs are at least as large as the minmax payoffs of G. □

In addition to not dealing with the interim stage, the mechanism above is not attractive for an additional reason. Wilson (1987) advocates that mechanisms should be restricted to rules and payoff functions that do not depend on the prior probability distribution of the game being implemented. The mechanism above violates the Wilson doctrine in two respects. First, in order to compute the optimal coordinated strategy c used in the definition of the protocol, the arbitrator must know the prior distribution. Second, in order to compute the value of G^{ad} used in allocating the protocol's payoffs, the arbitrator must know the prior as well.

In the next section, we show how these deficiencies may be overcome under the stronger revealed-payoff assumption.

4.2. **Interim implementation.** In this section we make the (unrestricted) revealed-payoff assumption: the realized payoff function $u : A \to \mathbf{R}^2$ (equivalently, the *state of nature,* if it is incorporated into the model) becomes known after the play of the game, and the players can compute what the realized payoffs $u(a)$ would have been for every chosen pair of actions a. In Section 6, we give examples where this assumption does not hold but the coco value can be implemented nonetheless.

Note that the revealed payoff assumption does not require that the signals themselves be revealed. To illustrate the distinction, think again of the vendors game with incomplete information, discussed earlier, and assume that both sellers receive weather forecasts (their signals) before deciding on a location. The payoffs in this example depend on the weather and not on the forecasts, and once the weather is observed, the profit in each location (whether chosen or not) is known. In other words, the entire payoff table for the realized state of nature becomes known, even if the signals (i.e., the weather forecasts, which could be quite long with data, graphs and probabilistic assessments) remain unknown.

Under the above assumption, one can design effective interim protocols to implement the coco value.

Interim partnership protocol for an arbitrary, finite, two-player Bayesian game $G = (A, Y, U, \mu)$.

(1) A triple (y_1, y_2, u) is drawn by the prior distribution μ, and each player i is informed of her *realized signal* y_i.

(2) Simultaneously, each player selects one strategy from the following two choices:

DO NOT PARTICIPATE: she declares **NO** and selects (commits to) an action $\tilde{a}_i \in A_i$, to serve as her *non-cooperative action*; or

PARTICIPATE: she declares **YES** and submits two sealed envelopes, one containing a reported signal $\tilde{y}_i \in Y_i$ and one containing a selected action (that she is committed to) $\tilde{a}_i \in A_i$, to serve as her *noncooperative action*.

The **YES**/**NO** declarations are revealed to both players and then:

- If either player declares **NO**, then the noncooperative pair of actions $(\tilde{a}_1, \tilde{a}_2)$ selected above is played, the protocol stops, and the players collect their respective G-payoffs, $u(\tilde{a})$.
- But if both declare **YES**, then the *reported signals* $\tilde{y}$ and noncooperative actions $\tilde{a}$ are revealed to both players, who are committed to continue as follows.

(3) Simultaneously, the players choose "*cooperative*" actions $a_i \in A_i$ and play G using a. Both $u(a) \in \mathbf{R}^2$ and the realized payoff function $u : A \to \mathbf{R}^2$ are then revealed.

(4) Based upon the cooperative actions a_i from stage 3 and the noncooperative actions $\tilde{a}_i$ from stage 2 (observed from the now open envelopes), side payments are made so that the net payoff to each player i is $u_i^{\text{eq}}(a) + u_i^{\text{ad}}(\tilde{a})$.

Note that in the protocol above, the participation decision is *voluntary* in that each player can force the play of the unmodified game by choosing not to participate,. However, this also means that any Nash equilibrium of G can be converted to a nonparticipatory equilibrium of the interim partnership protocol.

The validity of nonparticipatory equilibria varies with the game. For example, the reader can check that in the Prisoner's Dilemma game to participate and cooperate (i.e., declare YES, submit $\tilde{a}_i =$ *defect* and select $a_i =$ *cooperate*) is a dominant strataegy for each player. While it is possible, in general, to employ refinements and "implement away" nonparticipating equilibria,[19] it seems more reasonable to model the possibility that players may choose not to participate and to play noncooperatively, as shown by follow-up examples.

Before stating our theorem, we point out two practical considerations regarding the protocol.

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[19]One difficulty is evident even in a pure coordination game, such as

1,1	0,0
0,0	2,2

, where there is a (1,1) equilibrium. However, a *team-game refinement*, which is natural among cooperative players, could rule out such equilibria. A suitable implementation may then have *all* equilibria yielding the coco value in expectation.

(1) In stage 3 above, one might allow extra communication, in the form of cheap talk, to aid the players in selecting the same coordinated optimal strategy c. However, since Nash equilibrium allows for coordinated selection when multiple equilibria are available, this is not necessary for the formal theorem below. Similarly, for many games one might consider protocols with lower *communication complexity* (see, e.g., Kushilevitz and Nisan, 1996), which is defined as the number of bits transmitted in a binary communication. In many games, an optimal c can be computed using significantly less communication than when players reveal all of their private information.
(2) The definition of the protocol above is in no way dependent on the prior. Moreover, the implementing strategies rely on very solid solution concepts: to determine the pair $\widetilde{a}$, the players use the minmax solution (as opposed to just a Nash equilibrium); and to determine the actual action pair a, they use simple (one-person) optimization. Hence, the resulting solution inherits some appealing stability and polynomial-time computability properties from these more robust solution concepts.[20]

In the equilibrium discussed in this theorem, players choose to truthfully share information and to optimally coordinate, with threats defined through the relative-advantage game.

Definition 6. *Participatory, honest, and c-coordinated strategies.*

(1) *A strategy π_i of the partnership protocol above is* participatory*, if it declares YES (with probability one) for every y_i; and it is* honest*, if $\tilde{y}_i = y_i$ for every y_i.*
(2) *For an* optimal coordinated *strategy (see Definition 2) $c : Y \to A$ of the game G, a profile of strategies π in the partnership protocol is* c-coordinated *if:*
A. In stage 2 each player declares YES, uses a minmax strategy of G^{ad} to choose $\tilde{a}_i$, and truthfully selects $\tilde{y}_i = y_i$.
B. In stage 3 each player selects $a_i = c_i(\tilde{y})$, provided that she had reported truthfully ($\tilde{y}_i = y_i$), as planned in stage 2. If she failed to report truthfully in stage 2 ($\tilde{y}_i \neq y_i$, which is a probability zero event), then she selects an a_i which maximizes $\mathbf{E}_u \left[u^{eq}\big(a_i, c_{-i}(\tilde{y})\big) | (y_i, \tilde{y}_{-i}) \right]$.[21]

A *c-coordinated* strategy is clearly participatory, honest, and ex-post efficient; moreover, it enjoys the additional properties specified in the following theorem:

Theorem 3. *Consider the interim partnership protocol of a given finite two-player Bayesian game $G = (A, Y, U, \mu)$:*

(1) *Any c-coordinated strategy profile is a sequential Nash equilibrium of the partnership protocol with expected payoffs that equal the coco value of G, $\kappa(G)$.*
(2) *For any participatory Nash equilibrium of the partnership protocol, the expected payoffs are $\kappa(G) - (x, x)$ for some $x \geq 0$. In other words, all participatory equilibria are Pareto dominated by the coco payoffs.*
(3) *However, the equilibria of G also remain equilibria of the partnership protocol: for any mixed-strategy Nash equilibrium $\sigma = (\sigma_1, \sigma_2)$ of G, it is also a Nash equilibrium of the partnership protocol for both players to declare NO and select $\tilde{a}_i$ according to σ_i.*

[20]We thank Robert Wilson for pointing the gained stability.

[21]It is necessary to specify how to act under such zero-probability events in order to argue, as we do below, that we have a sequential equilibrium.

Proof of Theorem 3. For part 1 assume, for example, that player 2 uses his c-coordinated strategy. Notice first that by definition, in every one of player 1's stage-3 information sets, player 1 acts optimally, since her opponent tells the truth and follows the c-optimal selection.

So, for part (1), it remains to be shown that player 1 acts optimally at her first information set, namely, in stage 2. Suppose that instead of the above, she chooses not to participate, and to play $\widetilde{b}_1$. By the decomposition, her expected payoff, conditioned on y, is:

$$\mathbf{E}_u\left[u_1^{\mathrm{ad}}(\widetilde{b}_1, \widetilde{a}_2) + u_1^{\mathrm{eq}}(\widetilde{b}_1, \widetilde{a}_2) \mid y\right].$$

But if player 1 switches her strategy to a c-coordinated one, her payoff may be written as:

$$\mathbf{E}_u\left[u_1^{\mathrm{ad}}(\widetilde{a}_1, \widetilde{a}_2) + u_1^{\mathrm{eq}}(c(y)) \mid y\right],$$

where $\widetilde{a}_1$ is chosen by her advantage-game minmax strategy against player 2's $\widetilde{a}_2$, chosen according to his minmax strategy. Hence, it is easy to see that the switch to the c-coordinated strategy can only increase both terms in the above two expectations. Using the same decomposition argument above, we can also easily see that under participatory strategies, player 1 cannot obtain a higher payoff than she would by following any other c-coordinated strategy. Thus, part (1) of the theorem holds.

For part (2) the decomposition implies that at any participatory equilibrium the players' payoffs from the advantage component must equal their corresponding payoffs under the c-coordinated equilibrium. On the other hand, their payoffs from the equal-payoff component can only be smaller than the correponding payoffs under the c-coordinated equilibrium, and by the same amount.

Part 3 is obvious, since either player can declare NO, thereby forcing the game to be the original game G. □

4.3. ***Interim* individual rationality and conditional values.** Part (1) of Theorem 3 offers a positive equilibrium answer to the question of whether the unobligated players would choose to participate after they know their signals. For any pair of privately known signals, if one player participates, it is a best response for the opponent to participate. For complete-information games, this means in particular that the coco value is individually rational; as we already discussed, the coco value is also individually rational ex-ante, before the players know their signals.

One may ask whether the coco value is also individually rational *interim*, that is, after the players have observed their signals. To answer this, one must ask what the conditional coco values are, i.e., how much should the players expect, conditioned their signals, and what payoffs can they secure for themselves conditional on their signals? But as was illustrated in earlier literature on this subject (see for example Hart (1985), Chapter 6 of Myerson (1991), and Forges (2011)), these questions may not be well defined.

Consider, for example, the following zero-sum Bayesian game in which player 1 is completely informed (i.e., he knows the payoff table) and player 2 is completely uninformed.

wp .5

0, 0	1,-1
0, 0,	0, 0

wp .5

0, 0	-1, 1
0, 0	-1, 1

The (ex ante) minmax (and coco) value of the game, $(0, 0)$, can be guaranteed in two ways: (1) player 1 plays *up* in both games and player 2 plays *left*, or (2) player 1 plays *up* in both games and player 2 plays *right*. However, when player 1 knows that the payoff table is the one on the left, player 1 must play up but it is not clear what she should expect. Player

2 may be playing left, in which case player 1 should expect zero; or player 2 may be playing right, in which case player 1 should expect one. And since both *right* and *left* are minmax strategies for player 2, there are multiple conditional values for player 1.

In accordance with the papers cited above, answers to such questions must take the forms of vectors, rather than single numbers. In the example above, depending on the strategy of player 2, both (0,0) and (+1,-1) are conditional minmax (and conditional coco) payoffs for player 1. (0,0) are player 1's conditional payoffs (in the right and respectively the left game) when player 2 plays left, and (+1,-1) are player 1's conditional payoffs (in the right and respectively the left game) when player 2 plays right.

Notice also that player 1's strategy of always playing *down* does meet the following tempting definition of conditional individual rationality: it guarantees him the most he can guarantee, given each signal. This is because player 1 can guarantee a payoff of only 0 in the left payoff table. Similarly, in the right payoff table, player 1 can guarantee a payoff of only -1. The strategy of playing down does guarantee player 1 these minimal values. However, down is clearly an unsatisfactory strategy and does not even meet the definition of ex-ante individual rationality. Hence, the natural criterion of guaranteeing the most, conditioned on one's signal, is a poor definition of individual rationality.

Remark 2. *In certain cases the conditional value of a zero-sum Bayesian game is well-defined. For example, this is clearly the case when there are unique minmax strategies. The same is true for the coco value. In particular, when the advantage game admits unique minmax strategies, the conditional coco values are also well defined.*

5. Joint venture example: efficiency in a Myerson-Satterthwaite game

A manufacturer M can produce a certain item at cost $\$C$, and a distributor D can sell this item with a return of $\$R$. The pair of parameters (C, R) is generated by a known joint probability distribution π on the integers in $[0, 100]^2$; M knows the realized value of C, and D knows the realized value of R. Under the simple monetary utility assumed in this paper, if M manufactures the item, and sells it to D at a price P, who in turn sells with the return R, then M nets the payoff $P - C$ and D nets the payoff $R - P$.

The well-known impossibility result of Myerson and Satterthwaite (1983) implies that, in general, there is no mechanism that guarantees efficient outcomes: under any negotiation procedure M and D would fail to agree on a price P in some situation with $C < R$. However, under the strong revealed-payoff assumption in this paper, the coco value offers an efficient and fair solution that can be implemented in the interim sense discussed above.

For a strategic description of the situation above, we use a double-auction noncooperative Bayesian game G defined as follows: M submits a demanded price P^{dem}, and D simultaneously submits an offered price P^{ofr}. If $P^{\text{ofr}} < P^{\text{dem}}$, then there is no deal and each nets zero payoff. But if $P^{\text{ofr}} \geq P^{\text{dem}}$, then the item is manufactured by M and sold to D at the price $P^{\text{mid}} \equiv (P^{\text{ofr}} + P^{\text{dem}})/2$; M's payoff is then $P^{\text{mid}} - C$ and D's payoff is $R - P^{\text{mid}}$.

To compute the coco value of G, observe that when the signals are (C, R), the payoff $u^{\text{eq}}((P^{\text{dem}}, P^{\text{ofr}}))$ is $(R - C)/2$ if $P^{\text{ofr}} \geq P^{\text{dem}}$, and it is zero otherwise. Thus, the team optimum of the game is: team-opt$(G) = \mathbf{E}[\max\{R - C, 0\}]$.

As for the advantage component, consider the constant strategies $P^{\text{dem}} = 100$ and $P^{\text{ofr}} = 0$ played by M and D, respectively, regardless of their values of C and R. Under these strategies in the game G, each player guarantees two things: (1) his own payoff is at least zero, and (2) the opponent's payoff is not greater than zero. This means that in the advantage game, they guarantee themselves payoff advantages of zero, and $(0,0)$ is the minmax value of the advantage component game.

Under the definition of the coco value, the two paragraphs above imply the following:

Proposition 1. *The coco value of the joint venture game above is:*

$$\left(\frac{1}{2}\mathbf{E}[\max\{R-C,0\}]\ ,\ \frac{1}{2}\mathbf{E}[\max\{R-C,0\}]\right).$$

To illustrate the interim implementation of the coco payoffs above, consider the following strategies in the partnership game. After learning their true respective parameters, C and R, both players declare YES, submit the noncooperative strategies $P^{\text{dem}} = 100$ and $P^{\text{ofr}} = 0$, and report their individual parameters truthfully: $\widetilde{C} = C$ and $\widetilde{R} = R$. If the reported cost is greater than the reported return, $\widetilde{C} > \widetilde{R}$, the item is not produced, and each nets a zero payoff. But if $\widetilde{C} \leq \widetilde{R}$, then M produces the item (at a cost C), D sells it (with a return of R), and a transfer is made so that they each net $(C-R)/2$.

In general, it is difficult to compute a Bayesian-Nash equilibrium of a bargaining game like the one above, especially when it involves an asymmetric prior probability distribution over correlated signals. In contrast, the computation and implementation of the coco solution above is relatively simple.

Moreover, in this example one does not need to know ex-post the entire payoff functions, as was assumed by the revealed-payoff assumption used in the previous section. Knowledge of the realized production cost and realized net revenue is sufficient. Of course, obtaining only this knowledge may still be difficult. And indeed, as seen in real-life partnerships and joint ventures, accountants on both sides, with the possible help of "neutral" accounting firms, often struggle to assess such parameters.

Additional understanding of the solution and its ease of computation can be gained from the next example, where we now break the *structural* symmetry of the joint venture game above.

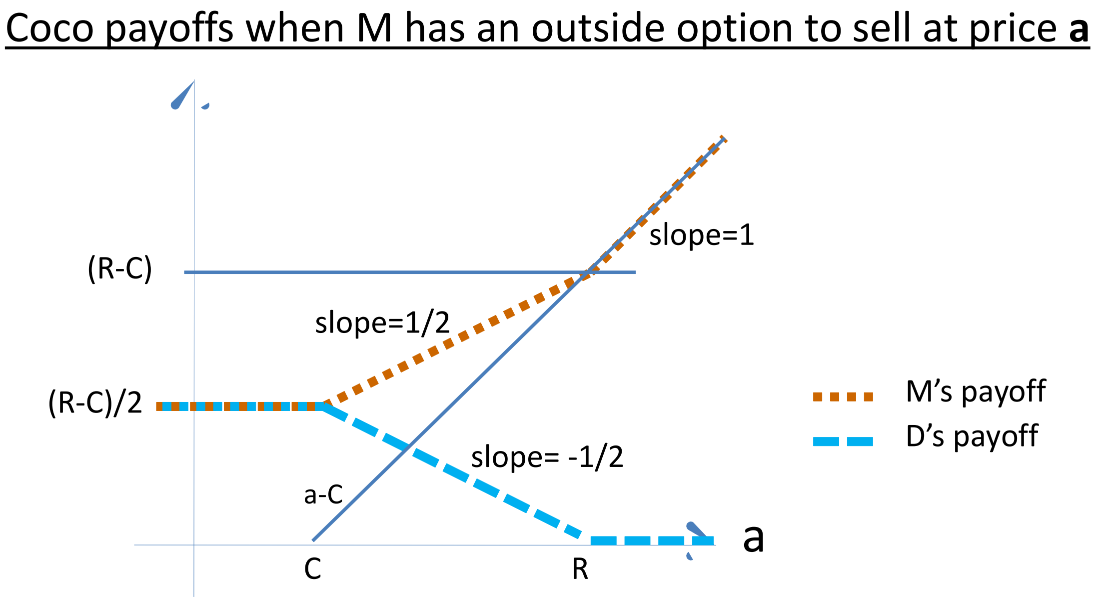
5.1. **One-sided outside options.** Assume that M has an option to produce the item and sell it to some outside buyer at an alternative price a. How does this affect the coco values of M and D? To keep the illustration simple, we assume complete information (i.e., C, R, and a are common knowledge), and that trade is possible, i.e., $C < R$. Figure 1 describes the coco payoffs of M and D as we increase a.

When the outside option is useless, $a < C$, it has no effect on the coco value. And when the outside option is sufficiently high to make D useless, M collects all the benefits and D is out. But in between, the coco value changes in a continuous (and piecewise linear) manner. Every extra dollar above the cost adds 50 cents to M and takes 50 cents away from D.

6. Weakening the (unrestricted) revealed-payoff assumption

For the purpose of achieving a general result, the interim implementation theorem uses sufficient conditions that are stronger than needed for many games. Moreover, the assessment of the player's relative advantage can sometimes be achieved in ways that differ from the interim protocol above. The following examples illustrate such situations.

- **Joint venture example: weak revealed payoffs suffice.** In general, the revealed-payoff assumption is only needed to assess the minmax value of the advantage game. This means that in games in which the minmax values can be assessed by other



means, the weak revealed-payoff assumption suffices. For example, in the joint venture example it is clear that the minmax value of the advantage game is zero, hence, the two players can simply form a partnership and share their net profits equally. (As discussed there, they still need to verify M's cost C and the revenue R collected by D). Furthermore, the same holds where the players information consists only of forecasts of their own cost and revenue: when trade occurs, it is sufficient that the realized C and M (but not the forecasts) be revealed.

- **Vendors example with weak revealed payoffs**. Consider a version of the hot-dog sellers example in which it is known that the per-hot-dog profit of one player is ten times as much as the other's. However, for any distribution over the number of buyers at the beach and airport, and for any forecasts that these players have, the weak revealed-payoff assumption suffices. The unrestricted revealed-payoff assumption requires that if both players go to the beach or airport, they would still know how many buyers were at the other location, which may be unreasonable. However, since in our implementation one player will be at either location, the revealed pair of profits will reveal the number of buyers at each location.
- **Pro-wrestling game**. Two professional wrestlers are about to participate in a match for which \$1000 will be awarded to the winner and nothing to the loser. Moreover, an extra \$500 bonus will be awarded to the two players, divided evenly, if the match is "a good show." We refer to this option as *dancing* since the sequence of moves to earn the bonus must be carefully choreographed. A high-level approximate model of this game is the following:

	Fight	Dance
Fight	1000p, 1000(1-p)	1000, 0
Dance	0, 1000	750, 750

Here p is the probability that player 1 would win if the two fought, and the expected payoffs are shown in the table. It is clear that *(fight, fight)* is a dominant strategy. A simple calculation shows that the coco value of the above game is $(1000p + 250, 1000(1 - p) + 250)$. In the case where p is common knowledge and there is no relevant private information, the players might adopt one of two simple agreements to yield the coco value. (For example, they might agree that $p \approx 1/2$, i.e., they have roughly equal chances of winning, and each agrees to dance.) While this may not be a legally binding contract, such an agreement may be enforced through reputation, repeated play, the use of a trustworthy third party, or other similar means.

However, in the case of private information, the players may not agree upon p. (For example, each player may have slept well the previous night and woken up feeling especially strong.) Instead, they could agree to engage in, say, a scrimmage wrestling match behind the scenes, whose sole purpose would be to determine the side payment in the real match. Presumably, the probabilities of winning in the scrimmage and the real match would be the same. The agreement would be that they would both dance in the actual match, but a side payment would be arranged so that the winner of the scrimmage would get a payoff of 1250 and the loser would get a payoff of 250. This has the property that it matches the coco value, in expectation. Note that this equality holds for *any* signal space and any distribution over prior information. Moreover, the protocol is simple enough to be understood by professional wrestlers.

Also note that they may choose any other means of determining a side payment, as long as they both agree to it. For example, it may be a convention that the two players merely arm wrestle rather have a full scrimmage match. Similar in spirit, such proxies for determining the stronger player are exhibited by animals in nature and in the Biblical story of David and Goliath.

An interesting feature of some of the examples above is that the protocols may make sense even if the players *do not have a common prior*. For example, when two wrestlers fight, the private information is in fact quite involved, including knowledge of what moves they are themselves particularly good at, beliefs about the opponent, and higher-order beliefs. The assumption that all of these probabilities are derived from a common prior is certainly questionable in such situations. While we do not provide a solution for zero-sum games without a common prior, in many real-world situations the protocols suggested by the coco decomposition may still be appropriate. For example, it may be perfectly plausible to tell two wrestlers to "go wrestle."

7. Elaboration on the axioms and additional properties

This section elaborates on properties of the coco value as a function of the game G that is being arbitrated, and on its relationships to the noncooperative arbitration protocols discussed earlier. Much of this discussion applies to games of complete information and hence to the NRKR values as well.

7.1. **On dominant strategies, monotonicity, and commitments.** Consider the following 2×1 **up/down game**.

0,0
1,5

coco value = (3,3)

At first look, it seems strange that P2 would be willing to settle for the coco payoff of 3, rather than the payoff of 5 that he can get by cutting out communication with P1 and letting her play her dominant strategy. While this intuition is clear in purely strategic environments, where commitments, communication, threats, and side payments are limited, in certain arbitrated cooperative environments the outcome (3,3) may be more reasonable. The following example illustrates this point.

Example 3. *The sprinkler game.*

Two neighbors, each having to decide whether or not to water a shared lawn, play the following game:

	not	water
not	0,0	5,1
water	1,5	0,0

coco value $= (3,3)$

In this payoff table, it seems fair and efficient that one of them will water and the other will compensate her with a transfer of 2, to obtain the coco value (3,3).

But what if P2's sprinkler breaks, so that he cannot water? Then we are back in the one-player up/down game above. And if the solution of the up/down game were (1,5), it would present two problems. First, there is a fairness issue, where the player who cannot water the lawn gets a higher benefit than the one who can. Second, there is the issue of incentives, where each of the two neighbors would have the incentive to break her sprinkler first, in order to increase her payoff. To avoid such difficulties, it is desirable that an arbitrated cooperative solution satisfy the "monotonicity in actions" axiom discussed earlier. Under this axiom no player can gain by eliminating options.

The example above also relates to issues of commitment. As stated above, it seems that in the up/down game, P2 should be able to walk away and expect a payoff of 5, instead of the coco value of 3. But notice that there is a similar counter commitment for P1: walk away first, after leaving a publicly-observed irrevocable instruction to her gardener to water if and only if P2 pays \$4. Now, under the same type of reasoning, P1 can expect a payoff of \$5, instead of \$3.

pay \$4	don't pay
5,1	0,0

The point is that in semi-cooperative environments there are competing contradictory commitments that must be balanced by an arbitrated value.

Remark 3. *It is worth noting that the coco value may be unnatural in other contexts. For example, the up/down game also describes the situation in which P1 owns the lawn, and P2 simply enjoys looking at it. Now it may seem unfair for P1 to demand a payment for watering her own lawn, and the noncooperative solution, with the payoffs (1,5), may be more reasonable.*

In this regard, we recall that the interim arbitration protocol discussed earlier has two types of equilibria: the participatory one, in which the players agree to arbitration and receive their coco payoffs; and the nonparticipatory one, in which arbitration is rejected and a Nash equilibrium of the underlying unarbitrated game is played. Whether we end in one type of equilibrium or the other may depend on focal point considerations. For example, *in the case of the jointly-owned lawn, fairness may lead to the selection of the arbitrated equilibrium; whereas in the sole-ownership scenario, fairness may lead to the selection of the noncooperative one.*

7.2. **On security levels, extortion, payoff dominance, and a competing value.** In a variety of applications of strategic games, authors use a common indirect approach to obtain cooperative payoffs through a "bridge" that connects strategic games with cooperative games. To every strategic game G, one associates a cooperative game V^G, and adopts an appropriate cooperative solution $\varphi(V^G)$ to yield cooperative payoffs for the players of G. While the coco value offers direct cooperative values for strategic games, without the need to construct such a bridge, it can still be viewed as a special case of the bridge approach as done below.

In particular, we proceed to compare the *coco bridge* with a popular *alternative bridge*, used early on by Aumann (1961) (see also Forges et al. (2002) for more references; see Carpente et al. (2005, 2006) for an axiomatization of a value that emerges under the alternative bridge solution).

Given a two-person TU bimatrix game $G = (A, B)$, the associated cooperative game is determined by three numbers: V_{12}, describing the maximal total payoff that the two players can generate if they cooperate; and V_1 and V_2, describing the respective maximal payoffs that the players can secure on their own. We use the Shapley (1953) value φ to associate to each cooperative game V the individual values: $\varphi_i \equiv V_i + \frac{1}{2}[V_{12} - (V_1 + V_2)] = \frac{1}{2}V_{12} + \frac{1}{2}(V_i - V_j)$.

For both bridges one defines the maximal total payoff that the players may collectively secure by $V_{12}^{\text{alt}} = V_{12}^{\text{coco}}$ =team-opt(G). But the coco bridge and the alternative bridge differ on the definition of the V_is, as follows: The maximal possible payoffs that the players may secure under the alternative bridge are defined by: $V_1^{\text{alt}} = \text{minmax}(A, -A)$ and $V_2^{\text{alt}} = \text{minmax}(B^T, -B^T)$. On the other hand, the maximal possible payoffs that they may secure under the coco bridge are defined by: $V_1^{\text{coco}} = \text{minmax}\left(\frac{A-B}{2}, \frac{B-A}{2}\right)$ and $V_2^{\text{coco}} = \text{minmax}\left(\frac{(B-A)^T}{2}, \frac{(A-B)^T}{2}\right)$.

Substituting these individual values into the Shapley formula above, we obtain for player 1, for example: $\varphi_1^{\text{alt}} = \frac{1}{2}V_{12} + \frac{1}{2}(\min\max(A) - \min\max(B^T))$ and $\varphi_1^{\text{coco}} = \frac{1}{2}V_{12} + \frac{1}{2}\min\max(A - B)$.

The difference between the solutions is illustrated by the two 2×1 games below, which differ only in the bottom left entries: 2 in the left-hand side (LHS) game, and a large negative number, $-m$, in the right-hand side (RHS) game. In both games $V_{12} = 4$ is obtained when P1 plays up. But P1's ability to make credible threats, in order to extort side payments from P2, is drastically different in the two games: in the LHS game, P1 can bring player 2's payoff down from 2 to 0 at no cost to himself; whereas in the RHS game, bringing P2's payoff down to 0 is very costly to P1.

Credible threat		Noncredible threat	
	2,2		2,2
	2, 0		**-m**, 0
coco value =	(3,1)	coco value =	(2,2)
alt. value =	(3,1)	alt. value =	(3,1)

Should such a difference be reflected in the solutions of the games? The coco value reflects this difference: it assigns P1 a payoff of 3 in the LHS game and only 2 in the RHS game. The alternative value, however, does not reflect the difference: it assigns P1 the payoff 3 in both games. The reason for the distinction is simple: the minmax($\frac{A-B}{2}$), used by the coco value, compares the individual payoffs cell by cell, while the minmax(A)$-$minmax(B^T) compares the two payoff tables separately, ignoring any externalities that depend on payoff relationships within cells.[22]

In the simple example above, the *payoff-dominance axiom*, used in the characterization of the coco value, plays an important role. It requires that for every value of $-m$ below zero, P1's payoff should not exceed the payoff of P2. In other words, when P1's cost of punishment exceeds the damage she can inflict on P2, she cannot benefit from a threat to punish.

7.3. **The cooperative value of information.** The coco value makes optimal use of information and compensates players for providing it, as was illustrated in the vendors example from Section 2.1.

The individual valuation of information has been well-studied in game theory. (See, for example, Kamien, Tauman, and Zamir (1990), and the more recent references in De Meyer, Lehrer, and Rosenberg (2009).) A measure of the cooperative value of information arises naturally from the coco value, if one considers how the values of the players change, as one changes the information of one or both players. This measure is fairly sophisticated, since it takes into account interactive aspects of the information: its provision, its use, and the direct and indirect benefits to both players.

For example, in the vendors game of Section 2.1, consider the possibilities that each player is either completely informed about the weather, or has no information about the weather. This gives rise to four different games, with (rounded-off) coco values described by the following table.

	P2 uninformed	P2 informed
P1 uninformed	95, 145	85,175
P1 informed	115,145	100,160

Starting from the case of no information at all, perfect weather information (PWI) acquired by player 1 yields the two players the respective values (20,0) ($=(115,145)-(95,145)$), whereas the respective values of PWI acquired by P2 are (-10,30) ($=(85,175)-(95,145)$). In general, the efficiency of the coco value guarantees that for any new information, the sum of the individual values is nonnegative; and the monotonicity property guarantees that any information obtained unilaterally by one player yields that player a nonnegative value.

7.4. **Computational complexity.** In the case of complete information, the coco value can be computed in polynomial time, that is, time which is polynomial in the size of a natural representation of the game. In the case of incomplete information, where each player has at most m signals, the coco value can be computed in time (size)$^{O(m)}$. More formally, suppose that a game is represented as follows. Let $|S|$ denote the size of finite set S. The sets of signals and actions for each player are taken to be the set $A_i = \{1, 2, \ldots, |A_i|\}$ and $Y_i = \{1, 2, \ldots, |Y_i|\}$, respectively. The prior distribution μ is represented by a list of triples, $y, u, \mu(y, u)$, where y is a signal profile, u is a matrix, and $\mu(y, u)$ is in $(0, 1]$. As is standard, we assume that all these numbers are rational numbers encoded as ratios of binary integers. The size of the game, $|G|$, is simply the total number of bits used to describe the game. We

[22] Again, readers familiar with the bargaining literature may associate the fixed-threat bargaining solution, extended by Harsanyi and Selten (1972) and Myerson (1984), with V^{alt}; and the variable-threat bargaining solutions, studied in this paper, with V^{coco}.

argue below that the coco value can be computed in time $|G|^{O(m)}$, but it is possible that there are faster algorithms.

Lemma 2. *The coco value can be computed in time* $|G|^{O(m)}$*.*

Proof. Computing the decomposition is algorithmically trivial: constructing the two games requires a few additions and divisions per payoff cell. Computing the value of the team game is also easy, since $\mathbf{E}_\mu[u(a)|y]$ is straightforward to evaluate, and the team optimum is:

$$\sum_{y \in Y} \Pr_\mu[y] \max_{a \in A} \mathbf{E}_\mu[u_1(a) + u_2(a)|y].$$

Hence, both the decomposition and the team-game value can be computed in time polynomial in the size of G. For the zero-sum Bayesian game G^{eq}, one first does the standard expansion into a complete-information game. Specifically, one constructs the $|A_1|^{|Y_1|} \times |A_2|^{|Y_2|}$ bimatrix game in which each strategy (a function from signals to actions) in G is an action in the new game, and the payoffs of the actions in the new game are the expected values of the payoffs in G from using the respective strategies. Computing this expected value in any particular cell can be done in time polynomial in $|G|$, but one must perform this computation $|A_1|^{|Y_1|} \times |A_2|^{|Y_2|}$ times. Finally, once one has constructed such a game, the value of a zero-sum bimatrix game is well-known to be computable by linear programming. Theoretical algorithms for linear programming are known to take time polynomial in the size of the input (see, e.g., Grötschel et al., 1988). (Algorithms that work fast in practice are also well-studied.) Hence, the total run-time of the algorithm is $|A_1|^{|Y_1|} \times |A_2|^{|Y_2|}\text{poly}(|G|)$, which implies the observation. □

7.5. **Composability and robustness.** Composability of protocols has become increasingly recognized as an important topic in computer science and specifically within cryptography games. While cryptographic protocols have typically been shown to be secure when run in isolation (such as when encrypting a single message or signing a document), Canetti (2001) proposes that cryptographic protocols should be universally secure when executed concurrently in an environment with many other protocols running simultaneously. That is, a secure program for encrypting messages and a secure program for signing documents are of limited utility if the two of them are not secure when they are both used.

Similarly, an analysis of a single game is arguably of less value if it does not apply when the game is played in a larger context (for example, the solution of a one-shot prisoners' dillema game does not predict the possible outcomes of the repeated prisoners' dilema game).

Two-person zero-sum games exhibit *universal composability.* First, optimal play in a repeated zero-sum game is simply optimal play in each stage. Moreover, suppose two players are to play m fixed zero-sum games $G_1, G_2, \ldots, G_m$, either in parallel or serially, or by some combination thereof. This can be viewed as one large extensive-form game G, where moving in G corresponds to moving in some subset of the constituent games, and the payoffs in G are the sums of the payoffs achieved in the constituent games. The minmax value of G, regardless of the particular order in which moves in G_i's are played, is equal to the sum of the minmax values of the constituent games. Put another way, suppose you were to play a game of tic-tac-toe, a game of chess, and a game of poker, all against the same opponent. If we ignore time constraints and concerns of bounded rationality, the order in which you make your moves in the various games is irrelevant: optimal play is simply to play each game optimally.

Similarly, optimal play in the composition of team games is simple. The coco value inherits the appealing composability properties of both team and zero-sum games. Suppose a Bayesian game G is played repeatedly, with signals drawn freshly each round. Then the coco value of the infinitely repeated game is equal to the value of G.

8. Conclusion

Much of human interaction is semi-cooperative in that strategic players cooperate in order to improve their payoffs. Laboratory experiments also suggest that in games with multiple of equilibria, fairness often serves as a focal point that leads to final selection. For these reasons it is important for game theory to offer a solution that is (1) efficient, (2) compatible with individual incentives, (3) fair, and yet (4) still easy for real players to compute and comprehend – quite an ambitious task.

The coco value is constructed to satisfy these goals, but its applicability is limited to two-person Bayesian games with transferable utility. Moreover, its general "first-best" interim implementability result is obtained under restrictive conditions such as ex post observability. The removal and relaxation of these restrictions present challenges for future research. Ideally, under each relaxation of the restrictions, one would like to preserve all three aspects of the theory presented in this paper: the implementation, the axiomatic justification, and the description of the solution by a simple, computable, closed-form expression. But at this time, it may be too ambitious.

For extensions to more than two players, even the case of three players is interesting, and it is not clear whether there will be a single solution that possesses the great number of appealing properties shared by two-player zero-sum and team games.

Similar difficulties may be encountered in the extension to games with no transferable utility (NTU games). This direction may require a substantially more refined discussion of the various axioms, as suggested by the conflicts among the many bargaining solutions in NTU cooperative games. But this direction is important to explore if we wish to study applications where cooperation involves the optimal allocation of risk in Bayesian environments.

As already mentioned in the introduction, it is important to have an extension to a "second-best" semi-cooperative value that can be implemented in environments that do not satisfy ex-post observability. conditions.

Finally, it would be useful to experimentally test the solution across a number of two-player games, as well as to try to identify experimentally which axioms are most violated in real-world or experimental play.

We refer the reader to the recent survey of Forges and Serrano (2011) for additional questions for future research.

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